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NUMERICAL ANALYSIS ON MAGNETIC SHIELDING OF AXIALLY SYMMETRIC SUPERCONDUCTING PLATE

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Abstract

The effects of the superconducting plate shape on the magnetic shielding are investigated. The plate shape is assumed to be axially symmetric and the permeability of the plate is assumed non-zero. A numerical code to calculate the magnetostatic field around the plate is developed and the spatial distributions of the decay factors are calculated by means of the code. The results of computations show that the magnetic shielding becomes large with the increase of triangularity and with the decrease of ellipticity.

Keywords: magnetic shielding, oxide superconductor, boundary element method.

1. Introduction

In detecting the weak magnetic field, even the earth magnetism and the other magnetic signals from exterior become noise. In this sense, the magnetic shielding is an essential problem for the measurements of the magnetic field from the brain. Recently it has been proposed that the superconducting plate should be used to shield the brain from external magnetic fields. However, it costs us much time and labor to shape the superconducting material into arbitrary shapes. If we could investigate the magnetic shielding effect of the superconducting plate by using the numerical simulation, the results of computations would play an important role in the design of the shielding plate.

The purpose of the present paper is to develop the numerical code for analyzing the magnetostatic field around the axially symmetric superconmagnetic shielding by use of the code. In the next section, we introduce the model of the magnetic shielding plate, and the governing equation is formulated by using the boundary element method. In the third section, we analyze the magnetic shielding effect of the superconducting plate. Conclusions are summarized in the final section. The SI units are used throughout the present paper.

2. Model of Magnetic Shielding Plate and Formulation by Boundary Element Method

We assume, at first, the shielding plate is axially symmetrical. Let us use the cylindrical coordinate system (z, r, ϕ) and take the symmetry axis as z-axis. As the source of the magnetic field, we place an 1-turn coil at (z_c, r_c). The plate surface is assumed as

$$\left(e\frac{z}{b} - \delta\frac{r^2}{b^2}\right)^2 + \left(\frac{r}{b}\right)^2 = 1, \tag{1}$$

where e and δ are the ellipticity and the triangularity, respectively (NAITOU and YAMAZAKI, 1987). Under this assumption, the plate shape is determined by two parameters, e and δ. The ellipticity e is twice as large as the ratio of a radius b to a thickness. On the other hand, the triangularity δ is closely related to the inclination of the plate toward z-axis.

In *Fig. 1*, we show the model of the magnetic shielding plate, where $\partial\Omega_s$ denotes the plate surface. Further, the region bounded by $\partial\Omega_s$ and the z-axis is denoted by Ω_2 and the region outside Ω_2 is represented by Ω_1.

Under the above assumptions, the behaviour of the magnetic field can be represented by

$$-r\nabla \cdot \left(\frac{1}{r^2}\nabla\psi_m\right) = \mu_m j_{m\phi}, \tag{2}$$

where μ_m is magnetic permeability and $j_{m\phi}$ denotes ϕ-component of the current density. In addition, ψ_m is the magnetic flux function which is a function of z and r, and is associated with magnetic induction $\mathbf{B}$ through the relation

$$\mathbf{B} = \frac{\nabla\psi_m}{r} \times \mathbf{e}_\phi.$$

Subscript m denotes the domain number. The axially symmetric magnetostatic field is determined by solving the boundary value problem of *Eq.* (2).

From the straightforward algebra, it is easy shown that the boundary value problem of *Eq.* (2) is equivalent to the following proposition. That

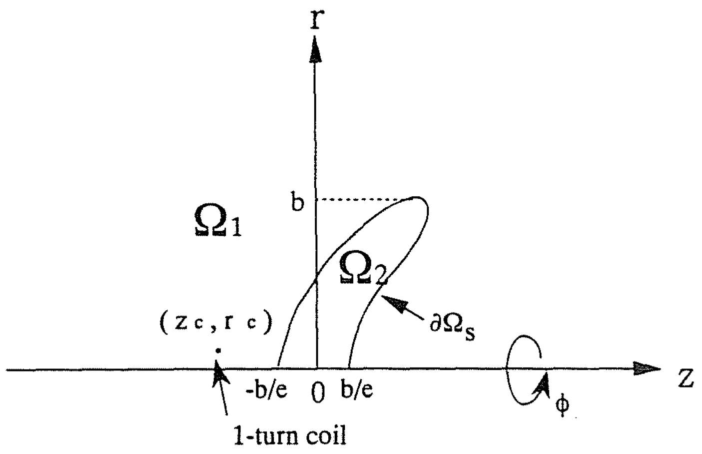


Fig. 1. The model of the magnetic shielding plate, where $\Omega \equiv \Omega_1 \cup \Omega_2$

