

# FEM CODE WITH PREISACH MODEL FOR RAIL IMPEDANCE

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## Abstract

This paper deals with the determination of the rail impedance by the help of field calculation taking into account the ferromagnetic materials. The Preisach model is applied for modeling the ferromagnetic nonlinearity in the framework of the finite element method. There is examined the effect of different magnetic parameters for the impedance and the effect of the premagnetisation current.

*Keywords:* magnetic hysteresis, Preisach model, FEM code, rail impedance.

## Introduction

It is important to know the impedance of railways for transient studies and for signaling circuits. The impedance changes by the variation of the frequency and the amplitude of the current because of the skin effect and the hysteresis phenomena in the ferromagnetic material of the railways.

Practically it is very difficult to measure the internal impedance. To measure the self impedance, the sum of internal and external impedance seems to be much more comfortable, because in this case the wall thickness of the insulation does not disturb, it can be disregarding.

A cylindrical aluminium tube, as a current return path around the rail conductor, was applied [1]. The field was determined for this arrangement. No magnetic field was supposed outside the aluminium tube. The magnetic field was examined only in the cross-section (*Fig. 1*) because the variation of the magnetic field was neglected in the direction of the conductor axis ( $\partial/\partial z = 0$ ). The variation of the electric field was supposed to change linearly along this axis, and this causes a 'two and a half dimension' problem.

In *Fig. 1* the internal region  $\Omega_1$  is supposed to be built up from ferromagnetic material with a hysteresis loop and the external region  $\Omega_2$  represents air with a permeability  $\mu_0$ . The internal region has  $\sigma$  conductivity. The excitation current is distributed homogeneously on the total surface of

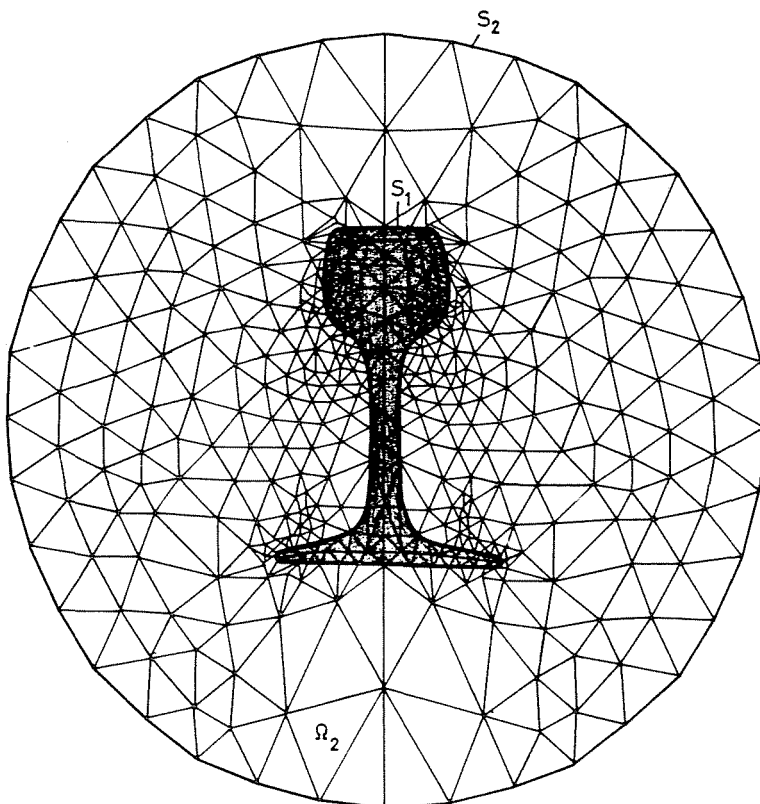


Fig. 1. The cross-section of the model with the finite element mesh

the  $\Omega_1$  region, so the  $n$  current and the eddy current component, calculated from the field produce the skin effect together.

The tangential component of the field intensity and the normal component of the flux density on the internal surface  $S_1$  are continuous. On the external surface the magnetic flux density has no normal component.

### The Model of the Electromagnetic Field

For the calculation the field and material equations are the following

$$\begin{aligned}
 \nabla \times \mathbf{H} &= \mathbf{J}_t = \mathbf{J}_s + \sigma \mathbf{E}, \\
 \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
 \nabla \cdot \mathbf{B} &= 0, \\
 \mathbf{B} &= \mu_0 \mathbf{H} + \mathbf{M}(\mathbf{H}),
 \end{aligned} \tag{1}$$

where  $\mathbf{H}$  is the magnetic field intensity,  $\mathbf{B}$  is the flux density,  $\mathbf{J}_t$  is the total current density,  $\mathbf{J}_s$  is the source current density,  $\mathbf{E}$  is the electric field intensity and  $\mathbf{M}$  is the magnetic polarization vector generated by the Preisach model in the internal region. It has zero value in the air.

