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MODELLING OF ELECTROMAGNETIC FIELD IN FERROMAGNETIC MATERIALS

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Abstract

The paper presents some results and ideas in the field of electromagnetic field calculation in the case of ferromagnetic materials and eddy currents. The hysteresis phenomenon is modelled by the classical Preisach model and the field is calculated with finite element method in time domain. A convergent and physically good scheme is given also for the solution.

Keywords: finite element method, Preisach model, convergency.

Introduction

A lot of methods are developed for eddy current calculation by many authors. So, nowadays it is time to solve problems with nonlinear materials. The aim of our work was to develop a Preisach-type model and apply it in the framework of the finite element method for a more accurate determination of electromagnetic field in ferromagnetic materials. To get an efficient method the theoretical work was made together with some practical development such as

- the effect of some changes in the geometry and in the parameters of magnetic material for the torque in hysteresis motors,
- railway road impedance characteristics in the case of different premagnetization current and superposed upper harmonics,
- magnetic bearing problem – the field and force between two static magnets.

These problems were examined in two-dimension, and the space was closed and no magnetic field was supposed outside the arrangement. In the above examples the nonlinear constitutive equation causes some convergency problem during the solution. In the latest time many efforts have been done to solve this [1], [2]. Now we introduce our method to find a convergent solution.

1. Nonlinear Quasi-Static Electromagnetic Field

1.1 Governing Equations

Using the usual field vectors the Maxwell's equations for quasi-static electromagnetic field are the following [3]

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J}_s + \mathbf{J}_e \ , \\ \nabla \mathbf{B} &= 0 \ , \\ \nabla \times \mathbf{E}_e &= -\frac{\partial \mathbf{B}}{\partial t} \ , \end{aligned} \tag{1}$$

where $\mathbf{J}_e$ denotes an eddy current density component and $\mathbf{J}_s$ is a homogeneously distributed source current density component. Now the Gauss's law is not necessary and the displacement current is neglected. Let us consider the constitutive equations in the following forms

$$\begin{aligned} \mathbf{J}_e &= \sigma \mathbf{E}_e \ , \\ \mathbf{B} &= \mu_0 \mathbf{H} + \mathbf{M}(\mathbf{H}) \ , \end{aligned} \tag{2}$$

where $\mathbf{M}(\mathbf{H})$ is generated with the Preisach model. For the solution of the Maxwell's equations we can introduce the magnetic vector potential $\mathbf{A}$ and the scalar electric potential φ

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \ , \\ \mathbf{E}_e &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi \ . \end{aligned} \tag{3}$$

In the case of one-component magnetic vector potential in two-dimensional problems the divergence of $\mathbf{A}$ automatically vanishes, so

$$\nabla \mathbf{A} = 0 \ , \quad \nabla \varphi = 0 \ , \tag{4}$$

and the differential equation has the following form

$$\nabla \times \left(\frac{\nabla \times \mathbf{A}}{\mu} - \frac{\mathbf{M}}{\mu} \right) + \sigma \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_s \ . \tag{5}$$

Supposing no magnetic field outside the arrangement, the boundary condition for the magnetic vector potential $\mathbf{A}$ is a Dirichlet one on the external surface and its value can be selected to be zero

$$A(r_{\text{external}}) := 0 \ . \tag{6}$$

One can propose to use not the $\mathbf{A} - \Phi$ method, but the $\mathbf{T} - \Psi$ current vector potential and scalar magnetic potential method because in this case it is easy to force current and we have to calculate the flux intensity $\mathbf{B}$ from the field density $\mathbf{H}$. But in this case it is a great disadvantage that three times more variables are necessary in 2D and solving the gauge problem the equations cannot be separated. It is true that (5) gives only a voltage force, but KONRAD [4] gave a solution for the current force substituting the source current once more back into the differential equation

$$\mathbf{J}_s = \frac{I + \int_{T_{\text{conductor}}} \sigma \frac{\partial \mathbf{A}}{\partial t} ds}{T_{\text{conductor}}} . \tag{7}$$

Of course this is a nonlinear problem also. The other advantage of the $\mathbf{T}-\Psi$ method, calculating $\mathbf{B}$ from $\mathbf{H}$, that is can be solved with a simple iteration at the $\mathbf{A}-\varphi$ method, too. Because of these we preferred the $\mathbf{A}-\varphi$ method.

