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A NEW 'FLOATING NODES METHOD' FOR 2D h-ADAPTIVE MESH REFINEMENT USING ELECTRIC FIELD INTENSITY VALUES AS A CRITERION

Vlatko ČINGOSKI, Kiyomi TOYONAGA, Kazufumi KANEDA and Hideo YAMASHITA

Electric Machinery Laboratory, Faculty of Engineering
Hiroshima University, Kagamiyama 1-4-1
Higashi-hiroshima 724, Japan
Phone: +81 824 24 7665,
Fax: +81 824 22 7195
email: vlatko@eml.hiroshima-u.ac.jp

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Abstract

Suitable fine mesh divisions are essential to obtain fast and accurate solutions in two-dimensional electric field analysis. However, the development of such fine division meshes always requires considerable technical knowledge and experience. A way of solution which proves highly effective is the application of adaptive methods for refinement of the division meshes. Since the researchers in the analysis of various electric field problems are usually interested in the values of electric field intensity and its distributions, it seems natural to employ these values as a criterion for mesh refinement. In this paper, we propose an h-adaptive refinement procedure by generating new nodes inside initial rough mesh. Also, we propose the improvement of the shape of finite elements by using a 'floating nodes method'. For both procedures, electric field intensity values were used as criteria.

Keywords: h-adaptive mesh refinement, 2D finite element analysis, electric field.

Introduction

In finite element analysis, generating an optimal dense mesh is necessary to maintain accurate solutions by low cost analysis, which ordinarily demands of researchers much experience and knowledge. One means of resolving this problem is the development of adaptive mesh refinement methods. Using these techniques, even relatively unexperienced researchers can obtain solutions with acceptable accuracy. In general, adaptive techniques can be divided in three main groups: developing dense division maps in areas with large computational error (h-adaptive); for the same division maps, increasing the order of interpolation polynomial – shape functions (p-adaptive); and mixed techniques from both. The p-adaptive method is free from generating a new division map for any refinement step, but requires the development of a new program constructing matrix of the system for any step of increase in the order of interpolation polynomial. On the other hand, in the h-adaptive method for the same order of interpolation functions, it is necessary to develop a new division map for each step of refinement. Several procedures for h-adaptive refinement have already been proposed: adding or deleting nodes to obtain a uniform variation of the energy for each finite element (SAITO et al., 1990); minimization of the energy by adding new nodes, eliminating some edges of performing 'swaps' on some of them (HOPPE et al., 1993), etc.

Regarding electric field analysis, adaptive techniques based on the aforementioned procedures can also be applied. The main interest of researchers, however, does not concentrate on electric potential distribution, but on the intensity of electric field strength and its distribution. Therefore, to obtain physical quantities with high accuracy, it is beneficial to use physical quantities directly as criteria for adaptive procedures.

To preserve accurate results in the analysis of electric field intensity distribution, in this paper, we propose a new procedure based on re-division of the initial mesh using electric field intensity as a criterion and modification of the shape of the finite elements by the 'floating nodes method'. Through this procedure, good results were obtained. Here, 2–D finite element analysis using a second-order triangular mesh and electric potential as an unknown variable is examined.

Outline of Proposed Algorithm

In *Fig. 1*, an outline of the proposed h-adaptive algorithm is presented. Initially, the analysis area is divided into a rough mesh, and the first step of electric field analysis is then performed. Following this procedure, new nodes are added inside the appropriate finite elements with high electric field intensity. Over this newly generated group of nodes, we generated a new triangle mesh based on the Delaunay triangulation method. The mesh generated by this procedure, however, does not always provide the desired shape of the triangular elements – equilateral or nearly equilateral triangles. This method is used for fixed positions of the generated nodes. For correcting triangle shapes, the Laplace method (TANIGUCHI, 1992) is commonly used, where the node position is determined as a centroid of the polygon generated from all adjacent nodes. Through this procedure, nearly equilateral triangles can be generated. In this paper, we propose the determination of the node position not only by the coordinates of the adjacent nodes but also using the values of electric field intensity as a weighted function. This procedure allows uniform distribution of electric field intensity values by adaptively generating a mesh that is dense in the high field area while, at the same time, rough in the low field area. Described below is a detailed ex-