is, the integral equation

$$-\int_{\Omega_m} \psi_m r \nabla \cdot \left(\frac{1}{r^2} \nabla w_m \right) \mathrm{d}z\mathrm{d}r = \int_{\Omega_m} \mu_m j_{m\phi} w_m \mathrm{d}z\mathrm{d}r +$$

$$+ \int_{\partial\Omega_s} \left(\frac{w_m}{r} q_m - \frac{\psi_m}{r} \frac{\partial w_m}{\partial n} \right) \mathrm{d}l \tag{3}$$

holds for an arbitrary function $w_m(z, r)$ and the boundary conditions are fulfilled. Here, $\partial/\partial n$ stands for the directional derivative whose direction is outward normal to $\partial\Omega_s$, and $\mathrm{d}l$ is the line element on $\partial\Omega_s$. In addition, $\partial\psi_m/\partial n$ is denoted by q_m. As it is well known, one of the particular solutions of

$$-r\nabla \cdot \left(\frac{1}{r^2} \nabla w_m \right) = \delta(z - z_i)\delta(r - r_i) \tag{4}$$

is

$$w_m(z, r; z_i, r_i) = \frac{\sqrt{r_i r}}{2\pi k} \Big[(2 - k^2)K(k) - 2E(k) \Big], \tag{5}$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and the second kind, respectively, and k is defined by

$$k^2 = \frac{4 r_i r}{(r + r_i)^2 + (z - z_i)^2}. \tag{6}$$

Adopting the particular solution (5) as the weighting function w_m in *Eq.* (3), we can obtain

$$c_i^m \psi_m(z_i, r_i) = \int\limits_{\Omega_m} \mu_m j_{m\phi} w_m \mathrm{d}z\mathrm{d}r + \int\limits_{\partial\Omega_s} \left(\frac{w_m}{r} q_m - \frac{\psi_m}{r} \frac{\partial w_m}{\partial n} \right) \mathrm{d}l. \tag{7}$$

Here c_i^m depends on z_i and r_i, and is defined by

$$c_i^m = \begin{cases} 1; & (z_i, r_i) \in \Omega_m \\ \frac{\Delta\theta_i}{2\pi}; & (z_i, r_i) \in \partial\Omega_s \\ 0; & \text{otherwise} \end{cases} \tag{8}$$

Here $\Delta\theta_i$ denotes a solid angle formed by $\partial\Omega_s$. The axially symmetric magnetostatic field is determined by solving the integral equation (7).

In order to solve the boundary integral equation (7), let us divide the boundary $\partial\Omega_s$ into M elements with N nodes, and assume that ψ_m and q_m vary as

$$\psi_m(z, r) = \sum_k N_k \psi_k^{e(m)}, \tag{9a}$$

$$q_m(z, r) = \sum_k N_k q_k^{e(m)}, \tag{9b}$$

on the e-th element $\partial\Omega_e$, where N_k is the shape function and $\psi_k^{e(m)}$ and $q_k^{e(m)}$ denote the value of ψ_m and q_m at the node on $\partial\Omega_e$ of local node number k. Substitution of *Eqs.* (9a) and (9b) into *Eq.* (7) yields

$$c_i^m \psi_m(z_i, r_i) = b_i^m + \sum_{e=1}^{M} \sum_k \left(g_{ie}^{k(m)} q_k^{e(m)} - h_{ie}^{k(m)} \psi_k^{e(m)} \right), \tag{10}$$

where $g_{ie}^{k(m)}$, $h_{ie}^{k(m)}$ and b_i^m are defined by

$$g_{ie}^{k(m)} = \int\limits_{\partial\Omega_e} \frac{N_k}{r} w_m \mathrm{d}l, \tag{11a}$$

$$h_{ie}^{k(m)} = \int\limits_{\partial\Omega_e} \frac{N_k}{r} \frac{\partial w_m}{\partial n} \mathrm{d}l, \tag{11b}$$

$$b_i^m = \int\limits_{\Omega_m} \mu_m j_{m\phi} w_m \mathrm{d}z\mathrm{d}r, \tag{11c}$$