1.2 Finite Element Realization

Introducing the energy related functional

$$W(\mathbf{A}) = \int\limits_{\Omega} \left(\nabla \times \mathbf{A} \left(\frac{\nabla \times \mathbf{A}}{\mu} - \frac{\mathbf{M}}{\mu} \right) + \mathbf{A}\sigma \frac{\partial \mathbf{A}}{\partial t} - \mathbf{J}_s \mathbf{A} \right) d\Omega , \tag{8}$$

the extremal value of it yields the solution of the differential equation. The solution has to be done in time domain, because it is not usual to transform the Preisach model into the frequency domain. In our finite element process triangular elements with linear shape function were used. The first variation of the functional results is a set of nonlinear algebraic equations

$$[K_1]\mathbf{A} + [K_2]\dot{\mathbf{A}} = G , \tag{9}$$

where the time-derivative in the $(k+1)$-th time step can be expressed by the help of the following form

$$\Theta \dot{\mathbf{A}}^{k+1} + (1 - \Theta)\dot{\mathbf{A}}^{k} = \frac{\mathbf{A}^{k+1} - \mathbf{A}^{k}}{dt} , \tag{10}$$

where Θ is a constant [5].

A double iteration method is necessary because of the nonlinearity and the time-dependence so it is worse to develop a very fast solver. Here we have to mention that using a current force with (7), it will result unusual finite element matrix, because there will be long rows also. This has to be taken into consideration at the solver.

2. Description of the Preisach Model

2.1 Modelling of Hysteresis

Only the quantum physics describes correctly the hysteresis phenomena, but it is not usable in the practice. There exist some classical physical models, too, but in this case the knowledge of the micro structure of the material and a very large computer work would be necessary to solve a practical engineering problem. It is much more comfortable to use mathematical models with macroscopic parameters. The most popular are [6] the analytical models, Hudgdon model, Langevin function, Stoner–Wolfarth model, Preisach model, and the Chua model. Nowadays as a result of several researches the Preisach model gives the largest flexibility and easy usage.

The Preisach model regards a piece of magnetic material as a collection of different elementary rectangular hysteresis loops. This collection determined by the Preisach diagram produces an ideal hysteresis shape and behaviour. It is illustrated in *Fig. 1* for a discretised case. After this by the help of a weight function, constants and feedback this ideal model can be fitted onto a real curve. (Moving model, accommodation). So the classical model is the following

$$M = k \iint_{H_a \geq H_b} p(H_a, H_b)\gamma(H_a, H_b)\, dH_a\, dH_b\ , \tag{11}$$

where $\gamma(H_a, H_b)$ means an elementary hysteresis operator, $p(H_a, H_b)$ a weight function and k a constant.

2.2 How to Use the Preisach Model?

If somewhere a hysteresis phenomenon exists, this model can be used because it is a mathematical one. (Only a little modification is necessary.) But now we consider only the field of electrical engineering.

In network theory we have to measure the connection between an input and an output signal of a complete instrument and install one very punctual Preisach model.

But in the field theory one has to use many Preisach models in the framework of some numerical field calculation method because we have to take into consideration the locally different magnetized state of the material. For the field calculation the finite element method offers itself, dividing the material into little pieces, so it is easy to declare Preisach models for every finite element. Enough little pieces are necessary for the exact

