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NUMERICAL SIMULATION OF ELECTROMAGNETIC FIELD IN TYPE–II SUPERCONDUCTORS

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Abstract

A finite element simulation technique is presented for analyzing Type-II superconductors in time varying magnetic field. The technique is applied to a simple two-dimensional model with a superconductor slab of highly nonlinear characteristic. The resulting current and field distribution is determined and compared with known solutions.

Keywords: electromagnetic field, type-II superconductor.

1. Introduction

The recent fast development in the field of Type-II superconducting materials has made the common application of the superconductivity more possible, such as superconducting particle accelerator, tokamak fusion reactors, superconducting generators and magnets for levitated vehicles. The evaluation of the electromagnetic forces and losses occuring in superconductors when they are exposed to magnetic fields which steadily increase, decrease, or oscillate with time is necessary for the design of these devices. So the macroscopic theory and the numerical simulation of electromagnetic field in superconductor play a crucial role in engineering design. However, in numerical analysis some difficulties arise from the strong nonlinearity of the $J(E)$ characteristic of the Type-II superconductors. This problem has been studied by a number of researchers. SUGIURA and HASHIZUME (1991a) developed the method of electromagnetic field analysis in Type-II superconductors based on the critical state model, and used a hybrid FEM–BEM formulation (HASHIZUME et al., 1991b). TAKEDA et al.(1992) has presented a method based on BEM formulation exclusively . The purpose of our research is to develop a simulation method based on solely finite element formulation. To overcome the difficulty caused by the open boundary, and to reduce the number of the freedoms we propose to use the infinite elements (ZIENKIEWICZ and MORGAN, 1983). For the formulation we use the magnetic vector potential **A** and the constitutive relation between the current density **J** and the electric field strength **E** comes from the critical state model (WILSON, 1983). In this paper we present the recent state and results of this research.

2. Basic Equations

The basic equations of the electromagnetic field for Type-II superconductors consist of the Maxwell equations and the electric, and magnetic constitutive relations. The latter are strongly non-linear, which makes the problem very difficult. Let the examined region Ω be divided into two regions: the superconducting region Ω_{sc} and the free space region Ω_a with the interface Γ_{sc} (*Fig. 1*).

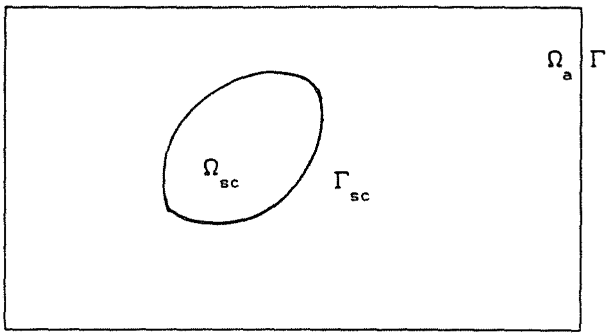


Fig. 1. Regions in general problem

The governing equations are in the superconductor region Ω_{sc}:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{4}$$

$$\left.\begin{array}{ll} \mathbf{J} = J_c(|B|)\dfrac{\mathbf{E}}{|\mathbf{E}|}, & |\mathbf{E}| \neq 0 \\ \dfrac{\partial \mathbf{J}}{\partial t} = 0 & |\mathbf{E}| = 0 \end{array}\right\}. \tag{5}$$

We applied the Bean model (BRECHNA, 1973) for the critical current density in the superconductor, that is

$$J_c(|\mathbf{B}|) = J_0 \tag{6}$$

was assumed.

The basic equations are for the free space Ω_a:

$$\nabla \times \mathbf{H} = \mathbf{J}, \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{8}$$

$$\mathbf{B} = \mu_0 \mathbf{H}. \tag{9}$$

The interface conditions for Γ_{sc} are:

$$\mathbf{n} \times (\mathbf{H}_{sc} - \mathbf{H}_a) = 0, \quad \mathbf{n} \cdot (\mathbf{B}_{sc} - \mathbf{B}_a) = 0, \quad \mathbf{n} \cdot \mathbf{J}_{sc} = 0. \tag{10}$$

We applied the $\mathbf{A} - \Phi$ method for the field representation, that is

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{11}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi. \tag{12}$$

In two-dimensional problems field vectors $\mathbf{J}$ and $\mathbf{E}$ have only one component. Then the magnetic vector potential $\mathbf{A}$ can also become a one-component vector parallel to $\mathbf{J}$ and $\mathbf{E}$. In case of zero transport current this choice eliminates the scalar potential Φ, so *Eq.* (12) yields:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}. \tag{13}$$

In this paper we consider problems where the applied magnetic fields are perpendicular to superconductors infinitely long in the z-direction. So the 2-D problems will be examined in the $x - y$ plane and the magnetic vector potential can be expressed by one component: $\mathbf{A} = \mathrm{A}\mathbf{e}_z$, where $\mathbf{e}_z$ is the unit vector in the z-direction.