respectively. Let $\tau(j,k)$ denote element number on which the node with global node number j and local number k is located. Introducing the matrix element by

$$G_{ij}^m = \sum_k g_{i\,\tau(j,k)}^{k(m)}, \tag{12a}$$

$$H_{ij}^m = \sum_k h_{i\,\tau(j,k)}^{k(m)} + c_i^m \delta_{ij}, \tag{12b}$$

we can rewrite *Eq.* (10) as

$$\sum_{j=1}^{N} H_{ij}^m \psi_j^m - \sum_{j=1}^{N} G_{ij}^m q_j^m = b_i^m \quad (i = 1, 2, \dots, N), \tag{13}$$

where ψ_j^m and q_j^m are the value of ψ_m and q_m at the node (z_j, r_j) on $\partial\Omega_s$. Further we express the *Eq.* (13) in the matrix form, and we get

$$\begin{aligned} \mathbf{H}_1\vec{\Psi}_1 - \mathbf{G}_1\vec{\mathrm{q}}_1 &= \vec{\mathrm{b}}, \\ \mathbf{H}_2\vec{\Psi}_2 - \mathbf{G}_2\vec{\mathrm{q}}_2 &= \vec{0}. \end{aligned} \tag{14}$$

As the boundary conditions, we usually impose the condition that the tangential component of the magnetic field and the normal one of the magnetic induction be continuous across the plate surface $\partial\Omega_s$. The conditions lead to

$$\vec{\Psi}_1 = \vec{\Psi}_2 \, (\equiv \vec{\Psi}_{\mathrm{I}}) \tag{15a}$$

$$\vec{\mathrm{q}}_1 = -\frac{1}{\mu_s}\vec{\mathrm{q}}_2 \, (\equiv \vec{\mathrm{q}}_{\mathrm{I}}). \tag{15b}$$

Here μ_s denotes relative magnetic permeability of the superconducting plate. Substituting the above boundary conditions into *Eq.* (14), we obtain

$$\begin{aligned} \begin{bmatrix} \mathbf{H}_1 & -\mathbf{G}_1 \\ \mathbf{H}_2 & \mu_s\mathbf{G}_2 \end{bmatrix} \begin{bmatrix} \vec{\Psi}_{\mathrm{I}} \\ \vec{\mathrm{q}}_{\mathrm{I}} \end{bmatrix} &= \begin{bmatrix} \vec{\mathrm{b}} \\ \vec{0} \end{bmatrix} \\ \mathbf{A} \qquad\qquad \vec{\mathbf{x}} \;\; &= \;\; \vec{\mathbf{b}}. \end{aligned} \tag{16}$$

Eq. (16) represents a set of $2N$ linear algebraic equations in the $2N$ unknowns, $\vec{\Psi}_{\mathrm{I}}$ and $\vec{q}_{\mathrm{I}}$.

It must be noted here that the uniqueness of the solution to the simultaneous equations (16) does not hold. The reason for this is explained as follows. On the i- and $(i+N)$-th row such that $r_i = 0$, all elements of matrix $\mathbf{A}$ become zero, because $w = 0$ and $\partial w/\partial n = 0$ there. Therefore, the inequality rank($\mathbf{A}$) $< 2N$ is satisfied, and the uniqueness of the solution to the simultaneous equations (16) does not hold.

In order to avoid these difficulties, we should take account of finiteness of magnetic induction $\mathbf{B}$ on the z-axis, which leads to

$$\lim_{r \to 0} |\nabla \psi| = 0. \tag{17}$$

We see from *Eq.* (17) that $\psi_i = 0$ and $q_i = 0$ at $r_i = 0$. From this fact, all matrix elements on the i- and $(i+N)$-th row, such that $r_i = 0$, must satisfy

$$A_{ij} = \delta_{ij}, \quad b_i = 0$$

and

$$A_{i+N,j} = \delta_{i+N,j}, \quad b_{i+N,j} = 0.$$

Under the above considerations, the simultaneous equations (16) can be solved and $\vec{\Psi}_{\mathrm{I}}$ and $\vec{q}_{\mathrm{I}}$ are determined uniquely.