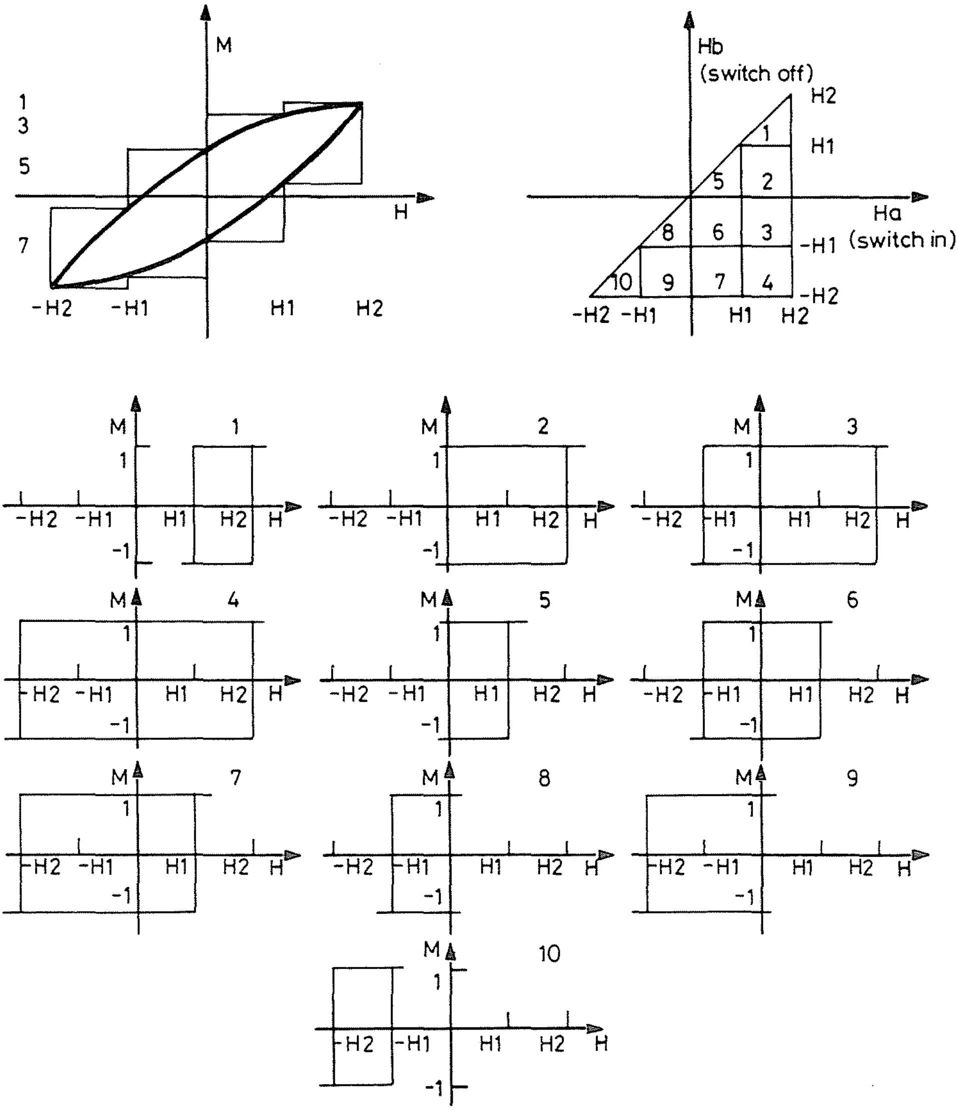

Fig. 1. A geometric interpretation to the Preisach model

determination of the field but enough large pieces to have all of the macroscopic behaviour of the material.

2.3 Connection between the Preisach Model and the Finite Element Method

The Preisach model is succeeded in 1D and the purpose of its developers was to describe the $\mathbf{B}-\mathbf{H}$ correlation exactly. But they usually have done their measurement on transformers, toroids and in the latest time on thin sheets. The model fitted onto this measurement is good only for network theory. In the field theory every finite element has an own Preisach model which has to have the global behaviour of the material without the effect of eddy current, shape and measures. These effects are taken into consideration in the field calculation.

Usually in the practice we can make only 2D or 3D field calculation models in the case of ferromagnetic material. Because of this a vector Preisach model is necessary. MAYERGOYZ [7] gave a vector model, but it is not easy to install and to use it in the framework of finite element method. We also have to take into consideration the anisotropy. ENOKIZONO [8] has done a lot of experiments and proposed the following connection

$$\begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix} . \tag{12}$$

In this field more research is necessary.

