MAGNETOHYDRODYNAMIC EQUILIBRIUM OF HELICITY-INJECTED SPHEROMAK BY COMBINATION OF FDM AND BEM

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Abstract

The sustainment of the spheromak has been successfully achieved by DC helicity injection in the FACT device at Himeji Institute of Technology. The flux conserver actually used in the experiments has the shielding wall to prevent the plasma from being in contact with the divertor bias coil. Equilibrium configurations of the spheromak in the flux conserver with the shielding wall and the divertor bias coil are numerically determined by using the combination of the finite difference and the boundary element method. Several results for equilibrium configurations and their equilibrium quantities are presented. On the basis of the results, the effects of the divertor bias coil on equilibrium configurations of the helicity-injected spheromak are investigated.

Keywords: MHD equilibrium, helicity, spheromak, BEM, FDM.

1. Introduction

The spheromak is a magnetic configuration whose toroidal and poloidal magnetic fields are produced by plasma currents only. It has many potential advantages that are not found for the tokamak: compactness and simplicity of design for external coils and vacuum vessels. In addition, the spheromak has a possibility of translation which allows the formation chamber to be separated from the burn chamber. In this sense, it has recently attracted attention as an energy source of the tokamak plasma.

A method of producing the spheromak is to use a magnetized coaxial plasma gun. The plasma produced in the gun is ejected into the metallic vessel which is called a flux conserver (FC). However, the plasma current decays due to resistivity of the plasma until it vanishes. In order to drive the plasma current and to steadily sustain the magnetic configuration, DC helicity has been injected to the spheromak by use of electrodes. Since this method is more effective and cheaper than the lower-hybrid wave and the neutral beam injection method, the experiments by this method have been carried out at Himeji Institute of Technology (NAGATA et al., 1991) and Los Alamos National Laboratory (JARBOE et al., 1983). Generally, the magnetic configuration of the spheromak sustained by DC helicity injection has to contain the open field lines that penetrate both the FC and the electrode. In the FACT device at Himeji Institute of Technology, the field lines are produced by means of the divertor bias coil and the poloidal current is applied along them. The toroidal magnetic flux is generated by the current and is partially converted into the poloidal one through the magnetohydrodynamic relaxation process.

The flux conserver actually used in the experiments has the shielding wall to prevent the plasma from being in contact with the divertor bias coil. The purpose of the present study is to numerically determine equilibrium configurations of the spheromak plasma in the flux conserver with the shielding wall and the divertor bias coil and to investigate their stability.

In the next section, the model used throughout the present study is introduced, and basic equations and their boundary conditions are given. In addition, the iterative procedure for solving the basic equations is explained in detail and, by means of the procedure, equilibrium configurations are numerically determined. In the third section, the MHD stability of the equilibrium configurations is investigated by evaluating the safety factor and the specific volume. Conclusions are summarized in the final section.

The MKS units with $\mu_0 = 1$ are used throughout the present study, where μ_0 is a magnetic permeability of vacuum.

2. MHD Equilibrium

2.1 Model of FC and Helicity Injector

A drum-type FC is used in the FACT device. The size of the FC is as follows: the radius and the height are 293.5 mm and 315.0 mm, respectively. The FC is joined with the helicity injector of radius 199.3 mm and height 316 mm. A divertor bias coil of rectangular cross-section is placed in the injector and is covered with a shielding wall so that the plasma may not be in contact with it. A cathode electrode of radius 30 mm is inserted into the injector. In the FACT device, a voltage is applied between the shielding wall and the cathode electrode. A poloidal current is expected to flow along the open field lines which penetrate both of them. In *Fig. 1*, we show the model of the FC and the helicity injector which will be used throughout this study.