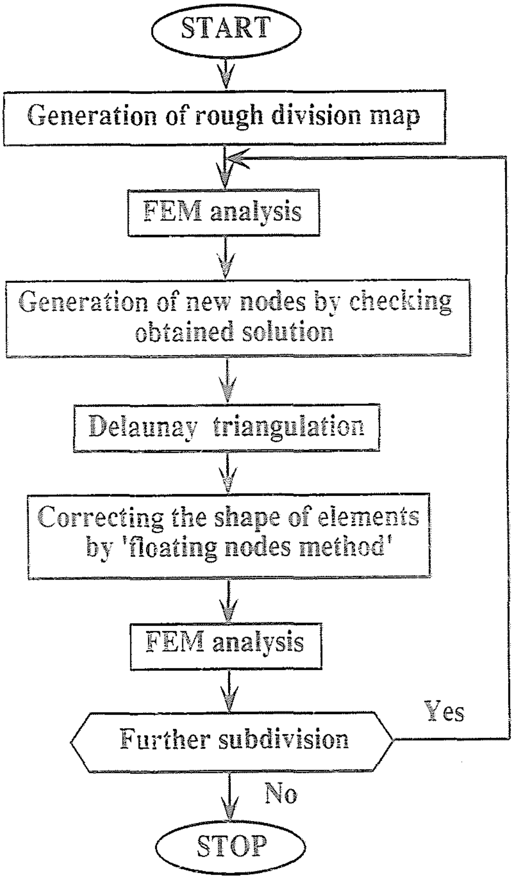


Fig. 1. Flow chart

planation of the methods for generating new nodes and the proposed 'floating nodes method' for correcting the shape of triangular finite elements.

Method for Generating New Nodes

To equalize the density values of electric field intensity in each element, we first choose finite elements with high electric field intensity values inside of which new nodes are generated. The main problem is how to determine where to place the new nodes in which finite element. Here, we use the values E_{ei}, the electric field intensity along edge i of the element e, which is calculated by dividing the difference between the electric potential values

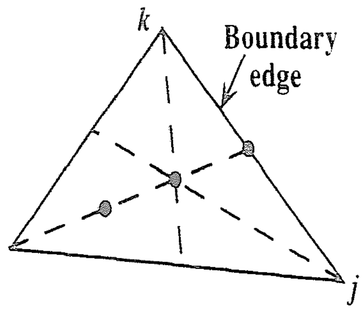


Fig. 2. Position of new nodes (boundary element)

on the terminal nodes of each edge of triangle e by its length. Assigning the maximum value of E_{ei} ($i = 1 \sim 3$) where i is three edges of an element as $E_{e\max}$, and from the relationship between $E_{e\max}$ and the threshold values E_{0j} ($j = 1 \sim 3$) ($E_{01} > E_{02} > E_{03}$), we developed a method for generating new nodes inside the element. To decrease the number of iterations in the iteration process in *Fig. 1*, we can put in an arbitrary number of new nodes inside each finite element (our maximum is 3). On the boundary of the analysis region and on the boundaries between different materials it is necessary to generate new nodes. Special treatment for the elements with boundary edges, therefore, must be considered in the procedure of generating new nodes.

1. For the finite element e including the boundary edge

a) if $E_{e\max} > E_{01}$, then three new nodes will be added at the following positions (see *Fig. 2*):
   - middle point of the boundary edge;
   - center of gravity of the finite element;
   - middle point of the line segment that connects the joint point of the two edges with larger values E_{ei} ($i = 1 \sim 3$) and the center of gravity;

b) if $E_{01} > E_{e\max} > E_{02}$, then two new nodes will be added at the following positions:
   - middle point of the boundary edge;
   - center of gravity of the finite element;

c) if $E_{02} > E_{e\max} > E_{03}$, then only one new node will be added:
   - at the middle point of the boundary edge.

2. For the finite element e without the boundary edge

a) If $E_{e\max} > E_{01}$, then three new nodes will be added at the following positions (see *Fig. 3*):

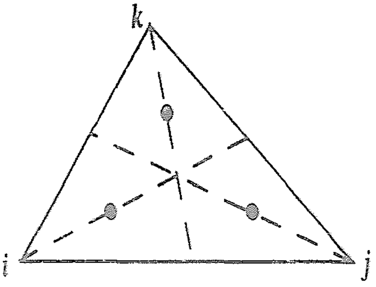


Fig. 3. Position of new nodes (inner element)

- middle points of the line segments that connect each node of the finite element and the center of gravity of the element.

b) If $E_{01} > E_{e\max} > E_{02}$, then two new nodes will be added at the following positions:
   - center of gravity of the finite element;
   - middle point of the line segment that connects the joint point of the two edges with larger values E_{ei} $(i = 1 \sim 3)$ and the center of gravity.

c) If $E_{02} > E_{e\max} > E_{03}$, then only one new node will be added:
   - at the center of gravity of the finite element.