2.4 Simple Installation of Preisach Model

First of all we have to determine the weight function. The best way is to measure it. Sometimes only the characteristic points are important, like remanent flux density B_r, coercive field H_c and the field H_s and flux density B_s at saturation. In this case the Gaussian distribution function gives comfortability in many cases. Using a 2D Gaussian distribution function usually we do not consider any correlation. Knowing that most of the hysteresis are centrally symmetrical we use the h_c-h_m coordinate system. In this case

$$\begin{aligned} p(h_c, h_m) &= p(h_c)p(h_m) = \\ &= \frac{1}{2\pi\sigma_1^*\sigma_2^*} \exp\left(-\frac{(h_c - a_1^*)^2}{2\sigma_1^{*2}}\right) \exp\left(-\frac{(h_m - a_2^*)^2}{2\sigma_2^{*2}}\right) , \end{aligned} \tag{13}$$

where $a_2^* = 0$ and σ_1^*, σ_2^* are constants. In *Fig. 2* from geometry $I_1 = I_2 = I_3$ so $I_4 = I_5$ and because of this $a_1^* = H_c$. At least two parameters have to be verified together with an iteration. So our Preisach model is the following

$$M = k \int\int_{H_a \geq H_b} p(H_c, H_m)\gamma(H_c, H_m)\; dH_c\, dH_m . \tag{14}$$