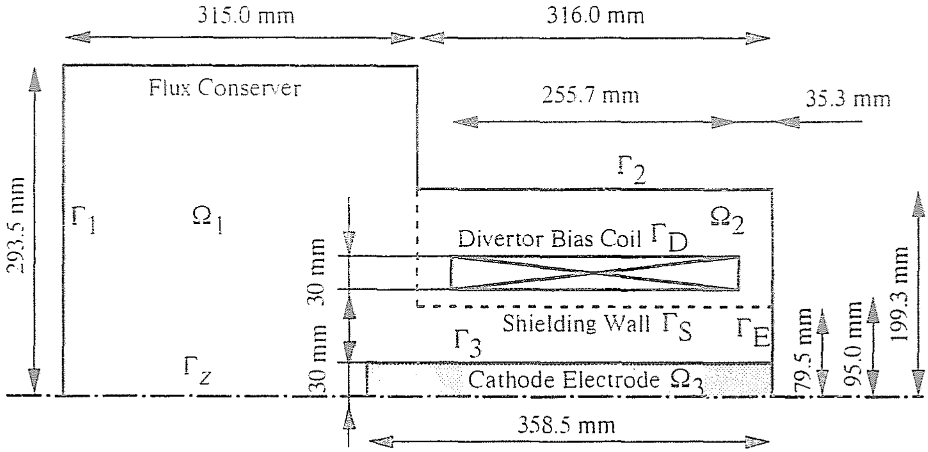

Fig. 1. The model of the flux conserver and the helicity injector

2.2 Governing Equations and Boundary Conditions

Let us use the cylindrical coordinate system (z, r, ϕ), take the symmetry axis as z-axis, and choose the bottom plate center on the FC as the origin. Since the equilibrium configuration of the spheromak is axially symmetrical, we can determine it by solving the Grad–Shafranov equation in the (z, r) plane. In the following, the region Ω in which the equilibrium configuration is determined is divided into unoverlapped subregions, Ω_1, Ω_2 and Ω_3, as shown in *Fig. 1*. The subregion Ω_1 is bounded by z-axis and the surface of the FC, the shielding wall and the cathode electrode whereas the subregion Ω_2 is enclosed by the surface of the helicity injector and the shielding wall (see *Fig. 1*). Furthermore, the cross-section of the cathode electrode is denoted by Ω_3. We assume that the plasma exists only in the subregion Ω_1 and also assume that the magnetic fields produced by the plasma extend over Ω_1 and Ω_2, and penetrate neither in Ω_3 nor outside the FC. Furthermore, we assume that the magnetic fields generated by the divertor bias coil extend all over the space. Under these assumptions, the Grad–Shafranov equation can be written in the form:

$$-\hat{L}\psi_D = 0 \, , \tag{1}$$

$$-\hat{L}\psi_p = \chi_{\Omega_1}(z,r)\left(r^2\frac{dp}{d\psi} + \frac{1}{2}\frac{dI^2}{d\psi}\right) \, , \tag{2}$$

where ψ_D and ψ_p denote the magnetic flux generated by the divertor bias coil and the plasma, respectively, and ψ is the sum of ψ_D and ψ_p. Furthermore, $\hat{L}$ is the differential operator defined as

$$\hat{L} \equiv r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} \, ,$$

and $\chi_{\Omega_1}(z,r)$ represents the characteristic function defined as

$$\chi_{\Omega_1} = (z,r) \equiv \begin{cases} 1, & ((z,r) \in \Omega_1) \, , \\ 0, & ((z,r) \notin \Omega_1) \, . \end{cases}$$

The pressure and the toroidal magnetic function are assumed as

$$\frac{dp}{d\psi} = 0 \, , \tag{3}$$

$$\frac{1}{2}\frac{dI^2}{d\psi} = \frac{\lambda}{c^2}\psi \, , \tag{4}$$

where λ is a constant and c denotes a representative length. In the present study, the radius of the FC is adopted as c. Under the assumptions (3) and (4), equilibrium configurations become so-called Taylor state (TAYLOR et al., 1974).

Now we consider the boundary conditions for *Eqs.* (1) and (2). Since the plasma exists in Ω_1 and it generates the magnetic fields only in Ω_1 and Ω_2, we can assume that $\psi_p = 0$ on $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_E$ and Γ_z. On the surface of the divertor bias coil, Γ_D, Ampere's law must be fulfilled. In terms of ψ_D and ψ_p, this law can be written as

$$\oint_{\Gamma_D} \frac{1}{r}\frac{\partial \psi_D}{\partial n} dl = I_D, \tag{5}$$

$$\oint_{\Gamma_D} \frac{1}{r} = \frac{\partial \psi_p}{\partial n} dl = 0 \, , \tag{6}$$

where $\partial/\partial n$ stands for the directional derivative whose direction is inward normal to Γ_D and I_D denotes the total current of the coil. Furthermore, ψ_p and ψ_D ought to be constant on Γ_D.

