PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 38, NO. 4, PP. 323–334 (1994)

MAGNETIC FIELD ANALYSIS FOR ANISOTROPIC MATERIALS BY USING MAGNETIC RELUCTIVITY TENSOR METHOD

M. ENOKIZONO and S. KAWANO

Dept. of Electrical and Electronic Engineering
Faculty of Engineering
Oita University
700 Dannoharu, Oita 870–11, Japan

Received: Dec. 16, 1994

Abstract

This paper deals with the numerical expression of magnetic properties on the silicon steel sheet. In conventional magnetic field analysis, the magnetic properties of the arbitrary direction have been modelled by magnetic properties of easy axis and its perpendicular direction. Such kind of modelling of the material cannot exactly express the phase difference between the magnetic flux density **B** and the magnetic field intensity **H**. Therefore we have measured the relationship between **B** and **H** as vector quantities under alternating and rotating flux condition by using two-dimensional magnetic measuring apparatus. Its properties are expressed by the magnetic reluctivity tensor, which is calculated by measured relationship between **B** and **H**.

Keywords: two-dimensional magnetic property, reluctivity tensor, anisotropy

1. Introduction

Studying soft magnetic materials that are commonly used in rotating machines and three-phase transformers is very important for saving energy. For overcoming the problem, the finite element method has been applied to the magnetic field analysis. The magnetic reluctivities were used as certain input data for the analysis. Though it is well known that **B** and **H** are vector values, the magnetic properties of material are measured assuming that the directions of **B** and **H** are parallel even if the directions are in fact different. Therefore, in the conventional magnetic field analysis, the magnetic reluctivity of arbitrary direction is modelled by using magnetic reluctivities of the easy axis and its perpendicular direction. This expression is partially effective for analyzing alternating magnetic field problems with weak anisotropy. However, it is not accurate in strong anisotropic problems and rotating magnetic field problems. Furthermore, the magnetic properties in the case of the rotating magnetic field excitation are quite different from those of alternating magnetic field. It is an important problem how to express the properties. In order to use appropriate magnetic properties to the arbitrary alternating and rotating magnetic field analysis, we have measured the **B** and **H** values as vector quantities using two-dimensional magnetic measuring apparatus [1] [2].

In this paper, the two-dimensional magnetic properties were expressed through tensor magnetic reluctivity that was used in the magnetic field analysis with the finite element method. As a result, it was shown that our new method was applicable to anisotropic magnetic materials.

2. Reluctivity Tensor

In magnetic field analysis, **B** and **H** are related by reluctivity as $\mathbf{H} = \nu\mathbf{B}$. It is supposed that reluctivity ν becomes tensor in anisotropic materials. In conventional expression off-diagonal terms have been usually assumed to be zero and diagonal terms ν_x and ν_y have only depended on B_x and B_y, respectively. Therefore, determination of tensor reluctivity was impossible by using conventional measuring method and **B** and **H** were measured as scalar values. There are other things to note. Though alternating field by one-dimensional excitation was applied, magnetic flux density produced the distorted rotational flux [3]. Therefore, to obtain the complete alternating magnetic flux condition we must control the applied magnetic field by two-dimensional excitation. Differences between one and two dimensional excitation are large. It is necessary to control and apply the rotating exciting field for complete alternating flux. And also, we can obtain the desired rotating flux by means of the control of wave form. Since the characteristics of the alternating and the rotating flux are quite different, reluctivity tensor was expressed individually.