The vector potential $\mathbf{A}$ (and the magnetic flux density $\mathbf{B}$) is composed by

$$\mathbf{A} = \mathbf{A}_s + \mathbf{A}_r, \tag{14}$$

where $\mathbf{A}_s$ means the impressed source term, and $\mathbf{A}_r$ is caused by screening currents in the superconductor.

Introducing the vector potential $\mathbf{A}$ into the basic equations, for the superconducting material we get

$$\Delta\mathbf{A}_r - \mu_0\sigma(|E|)\dot{\mathbf{A}}_r = \mu_0\sigma(|E|)\dot{\mathbf{A}}_r, \quad \text{in}\,\Omega_a \tag{15}$$

and for the free space have to be solved the Laplace equation:

$$\Delta\mathbf{A}_r = 0, \quad \text{in}\,\Omega_a. \tag{16}$$

In the *Eq.* (15) the time derivative of $\mathbf{A}$ denoted by $\dot{\mathbf{A}}$, and the superconducting material are characterized by a fictitious special conductivity defined by

$$\sigma = \frac{J_c}{|\mathbf{E}|}, \quad \text{if}\,|\mathbf{E}| \neq 0. \tag{17}$$

3. Test Problem

We used For verifying the method we used a 2–D model of a superconducting slab exposed to an external magnetic field. The arrangement is shown in the *Fig. 2.*

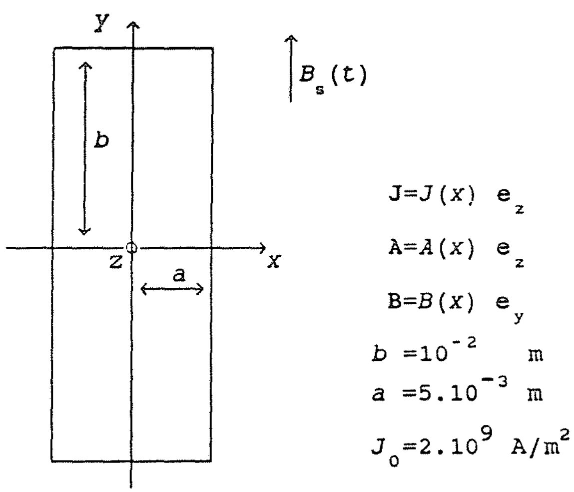


Fig. 2. The scheme of test problem

The applied extensive magnetic field was assumed to be uniform in space and varied in time by a triangle-shape impulse (*Fig. 3*).

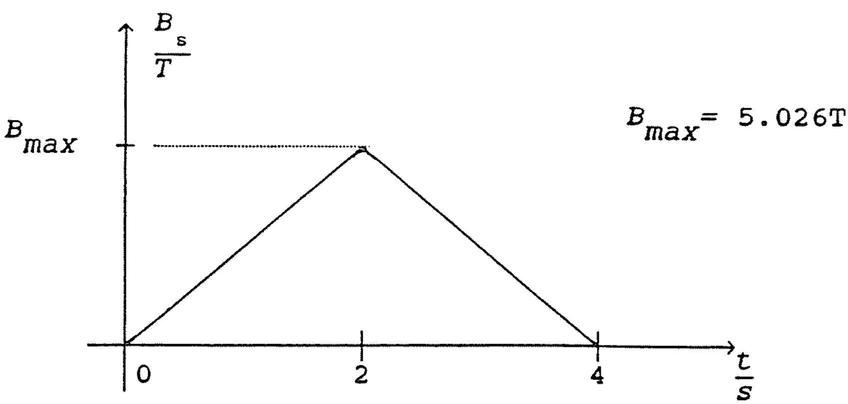


Fig. 3. The shape of the exciting field

4. Numerical Technique

It was assumed there are symmetry lines in the geometrical structure of the problem, and should be analyzed only a quarter of the whole region. The boundary conditions for the vector potential **A** and the macro-element structure are shown in the *Fig. 4*.