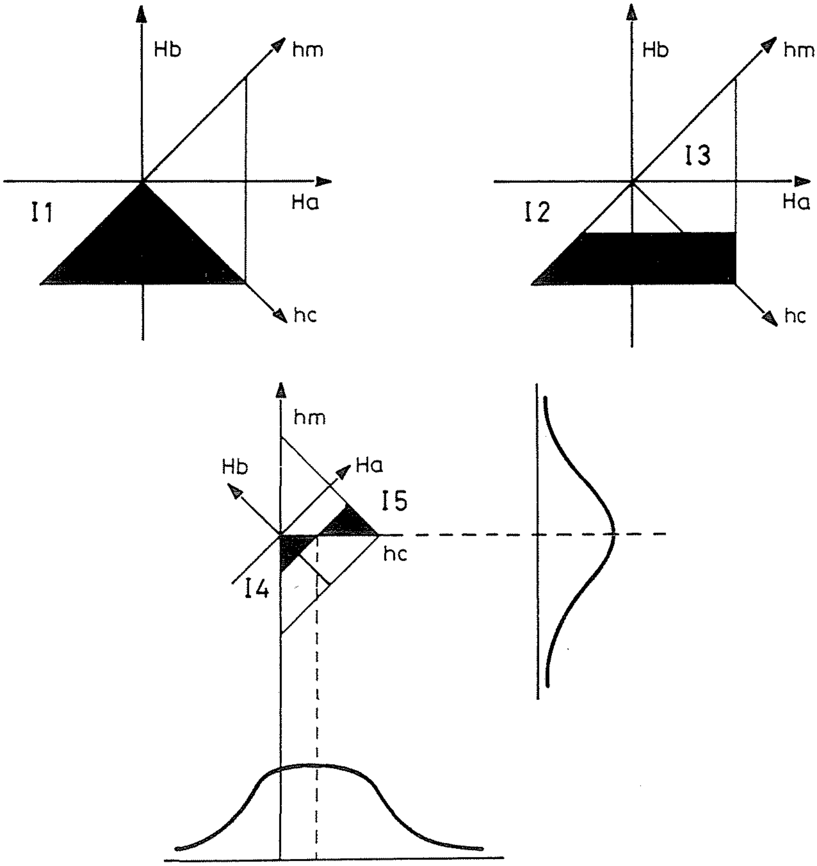

Fig. 2. Using a 2D Gaussian distribution function

3. Convergence

3.1 Test Magnetization Method

A developed scheme for the solution of the set of time-dependent nonlinear equations is given in *Fig. 3*. The external cycle is an iteration process in time domain and the inner one is a quasi-static solver for 'iron'. This is not the well-known single iteration method. Let us suppose that we know the exact solution for the magnetic vector potential $\mathbf{A}_k$, the magnetization vectors $\mathbf{M}_k$ and we know the magnetic pre-history of the ferromagnetic material for every finite element in the k-th time step. (On the scheme $\mathbf{M}$ denotes the magnetization vector and the magnetic pre-history, too.) Increasing the time t_{k+1} the excitation changes and we get a first approximation for $\mathbf{A}_{k+1}$ with the old magnetization $\mathbf{M}_k$. Now let us do the Preisach process on every finite element. To $\mathbf{A}_{k+1}^1$, $\mathbf{B}_{k+1}^1$ we find the new $\mathbf{M}_{k+1}^1, \mathbf{H}_{k+1}^1$. Substituting them back to the equations of course A_{k+1}^2 will change. But now we may not use the Preisach process again to get a new $\mathbf{M}_{k+1}^2$ to the latest $\mathbf{A}_{k+1}^2$ because this only would give a mathematical but physically bad solution. In our experience two possible mathematically good, but really bad solutions may exist depending on the circumstances:

i The solution point will be the same or very close to the real one but we did many mathematically induce minor loops. In this case the flux lines are good, only the magnetic history is useless. (*Fig. 4b*).

ii The solution point will not be the same, this is the totally bad solution. (*Fig. 4a*).

iii The solution may not depend on the mathematical method. In this case it can be checked by the hysteresis losses. In every moment the energy related functional has to be minimized but the magnetic material through the Preisach model forces the possible way to realize this. The real material locally changes from one state to another one everywhere between two time steps little enough. Because of this the magnetization state M has to be fixed for every finite element in the k-th time step and every test magnetization has to be done from this state $\mathbf{M}_k$ again and again. Of course the new $\mathbf{M}_{k+1}^1, \mathbf{M}_{k+1}^2, \mathbf{M}_{k+1}^3$ ... values have to be put back for the new A_{k+1}^2 A_{k+1}^3 ... solution. If the solution is under a good error limit then we have to do the real magnetization process and change $\mathbf{M}_k$ for $\mathbf{M}_{k+1}$. This can be seen in *Fig. 4c*.

3.2 Problems

Sometimes the convergence is very slow and there are problems with the error estimation too. This problem organizes from the constitutive equation, because the $\mathbf{B} - \mathbf{M}$ difference is very small. If the current is being forced out into a little skin depth there is no problem with the error estimation using an average square difference for the vector potential, but in the opposite case there is. The calculation of the total energy is not easy, either. In this case the changes in the length of flux lines can be used. Usually maximum 30 iterations give the result.

We have to know the magnetic history of every finite element. Because of this, between states enough little time step is necessary. In our case it was much less than the sampling needed for the Fast Fourier Transformation which helped the post-processing. This was checked by using a two-times smaller time step.