2.1 Alternating Flux Condition

In the case of alternating flux, off-diagonal terms of reluctivity tensor are equal to zero because these terms produce a rotational flux. As shown in *Fig. 1*, **B** vector is not parallel to **H** vector. We must define reluctivity as vector quantity including phase angle. It depends on the magnetic flux density and the inclination angle as shown in *Eq.* (2). Accordingly, it can be defined with *Eq.* (3).

$$\begin{pmatrix} H_x \\ H_y \end{pmatrix} = \begin{bmatrix} \nu_{xx} & 0 \\ 0 & \nu_{yy} \end{bmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} , \tag{1}$$

$$\nu_{xx} = f\left(B^2, \frac{B_y}{B_x}\right), \quad \nu_{yy} = f\left(B^2, \frac{B_y}{B_x}\right) , \tag{2}$$

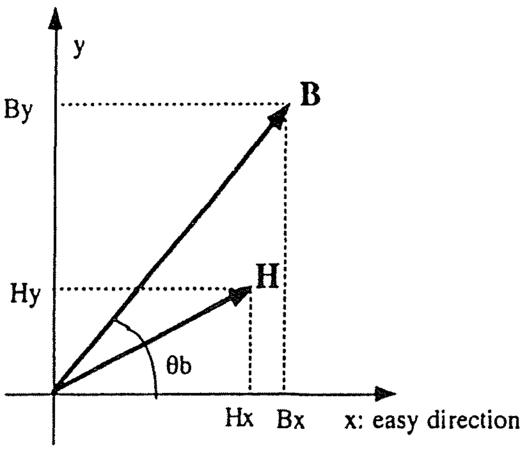

Fig. 1. Relationship between **B** and **H**

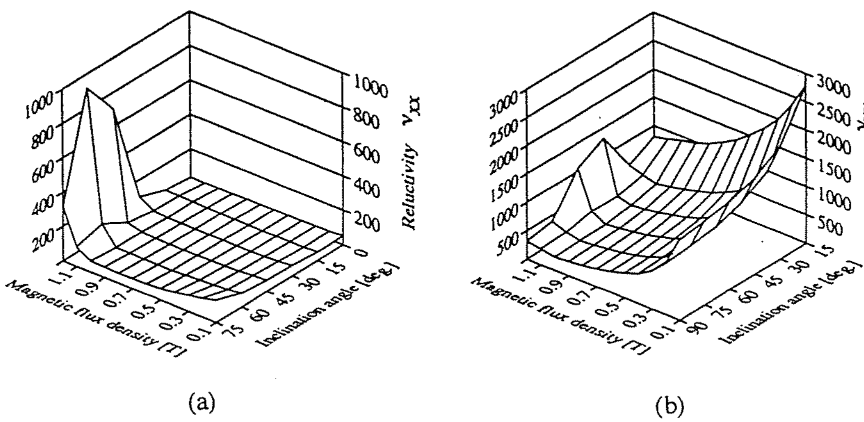

Fig. 2. The relationship between the magnetic reluctivity of the grain oriented sheet (23ZDKH), the magnetic flux density and the inclination angle, (a): ν_{xx}, (b) ν_{yy}

$$\nu_{xx} = \frac{H_x}{B_x} \,, \quad \nu_{yy} = \frac{H_y}{B_y} \,. \tag{3}$$

Since the ratio of components of flux density (B_y/B_x) is proportional to the inclination angle, we apply the ratio to this expression. The definition of this reluctivity is quite different from the conventional one because these values are measured under the control of complete sinusoidal flux condition by two-dimensional excitation and can express the magnetic properties of arbitrary direction. *Fig. 2* shows the relationship between the magnetic reluctivity of the grain-oriented steel sheet (23ZDKH90 produced by NSC, 0.23 mm thickness), the magnetic flux density and the inclination angle.