To determine the field of the arrangement the one component magnetic vector potential  $\mathbf{A} = A(x, y)\mathbf{e}_z$  and the scalar electric potential  $\varphi$  were introduced.

$$\begin{aligned}\mathbf{B} &= \nabla \times \mathbf{A}, \\ \mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi,\end{aligned}\quad (2)$$

The field equations yield a time dependent parabolic differential equation for the magnetic vector potential

$$\nabla \times \left( \frac{\nabla \times \mathbf{A}}{\mu} - \frac{\mathbf{M}}{\mu} \right) = \mathbf{J}_s - \sigma \frac{\partial \mathbf{A}}{\partial t}. \quad (3)$$

The boundary condition for the magnetic vector potential  $\mathbf{A}$  is a Dirichlet boundary condition on the external surface and it can be selected zero and a Neumann boundary condition on the internal surface must be fulfilled.

The stationary point of the energy related functional has

$$I(\mathbf{A}) = \int_{\Omega} \left( \nabla \times \mathbf{A} \left( \frac{\nabla \times \mathbf{A}}{\mu} \right) - \frac{2\mathbf{M}}{\mu} - 2\mathbf{J}_s \mathbf{A} + \mathbf{A} \sigma \frac{\partial \mathbf{A}}{\partial t} \right) d\Omega \quad (4)$$

to be determined by the help of the variation method, so this functional yields the solution of the differential equation (3) getting for the vector potential [2].

In the finite element process triangular elements are used with a linear shape function. The first variation of the functional results a set of nonlinear time dependent parabolic differential equations

$$[K_1]\mathbf{A} + [K_2]\dot{\mathbf{A}} = F. \quad (5)$$

The Preisach model simulation of the hysteresis characteristic for the magnetic material is the source of the nonlinearity in the right side of Eq. (5). The value of the derivative of the vector potential and the magnetization vector can be determined by the help of iteration. One of the following can be selected.

$$\Theta \dot{\mathbf{A}}^{k+1} + (1 - \Theta) \dot{\mathbf{A}}^k = \frac{\mathbf{A}^{k+1} - \mathbf{A}^k}{\Delta t} \quad (6)$$

$\Theta = \pm 1$	}	Euler scheme
$\Theta = 2/3$		Galjerkin scheme
$\Theta = 1/2$		Crank–Nicholson scheme

The Galjerkin and the Crank–Nicholson scheme are unconditionally stable but time-steps have to be chosen not too large to eliminate the numerically induced oscillations into the solution. In the  $i$ -th time-step for the iterative solution of the nonlinear equations the initial value of the magnetization vector was supposed to be zero. The described boundary condition yields the first approximation of the vector potential by the solution of the set of the equations.

The process can be terminated when two successive solutions differ within the prescribed tolerance. The error estimation was checked by the aid of average value of the magnetic energy.

The large number of the equations and the iteration need a very fast solver.

### Model of Hysteresis

In order to describe the nonlinearity of the magnetic material the scalar Preisach model is applied. By the superposition of the  $x$  and  $y$  direction scalar models the magnetic vector field can be modeled. Gaussian distribution function was used as a weighting function. Using a suitable discretisation the integrated value of the weight function can be calculated and stored preliminary. Using a memory vector and with the knowledge of the previous magnetization  $\mathbf{M}_i$  only the difference in the magnetization has to be calculated.

### Fitting the Calculations to the Reality

1. Using DC current the  $R_{DC}$  can be measured. Dividing this value with the territory of the railway we can get the  $\sigma$  conductivity.

2. It is very hard to determine the main points of the magnetic hysteresis, so a characteristic from the publication [3] was used to determine this points, the remanent induction ( $\mathbf{B}_r$ ), coercitive field ( $\mathbf{H}_c$ ), the magnetic field ( $\mathbf{H}_s$ ) and the induction ( $\mathbf{B}_s$ ) at saturation and the suscebility at demagnetized state. From this parameters the Preisach model can be installed (*Figs. 2, 3*).

3. The calculation causes different results depending on the original magnetic state, so to make the simulation more simply the material before the simulation was in demagnetized state.