2.3 Numerical Methods

In this subsection, we explain the method for solving *Eqs.* (1) and (2) under the above boundary conditions. First, let us solve *Eq.* (1). Since *Eq.* (1) is linear, we can easily derive the following boundary integral equation (MIYAUCHI et al., 1993) of *Eq.* (1):

$$c(z_i, r_i)\psi_D(z_i, r_i) = \oint_{\Gamma_D} \left(\frac{w^*}{r} \frac{\partial \psi_D}{\partial n} - \frac{\psi_D}{r} \frac{\partial w^*}{\partial n} \right) dl \ , \tag{7}$$

where $c(z_i, r_i)$ is the so-called shape coefficient. The fundamental solution w^* is now given by

$$w^*(z, r; z_i, r_i) = \frac{\sqrt{r r_i}}{2\pi k}[(2 - k^2)K(k) - 2E(k)] \ , \tag{8}$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and the second kind, respectively, and k is defined by

$$k^2 \equiv \frac{4 r r_i}{(z - z_i)^2 + (r + r_i)^2} \ .$$

Following the standard manner of the boundary element method, we can solve *Eq.* (7) under the boundary conditions described in the previous subsection and can determine ψ_D and $\partial \psi_D/\partial n$ on the surface of the divertor bias coil Γ_D. The values of ψ_D at an arbitrary location can be calculated by use of those of ψ_D and $\partial \psi_D/\partial n$ on Γ_D.

Next we solve *Eq.* (2). Since it is difficult to numerically express the characteristic function $\chi_{\Omega_1}(z, r)$ in *Eq.* (2), we solve *Eq.* (2) separately in Ω_1 and in Ω_2 in such a way that the tangential component of the poloidal magnetic field and the normal one of the magnetic induction are continuous across the shielding wall Γ_S. These continuity conditions lead to

$$[\![\ \psi_p \ ]\!] = 0 \ , \tag{9}$$

$$\left[\!\!\left[\ \frac{\partial \psi_p}{\partial n} \ \right]\!\!\right] = 0 \ , \tag{10}$$

where the bracket operator $[\![\ \cdot\ ]\!]$ denotes the gap of its operand across Γ_S. In order to fulfil *Eqs.* (9) and (10) consistently, we employ the iterative procedure composed of two steps. In the first step, we solve the boundary value problem of the equation

$$-\hat{L}\psi_p = 0 \ , \quad \text{in} \quad \Omega_2 \ ,$$

by use of the boundary element method and determine the values of $\partial\psi_p/\partial n$ on Γ_S. As values of ψ_p on Γ_S, those in the previous cycle are used. In the second step, the eigenvalue problem of *Eq.* (2) is solved in the subregion Ω_1 by using the values of $\partial\psi_p/\partial n$ on Γ_S evaluated in the first step. Since the shape of the subregion Ω_1 is complex, the boundary-fitted curvilinear coordinate system (BFCCS) (THOMPSON et al., 1974, 1977, 1985; KAMITANI et al., 1987) is constructed there and *Eq.* (2) is solved by its means. After finishing the second step, we have determined the values of ψ_p on Γ_S. These two steps are iterated until the values of ψ_P on Γ_S converge. *Fig. 2* shows the coordinate lines of the BFCCS. Boundary nodes used in the first step are generated on Γ_S so that they may overlap with grid points of the BFCCS on the boundary. Boundary nodes so generated are indicated by symbols ▩ in *Fig. 2*.

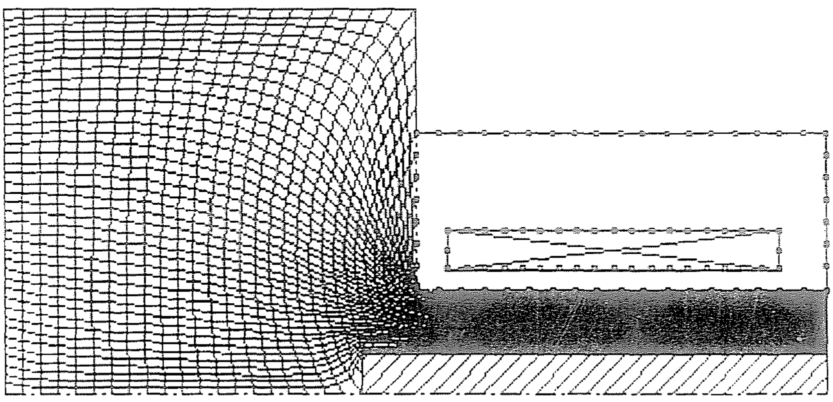

Fig. 2. Coordinate lines of BFCCS and boundary nodes. The symbols ▩ denote the boundary nodes that are used in the first and the second step of the iterative procedure