2.2 Rotating Flux Condition

Because of phase difference between **B** and **H** caused by rotational hysteresis, the reluctivity tensor becomes to be the full tensor. Therefore it is represented as:

$$\begin{pmatrix} H_x \\ H_y \end{pmatrix} = \begin{bmatrix} \nu_{xx} & \nu_{xy} \\ \nu_{yx} & \nu_{yy} \end{bmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} . \tag{4}$$

The elements of the tensor have been assumed to be constants in a single loop [4]. In this paper, this expression extended to high exciting condition was applied. Relationship between **B** and **H** has not been expressed well under this condition. In this case, the wave form of the measured H is affected by the third and the fifth higher harmonics as shown in *Fig. 3*. H_x and H_y including third harmonics can be written as follows:

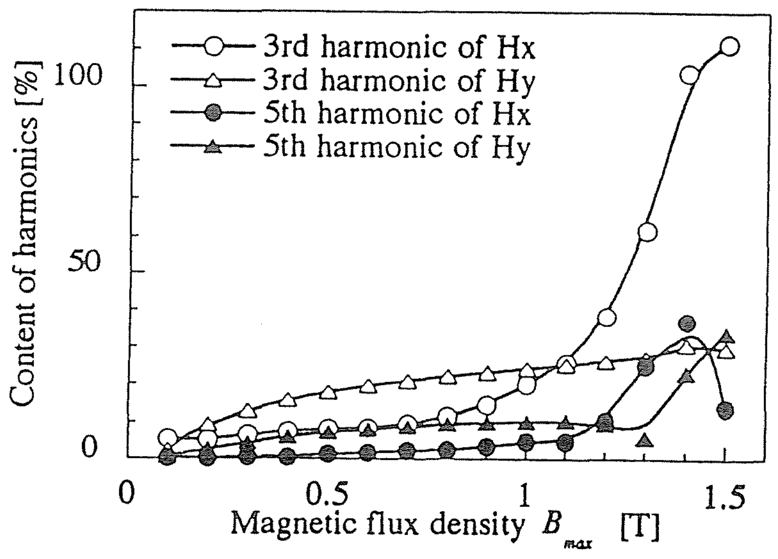

Fig. 3. Content of higher harmonics on H wave (H30, axis ratio: 1.0)

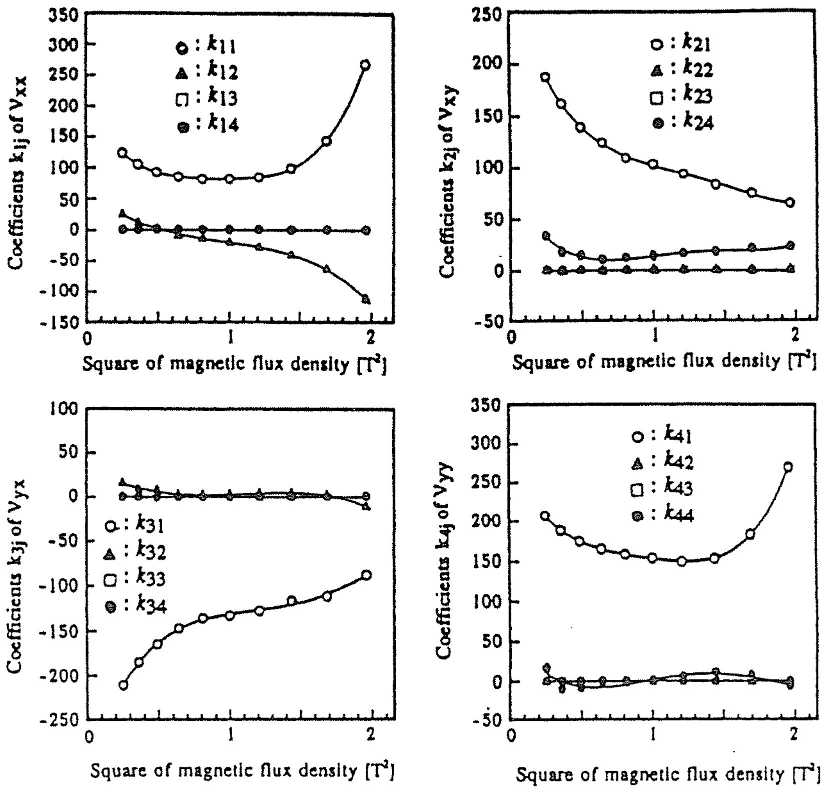

Fig. 4. Coefficient k_{ij} of the reluctivity tensor of non-oriented sheet (H30, axis ratio: 1.0)

