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VECTOR POTENTIAL FORMULATION FOR THE IMPEDANCE BOUNDARY CONDITION

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Abstract

The paper introduces a vector potential formulation applying the Impedance Boundary Condition for 3D eddy current problems involving multiply connected regions with exterior circuit conditions.

Keywords: impedance boundary condition, eddy currents, potential computation.

1. Introduction

The Impedance Boundary Condition (IBC) simplifies eddy current analysis by representing conductor regions with the following boundary condition on their surface:

$$\mathbb{n} \times \mathbb{E} = Z(\mathbb{n} \times \mathbb{H}) \times \mathbb{n} , \tag{1}$$

where $\mathbb{E}$ is the electric field, $\mathbb{H}$ is the magnetic field, $\mathbb{n}$ is the normal vector pointing inward the conductor and Z is the surface impedance:

$$Z^2 = \frac{j\omega\mu_i}{\sigma} , \tag{2}$$

where ω is the angular frequency, σ is the conductivity and μ_i is the permeability of the conductors. IBC is a good approximation when the penetration depth, δ,

$$\delta^2 = \frac{2}{\omega\mu_i\sigma} \tag{3}$$

is much smaller than the dimensions of the conductors [4, 14].

IBC was proposed first by SCHELKUNOFF [1]. It was further developed by RYTOV [2], LEONTOVICH [3]. The smooth surface model was extended to corners and slots by HOOLE [15], DEELEY [18], WANG, LAVERS and PEIBAI

[19]. Its experimental verification has been given by Hoole [15–17]. IBC is the 'brother' of the Absorbing Boundary Conditions (ABC). ABC has been used by D'ANGELO and I.D. MAYERGOYZ [20] as well as by RAMAHI and R. MITTRA [21] to simplify high frequency scattering problems. IBC has been used with success at boundary element method by AHMED, BURKE, DEELEY, FAWZI, HOOLE, LAVERS, MAYERGOYZ, SUBRAMANIAM, TSUBOI and XIANG, [5–13]. Using finite elements for 2D eddy current problems a natural boundary condition formulation was given by HOOLE et al [4, 29]. This principle has been applied recently by DARCHERIF et al [27]. SAKELLARIS et al have extended recently the formulation for 2D axisymmetric problems [28]. For the study of eddy current problems RODGER et al [22, 33] introduced surface elements which lead to an identical scheme as the surface impedance approach derived by PRESTON and REECE [23]. RODGER and ATKINSON [33] investigated eddy currents in thin multiply connected conductors. TANNEAU [31], DEELEY and XIANG [30] have proposed recently a hybrid FEM-BEM formulation with the IBC.

Using KRAHENBUHL's shell concept [24], SAKELLARIS [25, 26] successfully applied the IBC using reduced potential formulation. Using the general potential formulation [36–38], GYIMESI and LAVERS further extended the concept to multiply connected regions with exterior circuit conditions [35].

This paper introduces a vector potential formulation for 3D IBC problems with multiply connected conductors and exterior circuit conditions. The uniqueness of the formulation is achieved by BIRO and PREIS' A–V formulation [34]. The paper derives the weak formulation, too, from which the finite element matrices can be easily obtained. The pertinent proofs are also included.

2. Electromagnetic Plane Wave

Let us study the solution of the Maxwell equations in a half-space, $x > 0$, which is filled by a homogeneous isotropic conductive medium. According to the quasi-stationary assumptions, the displacement currents can be neglected in the conductors. Hence, the Maxwell equations take the following form using the complex formalism for stationary sinusoidal excitation:

$$\text{curl } \mathbb{E} = -j\omega\mu_i\mathbb{H} \ , \tag{4}$$

$$\text{curl } \mathbb{H} = \sigma\mathbb{E} \ , \tag{5}$$

$$\text{div } \mathbb{E} = 0 \ , \tag{6}$$

$$\text{div } \mathbb{B} = 0 \ . \tag{7}$$

After simple manipulations, the following equations can be obtained:

$$\Delta \mathbb{E} = j\omega\mu_i\sigma\mathbb{E} , \tag{8}$$

$$\Delta \mathbb{H} = j\omega\mu_i\sigma\mathbb{H} , \tag{9}$$

i.e. both the electric and the magnetic fields satisfy the Helmholtz equation.