Furthermore, the threshold values E_{01}, E_{02} and E_{03} can be freely defined by the user, but the default values are given as follows:

$$A = \frac{\text{Max. value of } E_{e\max} \text{ for all finite elements - Min. value of } E_{e\max} \text{ for all finite elements}}{4} \tag{1}$$

$$\begin{aligned} E_{01} &= \text{Min. value of } E_{e\max} + 3 \times A \\ E_{02} &= \text{Min. value of } E_{e\max} + 2 \times A \\ E_{03} &= \text{Min. value of } E_{e\max} + 1 \times A \end{aligned} \tag{2}$$

Moving Newly Generated Nodes Using 'Floating Nodes Method'

Generation of New Division Map

The new division map is generated using the nodes of the initial mesh and the newly added nodes from the previous step. To obtain as regular a division map (with equilateral triangles) as possible, we implemented the Delaunay triangulation method (TANIGUCHI, 1992). Using only Delaunay triangulation, however, is insufficient for obtaining a regular mesh of nearly equilateral triangles. On the contrary, the mesh developed by Delaunay triangulation keeps the prescribed position of the nodes and merely changes their connections. We have to perform some other procedure, therefore, in order to achieve the desired division map.

Improving the Shape of Finite Elements

In this paper, we also propose a new method for improving the shape of finite elements. We named this method 'floating nodes method'. To construct regular mesh in this procedure, corner nodes, which determine the shape of the analysis region, and the materials as far as the curved edges are fixed. Other nodes can move freely in the finite mesh, i.e. they 'float' in the analysis region depending on their initial coordinates and values of electric field intensity. Even nodes on the line segments forming the boundaries of the analysis region and boundaries between different materials can move but along those line segments only. In general, node movement is executed in connection with the values of electric field intensity at the surrounding nodes. This movement will put the node in the 'optimal' position concerning electric field intensity. The node coordinates after moving can be obtained by the following equations:

$$X = \frac{\sum_{i=1}^{n} x_i E_i}{\sum_{i=1}^{n} E_i} , \tag{3}$$

$$Y = \frac{\sum_{i=1}^{n} y_i E_i}{\sum_{i=1}^{m} E_i} , \tag{4}$$

where n is the number of nodes directly connected with point t, (x_i, y_i) are the coordinates of the surrounding node i, and E_i is the value of electric field intensity at node i.

In the case of boundary segment nodes, the following procedure is applied:

1. Definition of the x-coordinate from *Eq.* (3). Then defining the y-coordinate by substituting the already defined x-coordinate in the boundary segment equation.
2. Definition of the y-coordinate from *Eq.* (4). Then defining the x-coordinate by substituting the already defined y-coordinate in the boundary segment equation.
3. Finding the average value for x and y coordinates from the already defined values in steps 1 and 2.

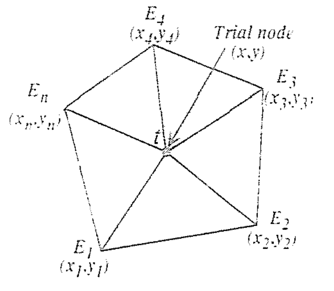


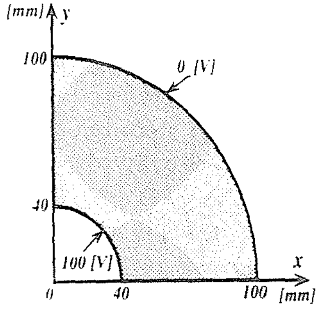
Fig. 4. 'Floating nodes method'

Results

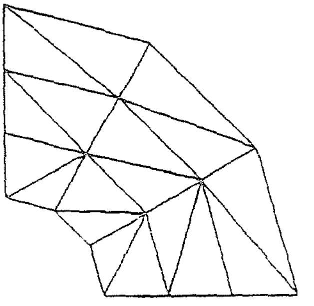
To examine the main properties of the proposed method, we analysed the model of an axis-symmetrical cylindrical condenser with infinite length (analysis region only 1/4 of the total area) presented in *Fig. 5*(a), together with the initial rough mesh *Fig. 5*(b). The results obtained by the proposed method are presented in *Fig. 6*. To point out the advantages of the proposed 'floating nodes method' for moving the nodes in the analysis region, we also present the generated mesh before and after applying it in *Figs. 6*(a) and (b), respectively.