$$
\begin{aligned}
H_x &= A_1 \cos(\omega t + \alpha_1) + A_2 \cos(3\omega t + \alpha_2) \\
H_y &= B_1 \cos(\omega t + \beta_1) + B_2 \cos(3\omega t + \beta_2) \ ,
\end{aligned} \tag{5}
$$

where $A_1, A_2, B_1, B_2, \alpha_1, \alpha_2, \beta_1$ and β_2 are constants obtained from the measured results. Thus, the magnetic reluctivity tensor considering the third higher harmonics is expressed as follows:

$$
\begin{pmatrix} \nu_{xx} \\ \nu_{xy} \\ \nu_{yx} \\ \nu_{yy} \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix} \begin{pmatrix} 1 \\ B_x^2 \\ B_x B_y \\ B_y^2 \end{pmatrix} , \tag{6}
$$

where k_{ij} is the coefficient matrix calculated from the measured result. *Fig. 4* shows coefficient k_{ij} of reluctivity tensor of the non-oriented sheet

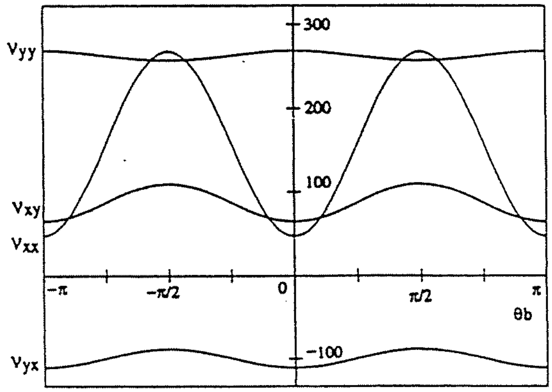

Fig. 5. Variation of the reluctivity tensor with the direction of **B** (H30, axis ratio: 1.0, $\mathbf{B}_{\max}$: 1.4 [T])

(H30 produced by NSC, 0.5 mm in thickness). *Fig. 5* demonstrates the variation of the reluctivity tensor of the non-oriented silicon steel sheet with the direction of **B**. The tensor value depends on the direction of **B**.

3. Formulation for Magnetic Analysis

3.1 Alternating Flux Condition

The two-dimensional governing equation including two-dimensional magnetic property is as follows:

$$\frac{\partial}{\partial x}\left(\nu_{yy}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{xx}\frac{\partial A}{\partial y}\right)=-J_0 \ , \tag{7}$$

where A is the magnetic vector potential, J_0 is the source current density and ν is a function of B^2 and (B_y/B_x). The functional for (7) is as follows:

$$\chi=\int\int\left(\int_0^B H\cdot dB\right)dxdy-\int\int J_0 A dxdy \ . \tag{8}$$

Applying the Newton–Raphson method to the non-linear analysis:

$$\begin{aligned}\frac{\partial^2\chi}{\partial A_{ie}\partial A_{je}} = \int\int\Bigg\{&\left(\nu_{xx}\frac{\partial N_{ie}}{\partial y}\frac{\partial N_{je}}{\partial y}+\nu_{yy}\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{je}}{\partial x}\right)\\ &+\sum_{k=1}^{3}\left(\frac{\partial \nu_{xx}}{\partial A_{je}}\frac{\partial N_{ie}}{\partial y}\frac{\partial N_{ke}}{\partial y}+\frac{\partial \nu_{yy}}{\partial A_{je}}\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{ke}}{\partial x}\right)A_{ke}-\\ &-\frac{\partial}{\partial A_{je}}(J_0 N_{ie})\Bigg\}\,dxdy\;.\end{aligned}\tag{9}$$