A particular solution of the Helmholtz equation in a conductive half-space is a plane wave penetrating into the medium in the positive x direction. The electric field has only a y component. The magnetic field points to the z direction. The searched particular solution is:

$$E_y = E_{y0}e^{-\gamma x} , \tag{10}$$

$$H_z = H_{z0}e^{-\gamma x} , \tag{11}$$

where E_{y0} and H_{z0} denote the values of the electric and magnetic fields, respectively, at the $x = 0$ plane. γ is defined by

$$\gamma = \sqrt{j\omega\mu_i\sigma} = \frac{1+j}{\delta} . \tag{12}$$

It can be seen that the field decays exponentially in the x direction. The field is concentrated in a thin region with a thickness characterized by the penetration depth, δ.

The relationship between the surface values, E_{y0} and H_{z0}, takes the following form:

$$E_{y0} = ZH_{z0} , \tag{1.a}$$

where Z is the surface impedance introduced in (2). This relationship can be expressed in the more general form of (1) which is valid when the direction of the field components don't coincide with the coordinate axis.

3. Impedance Boundary Condition

If the penetration depth, δ, is much smaller than the geometrical dimension of the conductor region, the electromagnetic field is concentrated near the surface of the conductor and penetrates nearly perpendicular to the surface. This behavior is similar to the case of a plane electromagnetic wave penetrating into a conducting half-space. Hence the IBC is a very good approximation.

$$\mathbf{n} \times \mathbf{E} = Z(\mathbf{n} \times \mathbf{H}) \times \mathbf{n} = Z\mathbf{H}_S , \tag{1.b}$$

$$\mathbf{E}_S = (\mathbf{n} \times \mathbf{E}) \times \mathbf{n} = -Z\mathbf{n} \times \mathbf{H} , \tag{1.c}$$

where $\mathbf{E}_S$ and $\mathbf{H}_S$ are the tangential components of the electric and magnetic fields, respectively.

Taking the vector product of (4) and (5) with the normal vector, $\mathbf{n}$, and regarding (1) gives

$$\mathbf{n} \times \text{curl } \mathbf{E} = -j\omega\mu\mathbf{n} \times \mathbf{H} = \frac{j\omega\mu}{Z}\mathbf{E}_S , \tag{13}$$

$$\mathbf{n} \times \text{curl } \mathbf{H} = \sigma\mathbf{n} \times \mathbf{E} = \sigma Z\mathbf{H}_S . \tag{14}$$

Applying the identity of the vector analysis,

$$\mathbf{n} \times \text{curl } \mathbf{E} = \text{grad } (\mathbf{n}\mathbf{E}) - \frac{\partial}{\partial n}\mathbf{E} , \tag{15}$$

to *Eq.* (13) leads to

$$\text{grad } (\mathbf{n}\mathbf{E}) - \frac{\partial}{\partial n}\mathbf{E} = \frac{j\omega\mu}{Z}\mathbf{E}_S . \tag{16}$$

If the electric field, $\mathbf{E}$, has no normal component anywhere, the normal derivative of the electric field on the air side can be expressed as follows:

$$-\frac{\partial}{\partial n}\mathbf{E} = \frac{j\omega\mu_0}{Z}\mathbf{E} , \tag{17}$$

where μ_0 is the permeability of the air.

(17) is a third type boundary condition. This boundary condition has been derived by many authors in the case of a 2D problem. In a general 3D case, however, the normal component of the electric field is not identically zero. (Though it is zero on the surface of the conductor.) Hence the more general (16) should be applied to 3D problems instead of (17). Note that (17) is not general enough even in the case of 2D problems when exterior circuit conditions are given for multiply connected conductors.

Taking the scalar product of (4) and (5) with the normal vector, $\mathbf{n}$, gives

$$\mathbf{n} \text{ curl } \mathbf{E} = -j\omega\mu H_n , \tag{18}$$

$$\mathbf{n} \text{ curl } \mathbf{H} = \sigma E_n . \tag{19}$$

Applying the identity of the vector analysis,

$$\text{div } (\mathbf{H} \times \mathbf{n}) = \mathbf{n} \text{ curl } \mathbf{H} - \mathbf{H} \text{ curl } \mathbf{n} , \tag{20}$$

to (18) and (19) with regard to curl $\mathbf{n} = 0$ leads to

$$\text{div } (\mathbf{E} \times \mathbf{n}) = -j\omega\mu H_n , \tag{21}$$

$$\mathrm{div}\ (\mathbb{H} \times \mathbb{n}) = \sigma E_n \ . \tag{22}$$

Taking into account (1) provides

$$\mathrm{div}\ [Z\mathbb{H}_S] = j\omega\mu H_n \ , \tag{23}$$

$$\mathrm{div}\ [\frac{1}{Z}\mathbb{E}_S] = \sigma E_n \ . \tag{24}$$

The surface divergence operator is defined as:

$$\mathrm{div}_S\ \mathbb{E} = \mathrm{div}\ \mathbb{E}_S = \mathrm{div}[(\mathbb{n} \times \mathbb{E}) \times \mathbb{n}] \ . \tag{25a}$$

The most important relationships of the surface differential geometry can be found in Van Bladel's excellent book [32].