From *Fig. 6*, we can see that the nearer the larger diameter of the cylinder, the larger the size of the finite elements becomes, but in the 'floating node method' movement, the shape of the generated finite elements becomes ever more regular and closer to the desired equilateral triangles.

Next, we will show the improvement in accuracy of the obtained results of finite element analysis using the division map generated by the proposed adaptive procedure and using the 'floating node method' (*Fig. 6*(b)).

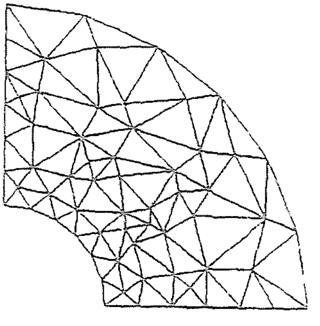


a) Model

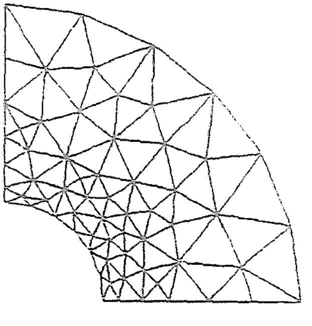


b) Rough mesh

Fig. 5. Analyzed model

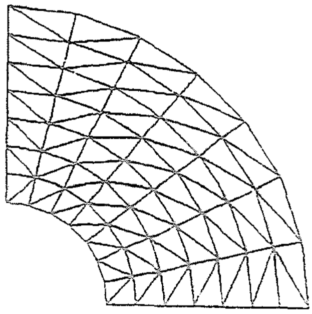


a) After Delaunay triangulation

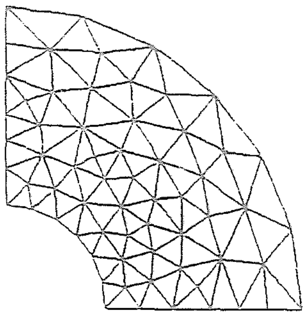


b) After 'floating nodes method'