Here,

$$\begin{aligned}\frac{\partial \nu_{xx}}{\partial A_{je}} &= \frac{\partial \nu_{xx}}{\partial B^2}\frac{\partial B^2}{\partial A_{je}}+\frac{\partial \nu_{xx}}{\partial(B_y/B_x)}\frac{\partial(B_y/B_x)}{\partial A_{je}}\,,\\ \frac{\partial \nu_{yy}}{\partial A_{je}} &= \frac{\partial \nu_{yy}}{\partial B^2}\frac{\partial B^2}{\partial A_{je}}+\frac{\partial \nu_{yy}}{\partial(B_y/B_x)}\frac{\partial(B_y/B_x)}{\partial A_{je}}\,.\end{aligned}\tag{10}$$

From (9) and (10) we can obtain the following equation:

$$\begin{aligned}\frac{\partial^2\chi^{(e)}}{\partial A_{ie}\partial A_{je}} = \int\int\Bigg\{&\left(\nu_{yy}\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{je}}{\partial x}+\nu_{xx}\frac{\partial N_{ie}}{\partial y}\frac{\partial N_{je}}{\partial y}\right)\\ &+2\frac{\partial \nu_y}{\partial B^2}\sum_{l=1}^{3}\left(\frac{\partial N_{je}}{\partial x}\frac{\partial N_{le}}{\partial x}+\frac{\partial N_{je}}{\partial y}\frac{\partial N_{le}}{\partial y}\right)A_{le}\sum_{l=1}^{3}\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{le}}{\partial x}A_{le}\\ &+\frac{\partial \nu_y}{\partial\left(\frac{B_y}{B_x}\right)}\frac{\sum_{l=1}^{3}\left(\frac{\partial N_{je}}{\partial y}\frac{\partial N_{le}}{\partial x}-\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{le}}{\partial y}\right)A_{le}}{\left(\sum_{l=1}^{3}\frac{\partial N_{le}}{\partial x}A_{le}\right)^2}\sum_{l=1}^{3}\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{le}}{\partial x}A_{le}\\ &+2\frac{\partial \nu_x}{\partial B^2}\sum_{l=1}^{3}\left(\frac{\partial N_{je}}{\partial x}\frac{\partial N_{le}}{\partial x}+\frac{\partial N_{je}}{\partial y}\frac{\partial N_{le}}{\partial y}\right)A_{le}\sum_{l=1}^{3}\frac{\partial N_{ie}}{\partial y}\frac{\partial N_{le}}{\partial y}A_{le}\\ &+\frac{\partial \nu_x}{\partial\left(\frac{B_y}{B_x}\right)}\frac{\sum_{l=1}^{3}\left(\frac{\partial N_{je}}{\partial y}\frac{\partial N_{le}}{\partial x}-\frac{\partial N_{ie}}{\partial x}\frac{\partial N_{le}}{\partial y}\right)A_{le}}{\left(\sum_{l=1}^{3}\frac{\partial N_{le}}{\partial x}A_{le}\right)^2}\sum_{l=1}^{3}\frac{\partial N_{ie}}{\partial y}\frac{\partial N_{le}}{\partial y}A_{le}\Bigg\}\,dxdy\;.\end{aligned}\tag{11}$$

Eq. (8) is minimized by the aid of the Newton–Raphson method.

3.2 Rotating Flux Condition

If the magnetic reluctivity is assumed to be a full tensor, the fundamental equation in the two-dimensional magnetic field problems is as follows:

$$\frac{\partial}{\partial x}\left(\nu_{yy}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_{xx}\frac{\partial A}{\partial y}\right)-\frac{\partial}{\partial y}\left(\nu_{xy}\frac{\partial A}{\partial x}\right)-\frac{\partial}{\partial x}\left(\nu_{yx}\frac{\partial A}{\partial y}\right)=-J_0\,. \tag{12}$$