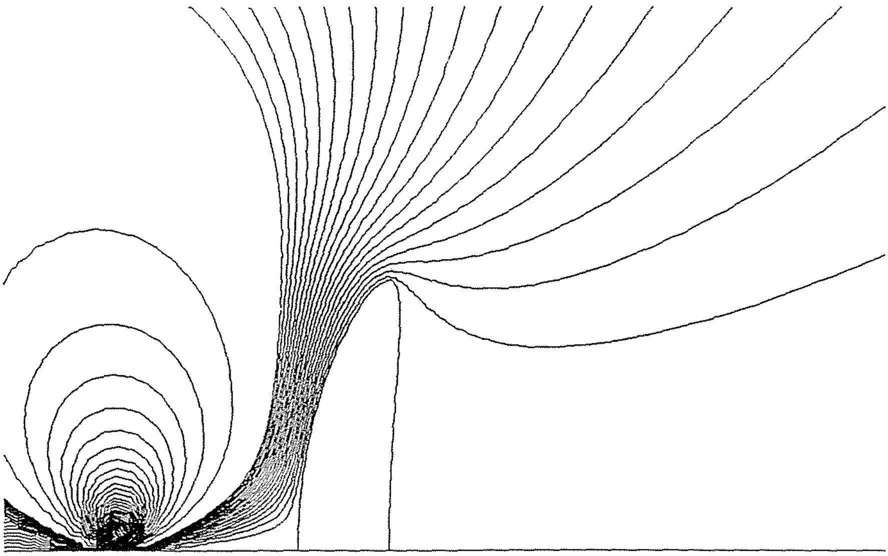
3. Analysis on Magnetic Shielding

The numerical code to analyze the axially symmetric magnetostatic field has been developed by using the method explained in the previous section. Using the code, we calculate the magnetic field around the axially symmetric superconducting plate and investigate the effects of the plate shape on the magnetic shielding.

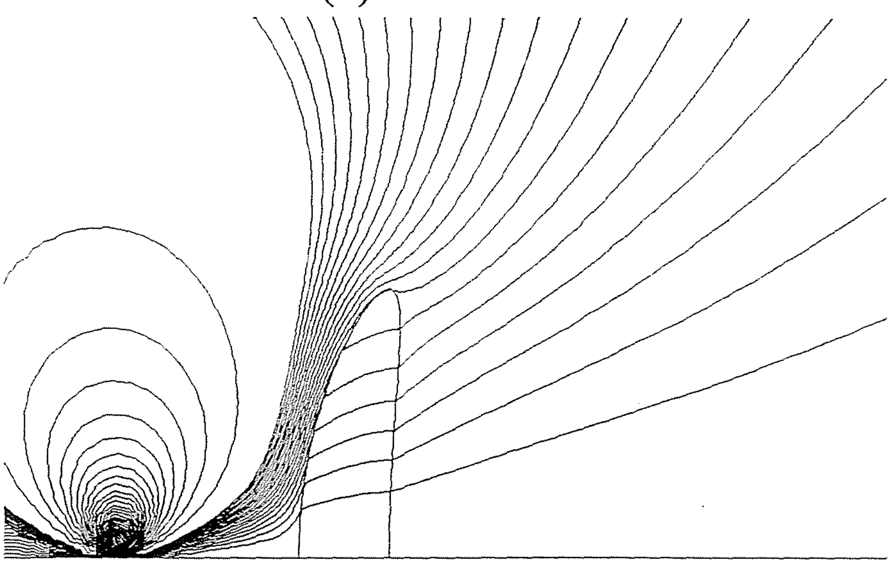
First, we investigate the influence of μ_s on the magnetic shielding. In *Fig. 2*(a) and (b), we show the contours of the magnetic flux function ψ, i.e., the magnetic field lines. We see from these figures that the magnetic field lines permeate the plate as μ_s becomes larger. In order to investigate this tendency quantitatively, we evaluate the decay factor defined by

$$\alpha = 20 \lg \frac{|\mathrm{B}|}{|\mathrm{B}_1|}, \tag{18}$$

where B_1 is magnetic induction generated by the 1-turn coil in free space. Typical examples of contours of the decay factor α are shown in *Fig. 3*(a) and (b). These figures suggest that the magnetic shielding effect becomes large with the decrease of the relative magnetic permeability μ_s. In *Fig. 4*, we show the dependence of the decay factors α on the relative magnetic permeability μ_s. We see from this figure that though the value of α is small when $\mu_s \approx 0$, it decreases with the decrease of μ_s. In the case of $\mu_s \leq 10^{-4}$, the value is nearly equal to the asymptotic value (in the limit of $\mu_s \to 0$).