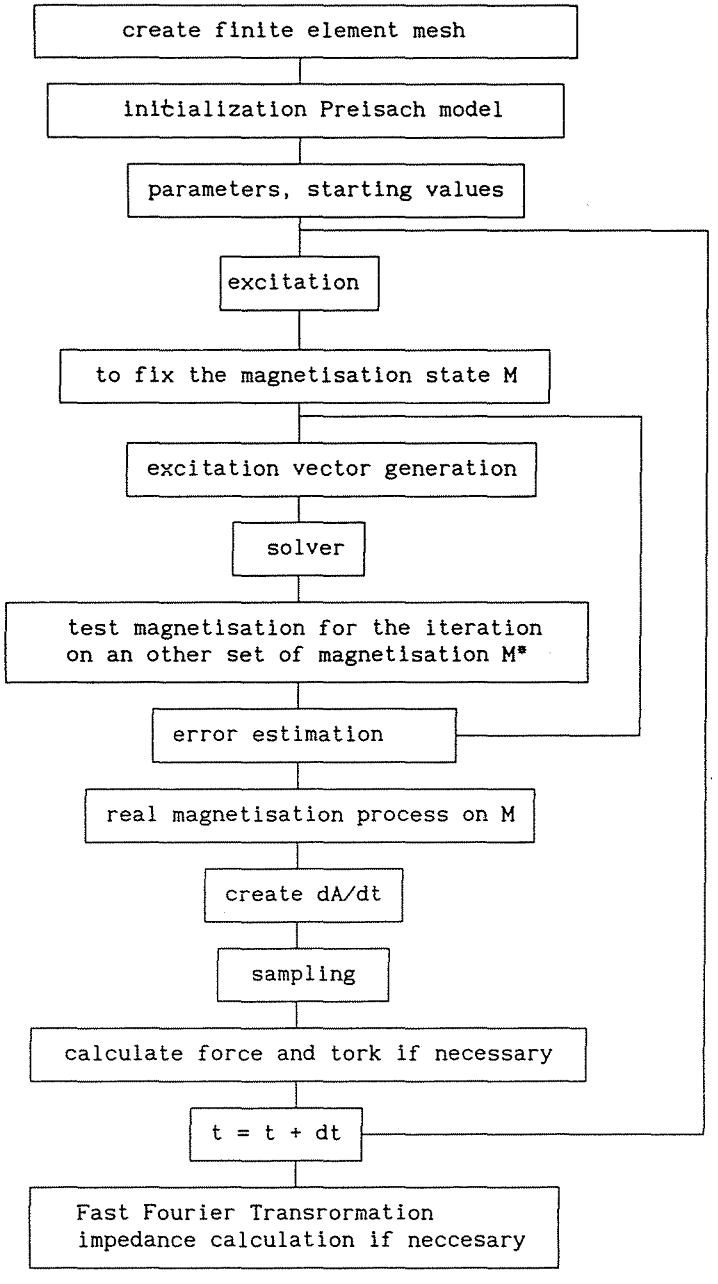

Fig. 3. Test magnetization method

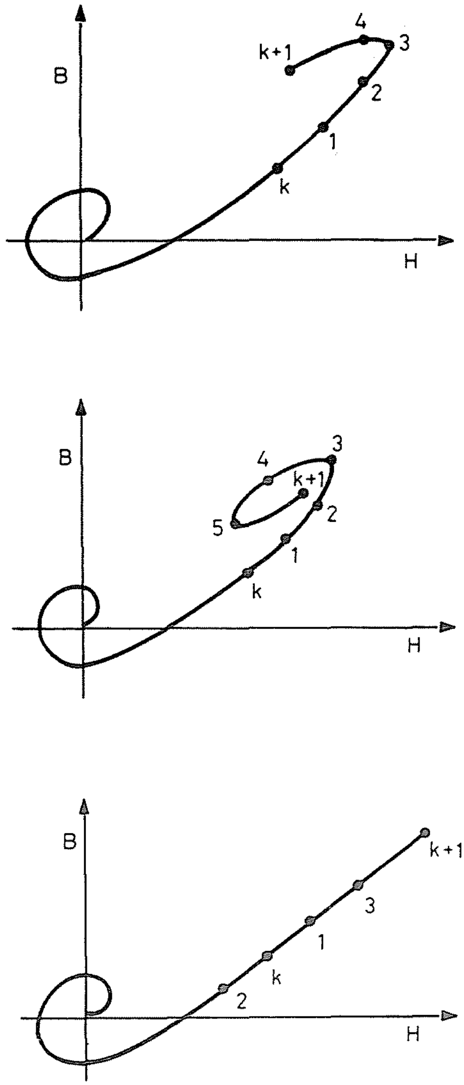

Fig. 4. a, b, c. The effect of the different iteration processes

4. Post Processing

- In the ferromagnetic material of electro-mechanical energy converters one part of the electrical energy will be heat, the other part of it mechanical energy. Because of this torque and force cannot be calculated from the changes of magnetic energy.

– The other problem is that the hysteresis losses can be introduced only for closed minor loops. So it is much better to use the Maxwell's stress tensor (or the $\mathbf{H} \times \mathbf{B}$ cross product). It says that torque and force will be there where a gradient is in the permeability. In ferromagnetic material it is almost in the whole volume because of the different magnetized state. This is the reason why we had to make effort to optimize our hysteresis motor.

5. Conclusion

i The ferromagnetic material has to be divided into little pieces which behaviour is macroscopic and can be modelled by the Preisach model.
ii At the installation of the model, one has to neglect the effect of the eddy currents, the shape and measures, because these are in the field calculation.
iii In the solution a test magnetization process is necessary to avoid the physically bad solution.
iv For the exact solution a good finite element mesh, an enough small time step and a good discretisation in Preisach model is necessary.

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