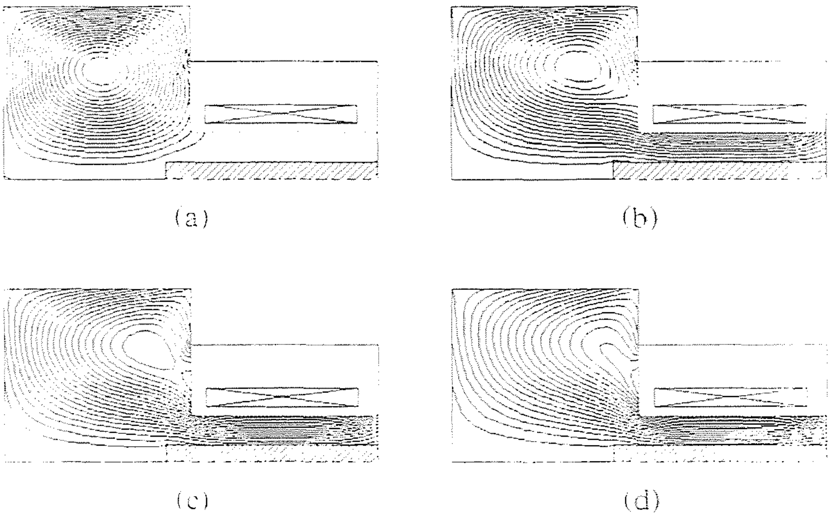

Fig. 3. Equilibrium configurations. (a) $I_D/I_p = 0.0$, (b) $I_D/I_p = 1.0$, (c) $I_D/I_p = 1.5$, (d) $I_D/I_p = 2.0$. Here I_p denotes the total plasma current.

2.4 Equilibrium Configurations

Equilibrium configurations are numerically determined by using the method explained in the previous subsection. Typical examples of results are shown in *Figs. 3(a), (b), (c)* and *(d)*. In these figures, the contours on which $\psi =$ const. are drawn. As it is well known, these contours represent magnetic field lines. We see from these figures that the magnetic axis becomes closer to the divertor bias coil with the increase of the coil current I_D. Although open field lines increase with the increase of I_D/I_p, there is no closed magnetic surface in the case of $I_D/I_p = 2.0$. Hence it might be said that the coil current must satisfy $I_D/I_p < 2.0$ for a magnetic configuration to have closed magnetic surfaces.

3. MHD Stability and Effects of Divertor Bias Coil

In this section, we investigate the stability of equilibrium configurations obtained in the previous section and study the effects of the divertor bias coil on them. Let us evaluate the safety factor $q(\psi)$ and the specific volume

$dV/d\psi$ defined by

$$q(\overline{\psi}) \equiv \frac{I(\overline{\psi})}{2\pi} \oint_{\psi=\overline{\psi}} \frac{dl}{r|\nabla\psi|}, \tag{11}$$

$$\frac{dV}{d\psi} \equiv -2\pi \oint_{\psi=\overline{\psi}} \frac{r\,dl}{|\nabla\psi|}, \tag{12}$$

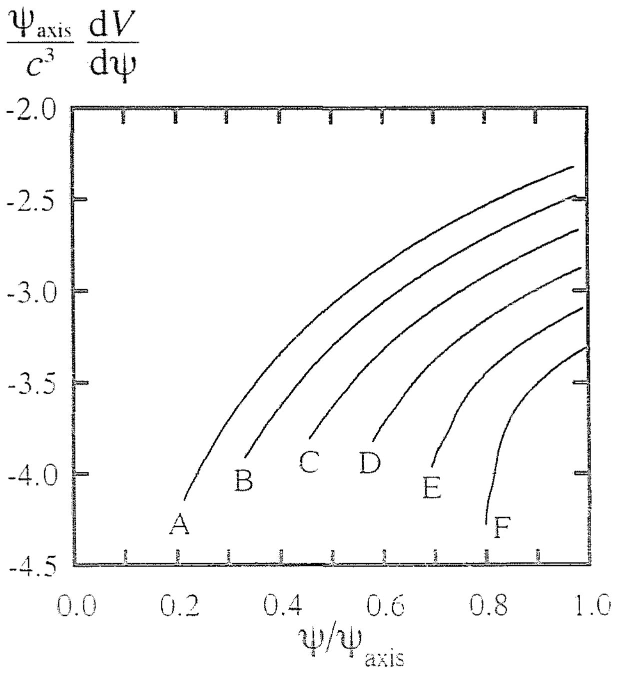

Fig. 4. Specific volume as function of ψ/ψ_{axis}. A: $I_D/I_p = 0.0$, B: $I_D/I_p = 0.2$, C: $I_D/I_p = 0.4$, D: $I_D/I_p = 0.6$, E: $I_D/I_p = 0.8$, F: $I_D/I_p = 1.0$