The required energy function is represented by (8). Therefore, we obtain the following equation with tensor ν

$$\frac{\partial \chi}{\partial A_{ie}} = \int\int \left(\nu_{xx} \frac{\partial A^{(e)}}{\partial y} \frac{\partial}{\partial A_{ie}} \frac{\partial A^{(e)}}{\partial y} - \nu_{xy} \frac{\partial A^{(e)}}{\partial x} \frac{\partial}{\partial A_{ie}} \frac{\partial A^{(e)}}{\partial y} \right.$$

$$\left. -\nu_{yx} \frac{\partial A^{(e)}}{\partial y} \frac{\partial}{\partial A_{ie}} \frac{\partial A^{(e)}}{\partial y} - \nu_{yy} \frac{\partial A^{(e)}}{\partial x} \frac{\partial}{\partial A_{ie}} \frac{\partial A^{(e)}}{\partial x} \right) dxdy - \int\int J_0 \frac{\partial A^{(e)}}{\partial A_{ie}} dxdy \ . \tag{13}$$

Eq. (14) is given for non-linear analysis with the Newton–Raphson method.

$$\begin{aligned} \frac{\partial^2 \chi^{(e)}}{\partial A_{ie} \partial A_{je}} = \int\int \Bigg\{ & \left(\nu_{xx} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{je}}{\partial y} + \nu_{yy} \frac{\partial N_{ie}}{\partial x} \frac{\partial N_{je}}{\partial x} \right. \\ & \left. - \nu_{xy} \frac{\partial N_{ie}}{\partial x} \frac{\partial N_{je}}{\partial y} - \nu_{yx} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{je}}{\partial x} \right) \\ & + \left(\frac{\partial \nu_{xx}}{B_x} \frac{\partial N_{je}}{\partial y} - \frac{\partial \nu_{xx}}{\partial B_y} \frac{\partial N_{je}}{\partial x} \right) \sum_{k=1}^{3} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{ke}}{\partial y} A_{ke} \\ & - \left(\frac{\partial \nu_{xy}}{B_x} \frac{\partial N_{je}}{\partial y} - \frac{\partial \nu_{xy}}{\partial B_y} \frac{\partial N_{je}}{\partial x} \right) \sum_{k=1}^{3} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{ke}}{\partial y} A_{ke} \\ & - \left(\frac{\partial \nu_{yx}}{B_x} \frac{\partial N_{je}}{\partial y} - \frac{\partial \nu_{yx}}{\partial B_y} \frac{\partial N_{je}}{\partial x} \right) \sum_{k=1}^{3} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{ke}}{\partial y} A_{ke} \\ & + \left(\frac{\partial \nu_{yy}}{B_x} \frac{\partial N_{je}}{\partial y} - \frac{\partial \nu_{yy}}{\partial B_y} \frac{\partial N_{je}}{\partial x} \right) \sum_{k=1}^{3} \frac{\partial N_{ie}}{\partial y} \frac{\partial N_{ke}}{\partial y} A_{ke} \Bigg\} dxdy \ . \end{aligned} \tag{14}$$

We can carry out the non-linear magnetic field analysis under rotating flux condition.

4. Results and Discussion

We applied the expression under alternating flux condition to the single-phase transformer core and analyzed the two-dimensional magnetic measuring apparatus model under rotating flux condition. The models of the single-phase transformer and the two-dimensional magnetic measuring apparatus are shown in *Figs. 6* and *7*, respectively.