Taking into account that $\sigma = 0$ in the air and consequently $E_n = 0$ on the surface of the conductor, (24) takes the following form:

$$\mathrm{div}_S(\frac{1}{Z}\mathbb{E}) = 0 \ . \tag{25b}$$

Exploring the identity

$$\mathrm{div}\ \mathbb{E} = \mathrm{div}_S\mathbb{E}_S + \frac{\partial}{\partial n}E_n \ , \tag{26}$$

and the fact that $\mathbb{E}$ is divergence free, when Z is constant

$$\frac{\partial}{\partial n}E_n = 0 \ . \tag{27}$$

Thus, on the surface of homogeneous conductors, both the normal component of the electric field and its normal derivative disappear.

Taking into account (1), (2), (3), (12) and (26), (23) can also be written as

$$\mathrm{div}_S(\Delta\mathbf{B}) = B_n \ , \tag{28}$$

where

$$\Delta = \frac{\mu_i}{\mu}\frac{\delta}{1+j} \ . \tag{29}$$

Note that, in general, μ jumps on the surface of the conductor. On the air side, of course, $\mu = \mu_0$ should be substituted.

(28) expresses the fundamental relationship that can be derived from IBC: The surface divergence is proportional to the normal component of the magnetic field. Similar equation can be derived in the case of a thin

magnetic shell. Exploring this similarity, **conductors for which the IBC is a good approximation can be substituted by a thin shell which has a complex thickness**, Δ. This similarity was first reported by KRAHENBUHL [24]. RODGER et al [22, 33] formulated the problem directly using shell elements and arrived to a scheme derived by PRESTON and REECE [23] applying the surface impedance approach.

If Δ is constant over the surface of the conductor, (28) can be reformulated by (7) and (26) as:

$$-\Delta \frac{\partial}{\partial n} B_n = B_n , \tag{30}$$

which is a third kind boundary condition for the normal component of the flux density.

If the penetration depth, δ, approaches to zero, the IBC reduces to a Neumann condition stating that the normal component of the magnetic field vanishes on the conductor surfaces.

(28),(29) and (30) characterize the behavior of a permeable conductor. If the frequency (or conductivity) increases, δ decreases; the normal component of the flux density becomes small; the conductive feature dominates. If the permeability increases, although the δ decreases Δ becomes larger; the tangential component decreases; the permeable feature dominates.

4. Vector Potential Formulation

The vector potential is introduced by the usual definition:

$$\mathbb{B} = \text{curl } \mathbb{A} . \tag{31}$$

Substituting this expression into (5), the following differential equation can be derived in the air domain, D_0:

$$\text{curl } \nu \text{ curl } \mathbb{A} = \mathbb{J} , \tag{32}$$

where $\nu = \frac{1}{\mu}$ is the reluctivity, and $\mathbb{J}$ is the current density.

(4) is identically satisfied if the electric field, $\mathbb{E}$, is derived by

$$\mathbb{E} = -j\omega A - \sum c_{0i} \mathbb{E}_{0i} , \tag{33}$$

where $\mathbb{E}_{0i}$ are derived from scalar potentials. c_{0i} are unknown coefficients which are computed by satisfying Kirchhoff's loop equations around the holes of the multiply connected conductors. For the derivation of these equations, we refer to [35].

For the sake of simplicity, we assume homogeneous Dirichlet boundary conditions on the far boundary, S_0, of the air domain, D_0. Thus, the tangential component of the vector potential, $\mathbb{A}$, disappears on S_0.

$$\mathbb{n} \times \mathbb{A} = 0 \quad \text{on} \quad S_0 \; . \tag{34}$$

On the surface of the conductor, S_i, IBC is prescribed. The following boundary condition can be obtained by substituting (33) into (1c) and regarding (2), (3) and (29):

$$\Delta \mathbb{n} \text{ curl } \mathbb{A} = \mathbb{n} \times (\mathbb{A} + \frac{1}{j\omega} \sum c_{0i} \mathbb{E}_{0i}) \times \mathbb{n} \; . \tag{35}$$

(35) is the most general boundary condition that can be derived from the IBC. To the best knowledge of the authors, this form has not been published yet. When the conductor region, D_i, is multiply connected, electrical circuit condition(s) should be specified. In this case the E_{0i} terms cannot be neglected, in general. Therefore, the full form of *Eq.* (35) should be prescribed. Note that the $\mathbb{E}_{0i}$ terms cannot be neglected even in the case of 2D problems if exterior conditions are prescribed.