In *Figs. 4* and *5*, we show the specific volume and the safety factor as functionf ψ/ψ_{axis}, where ψ_{axis} is a value of the flux function ψ on the magnetic axis. We see from *Fig. 4* that $d^2V/d\psi^2$ is always positive and we may conclude that the equilibrium configurations are not stabilized by the magnetic well. In addition, *Fig. 5* indicates that the value of safety factor on the separatrix increases remarkably with the increase of the coil current. In addition, the safety factor increases monotonically toward the magnetic axis in the case of $I_D/I_p \leq 0.4$, whereas it decreases monotonically in the case of

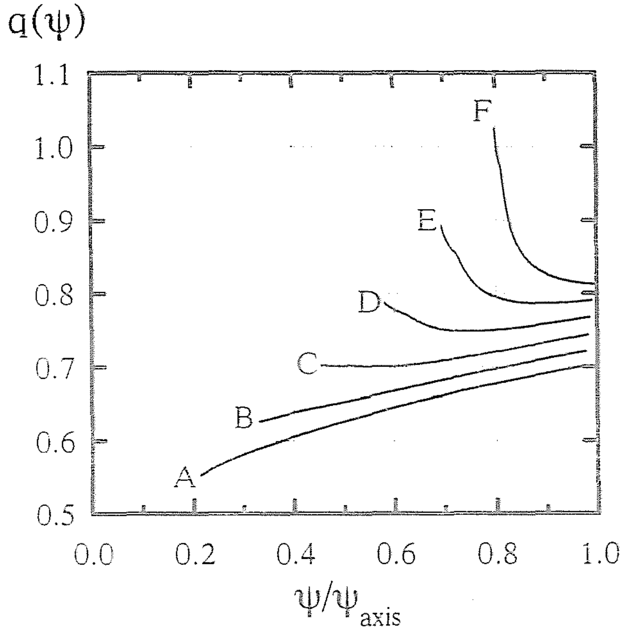

Fig. 5. Safety factor profile. A: $I_D/I_P = 0.0$, B: $I_D/I_P = 0.2$, C: $I_D/I_P = 0.4$, D: $I_D/I_P = 0.6$, E: $I_D/I_P = 0.8$, F: $I_D/I_P = 1.0$

$I_D/I_p \geq 1.0$. As it is well known, the safety factor of the spheromak takes a minimum value on the separatrix, increases monotonously, and takes a maximum value on the magnetic axis. Furthermore, the safety factor profile of the tokamak behaves to the contrary. Therefore, the distribution of the safety factor for the helicity-injected spheromak changes from the spheromak-like profile through the ultra-low-q one to the tokamak-like one with the increase of the coil current. Especially, in the case of $I_D/I_p = 0.6$, the equilibrium configuration has a pitch-minimum region near $\psi/\psi_{\text{axis}} = 0.75$. Therefore, it is possible that the equilibrium configuration becomes unstable against localized perturbations in the case of $0.4 < I_D/I_p < 0.8$.

4. Conclusions

We have numerically determined equilibrium configurations of the helicity-injected spheromak by the combination of the finite difference and the boundary element method, and have investigated the effects of the divertor bias coil on their stability. Conclusions obtained in the present study are as follows.

1. As the coil current is increased, the magnetic axis becomes closer to the divertor bias coil and, therefore, the number of open field lines increases. In order that a magnetic configuration has closed magnetic surfaces, the coil current must satisfy $I_D/I_p < 2.0$.
2. The helicity-injected spheromak has no magnetic well regardless of the value of I_D/I_p. Hence, it is stabilized by the magnetic shear only.
3. The value of safety factor on the separatrix increases remarkably with the increase of I_D/I_p. Thus, the distribution of the safety factor changes from the spheromak profile through the ultra-low-q one to the tokamak-like one with the increase of the coil current.
4. In the case of $0.4 < I_D/I_p < 0.8$ the equilibrium configuration has a pitch-minimum region and, therefore, becomes unstable against localized perturbations.

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