In the case of the single-phase transformer, the leakage flux was assumed not to exist. *Fig. 8 (a)* shows the flux distribution and the distributions of **B** and **H** vectors by using conventional expression when the average value of **B** is equal to 1.7 [T]. The large phase difference between **B**

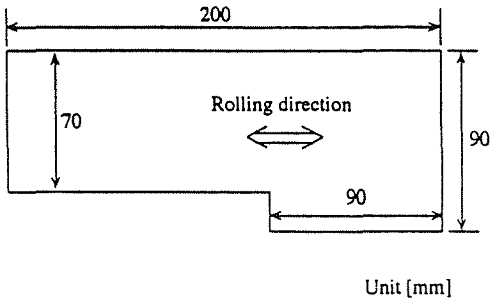

Fig. 6. The model of single-phase transformer core

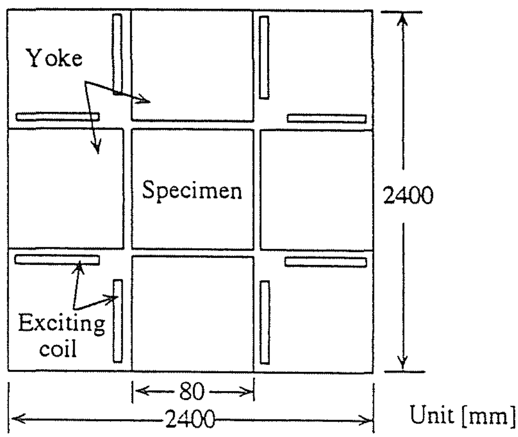

Fig. 7. The model of two-dimensional magnetic measuring apparatus

and **H** can be seen. The loci of **B** vector are shown in *Fig. 9, (a)* is result by using conventional expression, *(b)* is result by using our new expression. The difference between both expressions is very large, especially in the corner part of the core. Furthermore, the direction of **B** changes in high excitation condition in the corner part.

In the case of two-dimensional magnetic measuring apparatus, the magnetic flux density of the specimen in this model is nearly uniform, because there are air gaps between the specimen and each yoke. Therefore, since we have assumed that the magnetic properties on the internal parts of specimen are equal, the loci of **B** and **H**, and the phase differences between **B** and **H** in the mid point of the magnetic material are shown as calculated results. *Fig. 10* shows that calculated under rotating flux condition. In these figures, *(a)* is the measured result, *(b)* is the result

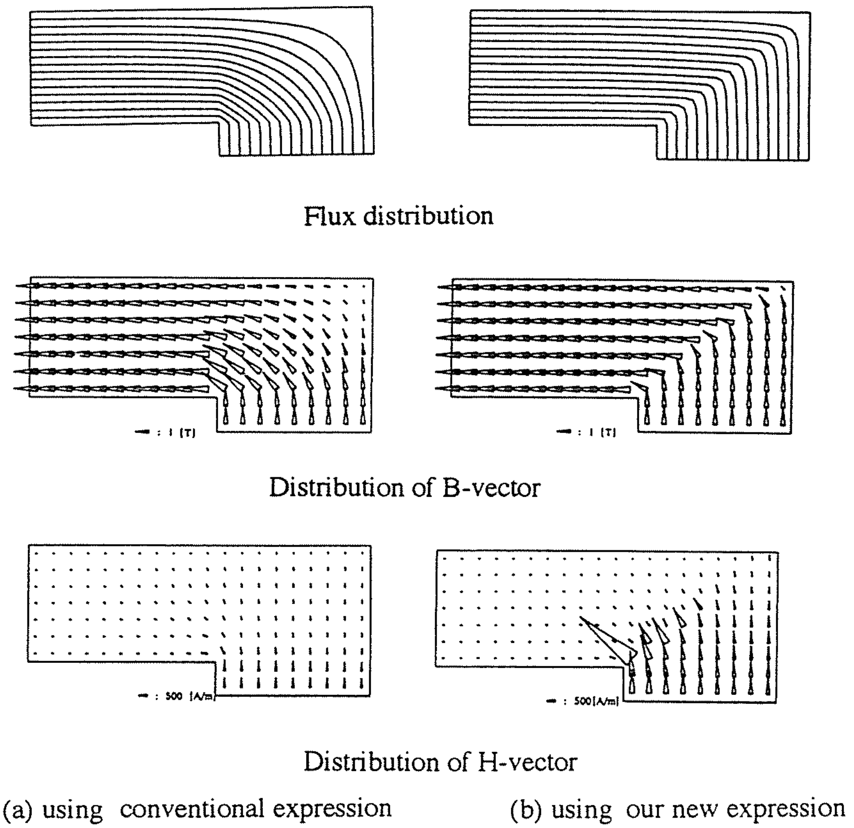

Fig. 8. Flux distribution and distributions of **B** and **H** vector (average value of **B**: 1.7 [T])