Fig. 6. Division map



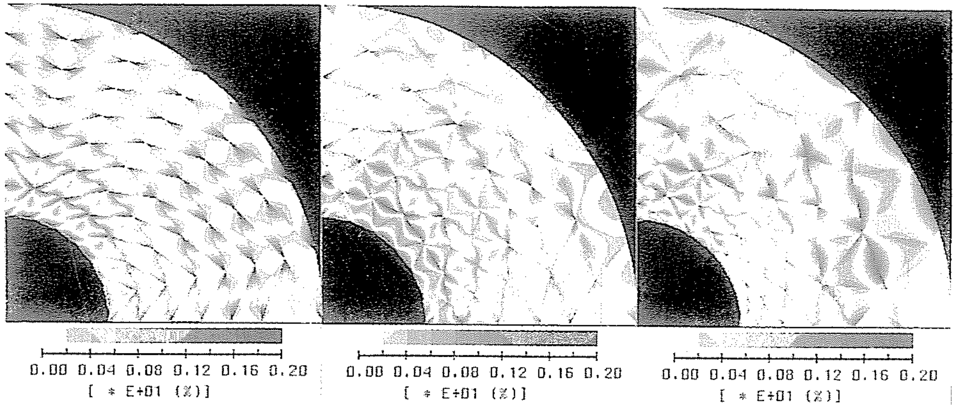
a) Regular mesh



b) Laplacian mesh

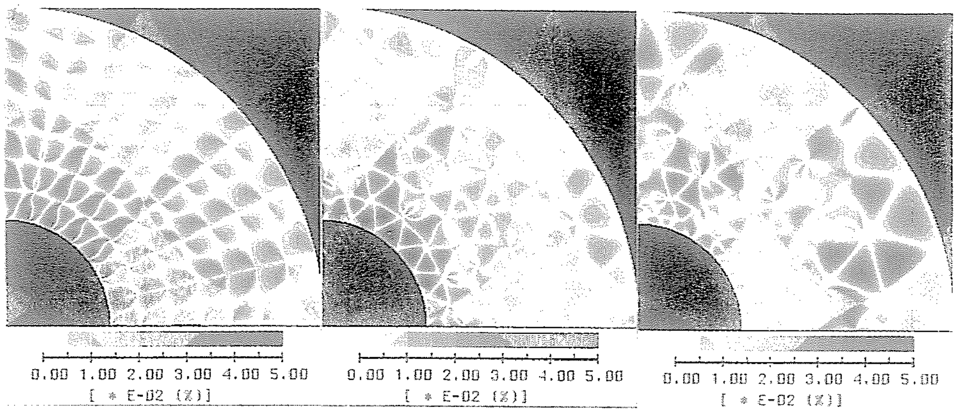
Fig. 7. Different division maps

Also, the improvement will be compared with results obtained by two other division maps: a uniform division mesh of 7×7 elements with approximately the same number of nodes and elements as the analysed mesh (see *Fig. 7(a)*), and another where movement of the nodes was facilitated using the Laplace method [3] (see *Fig. 7(b)*). The error distribution of the results obtained by all three meshes is presented in *Fig. 8*. From *Fig. 8*, the 7×7 uniform mesh and Laplacian mesh both have a tendency to show larger error distribution near the inside diameter of the cylinder and smaller error distribution near the outside diameter of the cylinder. The mesh generated by the proposed method, however, shows precisely the opposite trend. In electric field analysis, highly accurate results are necessary in the high electric intensity field area (in this model, around the inside diameter), a characteristic strongly satisfied by the proposed method. Finally, to investigate in detail the error distributions of the electric field in the high intensity field area (near the inside diameter), we analysed the distributions of



a) Regular mesh b) Laplacian mesh c) Proposed mesh

A. Electric field intensity.

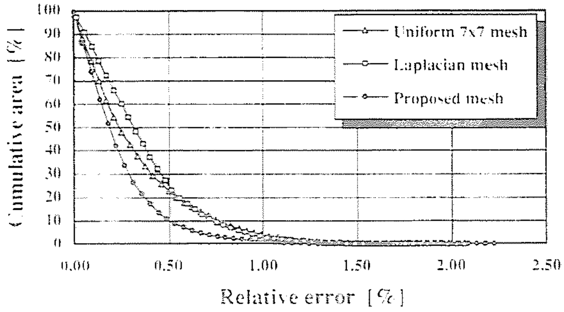


a) Regular mesh b) Laplacian mesh c) Proposed mesh

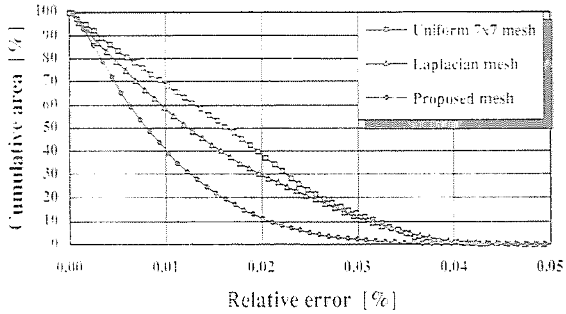
B. Electric potential.

Fig. 8. Error distributions

the cumulative error in an area 50 [mm] × 50 [mm] from the central axis (*Fig. 9*). These diagrams further prove that the proposed method is favorable to already existing methods. From the presented distributions of cumulative error for electric potential, for example the proposed method indicates that only 20% of the total analysed area has an error larger than



a) Electric field intensity



b) Electric potential

Fig. 9. Cumulative area vs. relative error

0.01%, while for the other two methods, the area with an error larger than 0.01% reached more than 60%, as expected.

Conclusions

In this paper, the authors proposed a new h-adaptive method for the analysis of electric field problems by generating new nodes in the analysis region using electric field intensity as a criterion and improving the shape of finite elements using the 'floating nodes method'. The main advantages of the proposed algorithm are:

1. By using as a criterion the physical values which researchers seek, the accuracy of the obtained results can be improved;
2. The shape of the generated finite elements is nearly a regular equilateral triangle;

3. The input data is simple and requires only an initial data set for generating the first rough mesh.

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