(a)



(b)

Fig. 2. Contours of the magnetic flux function ψ
(a) $\mu_s = 10^{-3}$, $(e = 6.0, \delta = 1.0)$ (b) $\mu_s = 10^{-1}$, $(e = 6.0, \delta = 1.0)$

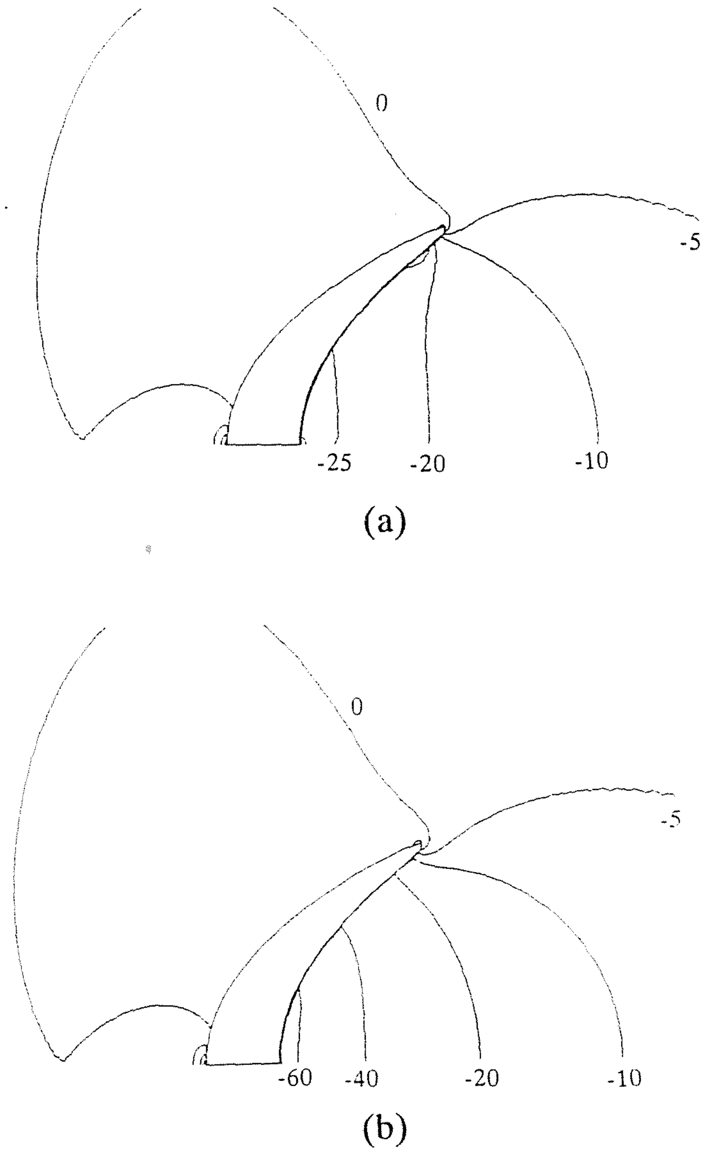


Fig. 3. Contours on which $\alpha = \text{const}$. Numerals beside the contours are values of α.
(*a*) $\mu_s = 10^{-2}$, $e = 6.0$, $\delta = 5.0$ (*b*) $\mu_s = 10^{-4}$, $e = 6.0$, $\delta = 5.0$

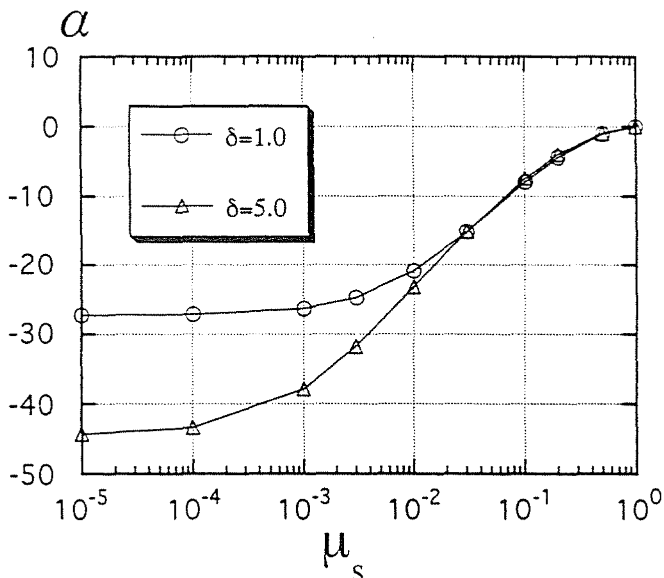


Fig. 4. The dependence of the decay factors α on the relative magnetic permeability μ_s, $(z/b = 0.5,\ e = 6.0)$