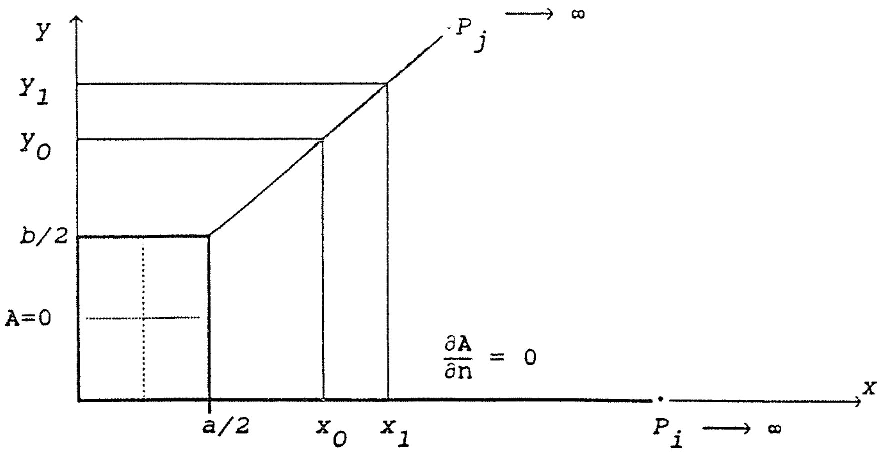


Fig. 4. Macro-element structure for the test problem

To introduce the infinite element we applied the isoparametric transformation (PETRE and ZOMBORY, 1988):

$$x = \frac{x_0(1-\eta) + x_1(1+\eta)}{1-\xi}, \tag{18a}$$

$$y = \frac{y_0(1-\eta) + y_1(1+\eta)}{1-\xi}, \tag{18b}$$

where ξ and η are the isoparametric coordinates.

To generate the finite-element formulation the Galerkin's method was applied. The resultant system of equation in matrix form written by:

$$\mathbf{M}_{sc} \cdot \mathbf{A}_{sc} + \mathbf{M}_{sa}\mathbf{A}_a + \mu_0\sigma(|\mathbf{E}|)\mathbf{D}_{sc} \cdot \dot{\mathbf{A}}_{sc} = \mu_0\sigma(|\mathbf{E}|)\mathbf{G}_{sc} \cdot \dot{\mathbf{A}}_s \tag{19}$$

for the superconducting material and

$$\mathbf{M}^*_{sa} \cdot \mathbf{A}_{sc} + \mathbf{M}_{aa} \cdot \mathbf{A}_a = 0 \tag{20}$$

for the air. In *Eqs.* (19–20) $\mathbf{A}_{sc}$ and $\mathbf{A}_a$ denote vector potential values in the superconductor and the free space, respectively. The $\mathbf{M}_{sc}, \mathbf{M}_{sa}, \mathbf{M}_{aa}$, $\mathbf{D}_{sc}$, $\mathbf{G}_{sc}$ matrices can be determined by integration of the shape function on the finite elements, and their values are independent of the material properties. To reduce the number of unknowns we can eliminate the values $\mathbf{A}$ in the *Eq.* (19) and we get formally:

$$(\mathbf{M}_{sc} - \mathbf{M}^*_{sa}\mathbf{M}^{-1}_{aa}\mathbf{M}_{sa})\mathbf{A}_{sc} + \mu_0\sigma(|\mathbf{E}|)\mathbf{D}_{sc} \cdot \dot{\mathbf{A}}_{sc} = \mu_0\sigma(|\mathbf{E}|)\mathbf{G}_{sc} \cdot \dot{\mathbf{A}}_s. \tag{21}$$