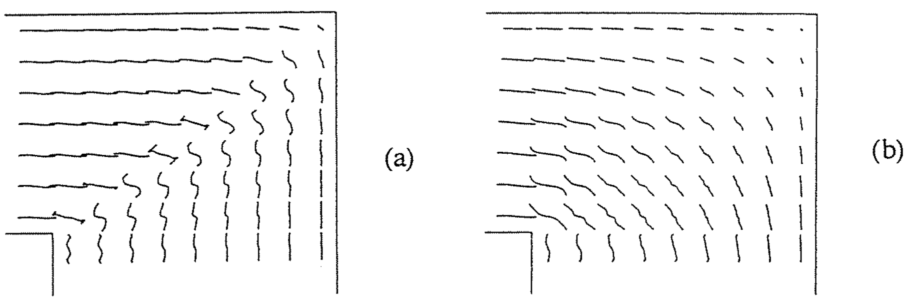

Fig. 9. Loci of **B** vector by using conventional and tensor method, (a) result by using tensor expression, (b) results by using conventional expression

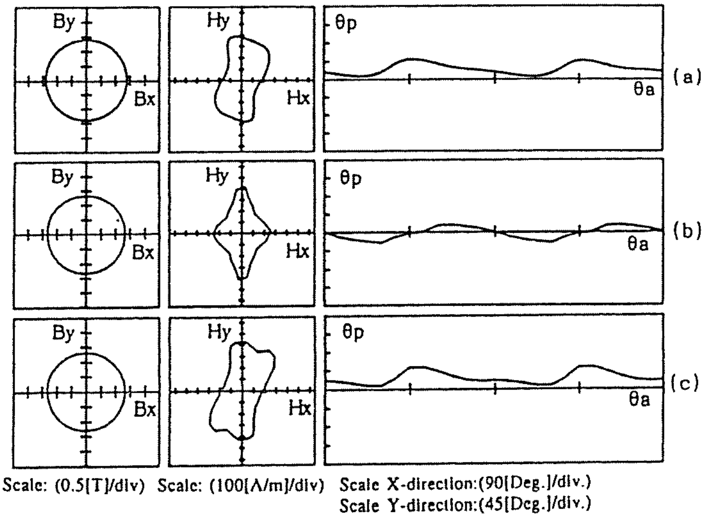

Fig. 10. Loci of **B** and **H** and the phase difference under rotating flux condition, (a) measured results, (b) results calculated by using conventional expression, (c) results calculated by using tensor expression

obtained by using the conventional analysis and *(c)* is the result obtained by using our new method. The differences between results *(a)* and *(b)* are very large as shown in *Fig. 10.* It is seen that phase difference between **B** and **H** is not exactly expressed by using conventional method. Result *(c)* is similar to the measured one *(a)* about the phase difference between **B** and **H**. It is shown that the results calculated by using tensor expression are in good agreement with the measured results.

5. Conclusion

In this paper, the modelling of the two-dimensional magnetic properties for the numerical non-linear analysis has been presented by using the magnetic reluctivity tensor. In rotating flux condition the higher harmonics were considered. In the example of this method, it was shown that the calculated results were in good agreement with the measured one. It can be said that the presented method is very useful to analyze the electrical machine and apparatus, the magnetic properties must be dealt with by the tensor form. Furthermore, we should calculate it with measured data which are obtained under the controlled pure alternating and rotating flux condition.

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