In case of 2D problems, the left hand term is the normal derivative. If $\mathbb{E}_{0i} = 0$, (35) would become a third type boundary condition. Because of the exterior circuit conditions, the E_{0i} term cannot be neglected. Consequently, an arbitrary constant (more precisely a gradient of a potential) cannot be added to the vector potential.

The solution of the IBC problem can be found by satisfying the differential equations (32) and (33) with the boundary conditions (34) and (35). The solution is obviously not unique since the divergence of the vector potential, $\mathbb{A}$, can be freely chosen. Moreover, if the conductor domain, D_i, is multiply connected, electrical circuit condition(s) should also be specified.

To make the solution unique, we follow BIRO and PREIS' formulation [34]. The divergence of the vector potential is specified zero. This is the Coulomb gauge. Thus, (32) becomes

$$\text{curl } \nu \text{ curl } \mathbb{A} - \text{grad } \nu \text{ div } \mathbb{A} = \mathbb{J} \; . \tag{36}$$

The normal component of the vector potential, $\mathbb{A}$, is zero on both boundaries, S_i and S_0.

$$\mathbb{n}\mathbb{A} = 0 \quad \text{on} \quad S_0 \quad \text{and} \quad S_i \; . \tag{37}$$

Note that without (37) the Coulomb gauge is not sufficient for uniqueness.

A unique solution of the IBC problem can be found by satisfying the differential equation (36) with the boundary conditions (34), (35) and (37). The parameters, c_{0i}, are determined by specifying exterior conditions. The uniqueness proof is given in Appendix A.

The weak formulation of (36) is derived in Appendix B. The result is:

$$\int \nu[\text{curl } \mathbb{A} \text{ curl } \eta + \text{div } \mathbb{A} \text{ div } \eta]dV - \int \frac{\eta\nu\mathbb{A}_S}{\Delta}dS$$

$$= \int \eta \mathbb{J} dV + \int \frac{\nu\eta}{j\omega\Delta} \sum c_{0i}\mathbb{E}_{0iS} dS \; , \tag{38}$$

where E_{0iS} is the tangential vector of $\mathbb{E}_{0i}$ and η is a test function. The finite element matrices can be easily obtained from (38). This system of equations should be completed by the exterior conditions to balance the number of unknowns and equations.

5. Appendix A

This Appendix gives the uniqueness proof of the vector potential formulation.

Let us suppose that two vector potentials, $\mathbb{A}_1$ and $\mathbb{A}_2$, satisfy the differential equation (36) with boundary conditions (34), (35) and (37). Then their difference,

$$\mathbb{A} = \mathbb{A}_1 - \mathbb{A}_2$$

satisfies

$$\text{curl } \nu \text{ curl } \mathbb{A} - \text{grad } \nu \text{ div } \mathbb{A} = 0 \quad \text{in} \quad D_0 \; , \tag{A.1}$$

$$\mathbb{n} \times \mathbb{A} = 0 \quad \text{on} \quad S_0 \; , \tag{A.2}$$

$$\mathbb{n}\mathbb{A} = 0 \quad \text{on} \quad S_0 \quad \text{and} \quad S_i \; , \tag{A.3}$$

$$\Delta \mathbb{n} \times \text{curl } \mathbb{A} = \mathbb{A}_S \quad \text{on} \quad S_i \; , \tag{A.4}$$

where $\mathbb{A}_S$ is the tangential component of the vector potential:

$$\mathbb{A}_S = (\mathbb{n} \times \mathbb{A}) \times \mathbb{n} \; . \tag{A.5}$$

Multiplying (A.1) by the conjugate of the vector potential, $\overline{\mathbb{A}}$, and integrating over the air domain, D_0, provides

$$\int \overline{\mathbb{A}}[\text{curl } \mathbb{H} - \text{grad } \nu \text{ div } \mathbb{A}]dV = 0 \; . \tag{A.6}$$