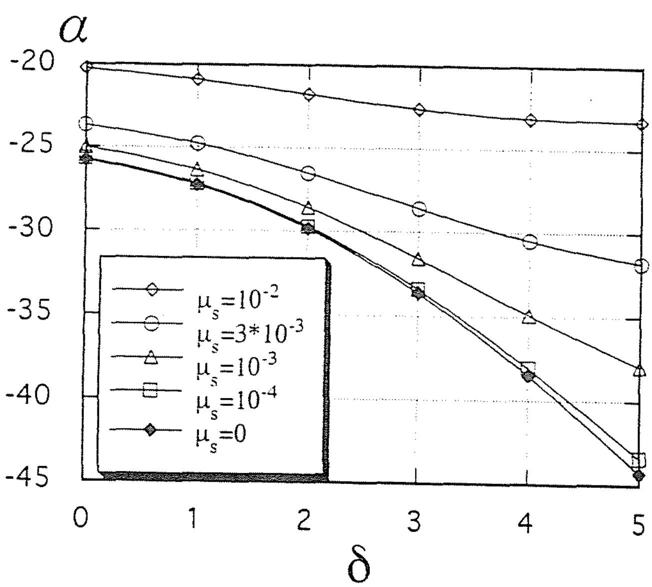


Fig. 5. Decay factors α on z-axis as a function of triangularity δ, $(z/b = 0.5,\ e = 6.0)$

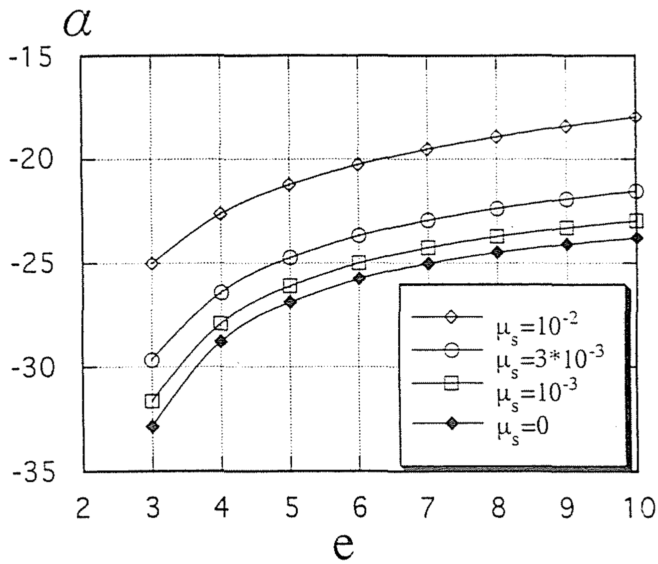


Fig. 6. Decay factors α on z-axis as a function of ellipticity e, ($z/b = 0.5$, $\delta = 5.0$)

Next, we investigate the effect of the plate shape on the magnetic shielding. *Fig. 5* shows the decay factor α on z-axis as a function of triangularity δ. Here the ellipticity of the plate is fixed as $e = 6.0$. This figure indicates that the magnetic shielding effect becomes large with the increase of δ and decrease of μ_s. *Fig. 6* shows the decay factor α on z-axis as a function of ellipticity e. Here the triangularity of the plate is fixed as $\delta = 0$. This figure indicates that the magnetic shielding effect becomes large with the decrease of e.

4. Conclusions

We have developed the numerical code for analyzing the magnetic shielding of axially symmetric superconducting plates and have investigated the magnetic field around the plate by using the code. Conclusions obtained in this paper are summarized as follows.

1. In case that the symmetry axis is partially contained in the domain, it is possible that the discretized integral equation becomes singular. In order to avoid these difficulties, we should take into account of finiteness of the magnetic filed on the symmetry axis.

2. The magnetic shielding becomes large with the decrease of relative magnetic permeability. We can regard the materials of relative magnetic permeability less than 10^{-4} as complete diamagnetic one.
3. The effects of the plate shape on the magnetic shielding is studied. Decay factors are shown to decrease with increase of triangularity and with the decrease of ellipticity.

References

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