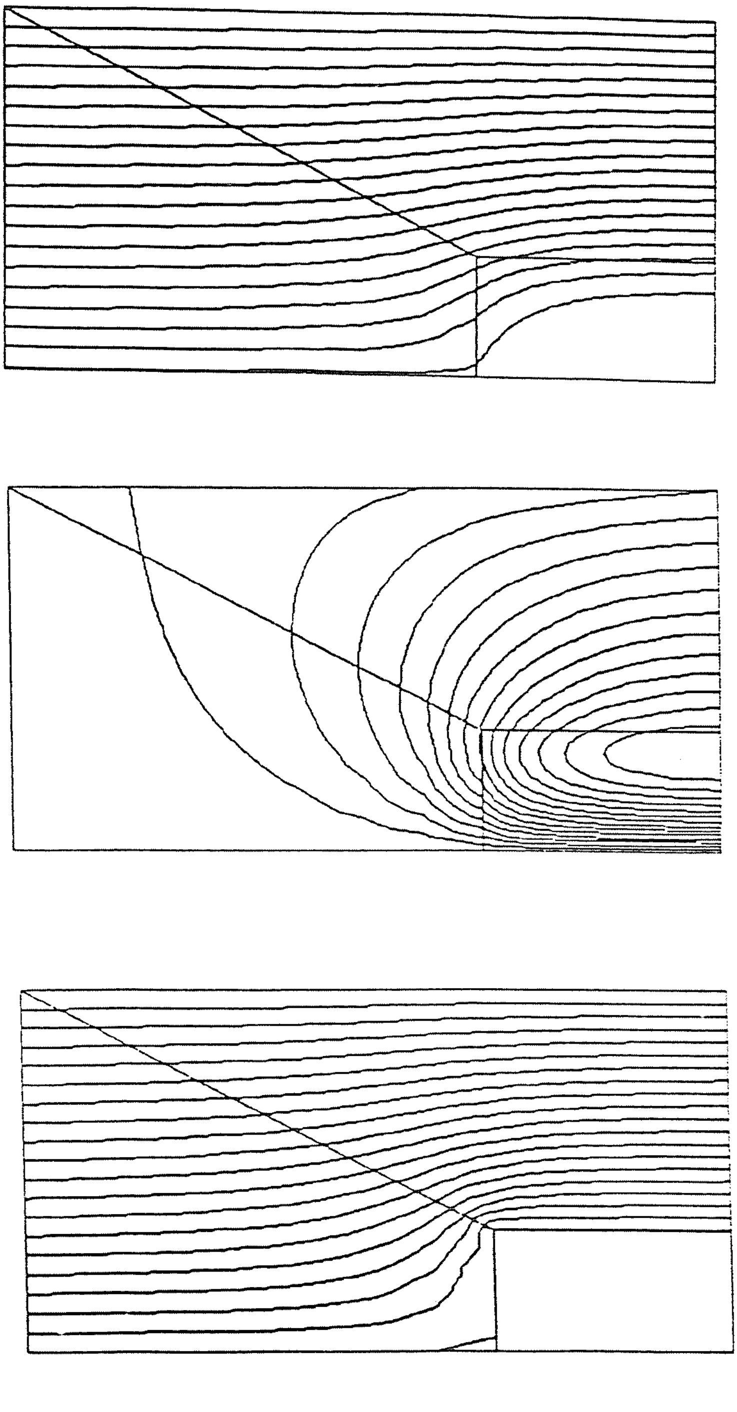
The determination of $\mathbf{A}_{sc}$ from this equation requires to solve a nonlinear system of equation in every time step. Since $|\mathbf{E}|$ can be determined by *Eq.* (12) the values of $\dot{\mathbf{A}}_{sc}$ are needed. This is done by using the iterative scheme proposed by UESAKA et al., (1992). The scheme consists of the following steps:

a) The initial values of conductivities σ are set to be uniform and have large values in the whole superconductor. $\mathbf{J}$ is obtained by solving equations (21) and (12).
b) We use the limitation $|\mathbf{J}| \geq J_c$ and modify the conductivities by

$$\sigma_{\text{mod}} = J_c/|\mathbf{E}| \qquad \text{if} \quad |\mathbf{J}| \geq J_c. \tag{22}$$

c) We repeat these steps with the modified conductivities σ_{mod} until the distribution of current density $\mathbf{J}$ approximately unchanged.

By taking this scheme it is possible to treat a Type-II superconductor as a normal conductor with nonlinear constitutive equations between $\mathbf{J}$ and $\mathbf{E}$.



c)

b)

a)

Fig. 4.

5. Numerical Results

We used the method outlined above to solve the test problem of Section 3. The calculations were done by using 40 eight-noded finite-elements, which leads to solve the nonlinear system of equations with 120 unknowns for the supercondutor. The numerical results are very close to the result of SUGIURA (1991a), and the previously published analysis (BADICS and SEBESTYÉN, 1992). Some pictures are shown in *Fig. 5* to demonstrate the field distribution.

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