Applying the identities of the vector analysis,

$$\text{div }(\overline{\mathbb{A}} \times \mathbb{H}) = \mathbb{H} \text{ curl } \overline{\mathbb{A}} - \overline{\mathbb{A}} \text{ curl } \mathbb{H} \ , \tag{A.7}$$

and

$$\text{div }(\overline{\mathbb{A}}U) = U \text{ div } \overline{\mathbb{A}} + \overline{\mathbb{A}} \text{ grad } U \tag{A.8}$$

yields

$$\int [\mathbb{H} \text{ curl } \overline{\mathbb{A}} + \text{div } (\mathbb{H} \times \overline{\mathbb{A}}) + \nu \text{ div } \mathbb{A} \text{ div } \overline{\mathbb{A}} - \text{div } (\overline{\mathbb{A}}\nu \text{ div } \mathbb{A})] dV = 0 \ . \tag{A.9}$$

Transforming (A.9) with the Gauss theorem

$$\int [\mathbb{H} \text{ curl } \overline{\mathbb{A}} + \nu \text{ div } \mathbb{A} \text{ div } \overline{\mathbb{A}}] dV + \int [\mathbb{H} \times \overline{\mathbb{A}} - \overline{\mathbb{A}}\nu \text{ div } \mathbb{A}] \mathbb{n} \, dS = 0 \ . \tag{A.10}$$

On the far boundary, S_0, the tangential component of the vector potential is zero according to (A.2). Moreover, the normal component of the vector potential disappears on both S_i and S_0, according to (A.3). Hence, the second term in the surface integral falls out and the first term integral reduces to the conductor surface, S_i . On this surface, however, the tangential component of the magnetic field, $\mathbb{H}$, satisfies the fundamental equation (A.4) of the IBC. Thus, the above integral takes the following form:

$$\int [\nu \text{ curl } \mathbb{A} \text{ curl } \overline{\mathbb{A}} + \nu \text{ div } \mathbb{A} \text{ div } \overline{\mathbb{A}}] dV + \int \frac{\nu}{\Delta} \mathbb{A}_S \overline{\mathbb{A}}_S \, dS = 0 \ . \tag{A.11}$$

Since Δ is a complex number and the volume integral is positive or zero, (A.5) can be satisfied only if the surface integral is zero. This involves that the tangential component of the vector potential is zero. From this it follows that the volume integral term is zero. Consequently, both the rotation and divergence of the vector potential vanish in the whole volume. According to (A.3) the normal component is also zero. These conditions can only be satisfied if $\mathbb{A} = 0$. Therefore, the solution is unique which proves the uniqueness of the vector potential formulation.

6. Appendix B

This Appendix derives the weak formulation (38) of the IBC problem from (36). Multiply (36) by a test function η satisfying the same boundary conditions as the vector potential, $\mathbb{A}$, and integrate the equation over the volume, D_0. This gives

$$\int \eta[\text{curl } \mathbb{H} - \text{grad } (\nu \text{ div } \mathbb{A}) - \mathbb{J}]dV = 0 \; . \tag{B.1}$$

Applying the identities of the vector analysis,

$$\text{div } (\eta \times \mathbb{H}) = \mathbb{H} \text{ curl } \eta - \eta \text{ curl } \mathbb{H} \tag{B.2}$$

and

$$\text{div } (\eta U) = U \text{div } \eta + \eta \text{ grad } U \tag{B.3}$$

yields

$$\int [\mathbb{H} \text{ curl } \eta + \text{div } (\mathbb{H} \times \eta) + \nu \text{ div } \mathbb{A} \text{ div } \eta - \text{div } (\eta\nu \text{ div } \mathbb{A}) - \eta\mathbb{J}]dV = 0 \; . \tag{B.4}$$

Transforming this with Gauss theorem

$$\int [\mathbb{H} \text{ curl } \eta + \nu \text{ div } \mathbb{A} \text{ div } \eta - \eta\mathbb{J}]dV + \int [\mathbb{H} \times \eta - \eta\nu \text{ div } \mathbb{A}] \, \mathbb{n} \, dS = 0 \; . \tag{B.5}$$

On the far boundary, S_0, the tangential component of the vector potential and the test function are zero according to (34). Moreover, the normal component of the vector potential disappears on both S_i and S_0, according to (37). Hence, the second term in the surface integral falls out and the first term integral reduces to the conductor surface, S_i. On this surface, however, the tangential component of the magnetic field, $\mathbb{H}$, satisfies (35). Thus, the above integral takes the following form:

$$\int [\mathbb{H} \text{ curl } \eta + \nu \text{ div } \mathbb{A} \text{ div } \eta - \eta\mathbb{J}]dV -$$

$$\int \eta[\frac{\nu}{\Delta}(\mathbb{n} \times [\mathbb{A} + \frac{1}{j\omega}\sum c_{0i}\mathbb{E}_{0i}]) \times \mathbb{n}]dS = 0 \; . \tag{B.6}$$

(38) can be easily obtained from (B.6) which concludes the proof.

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