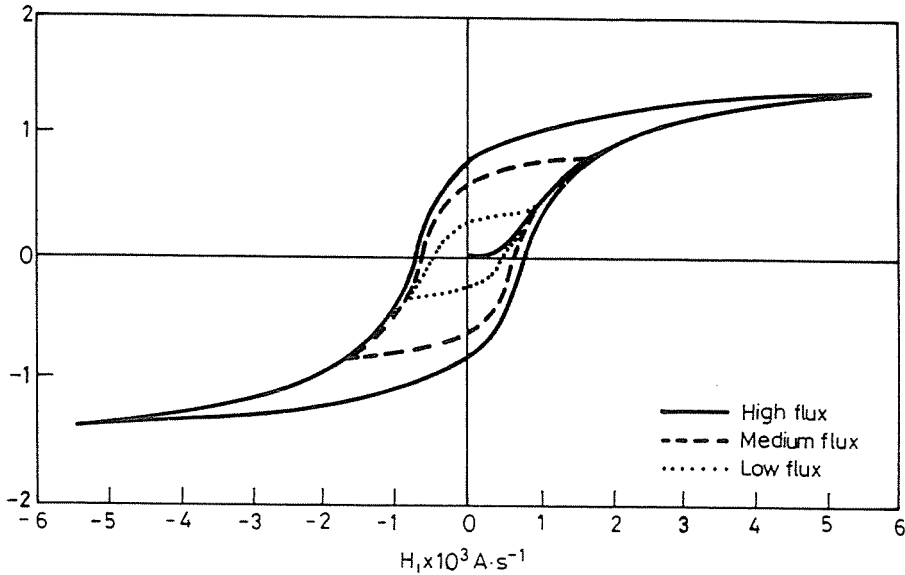


Fig. 2. The hysteresis curve from [3]

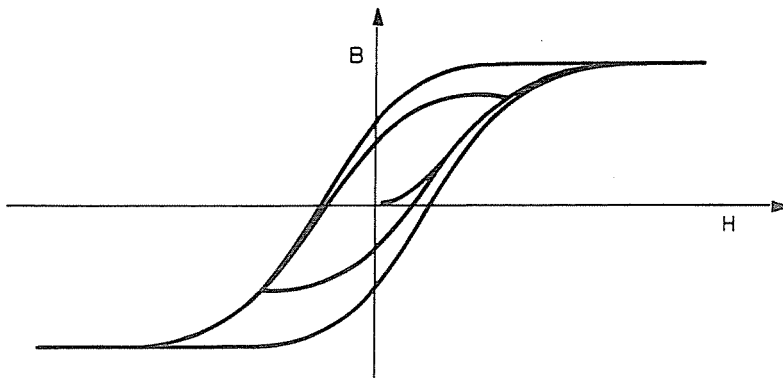


Fig. 3. The hysteresis curve generated by the Preisach model

4. Before the simulation the value of the resistance and the inductivity are not known. Turn a voltage on an R-L circuit some transient in current and voltage appear, so the calculation can be started after the second or third period, where the transients can be neglected. (Fig. 4)

### Determination of the Impedance

The voltage per unit length and the current per unit length are determined step by step in time (Fig. 5), then using the Fast Fourier Transformation

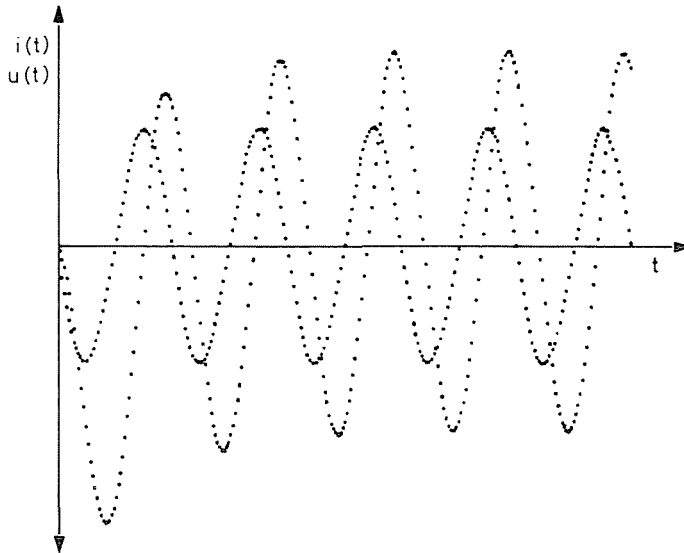


Fig. 4. The transients at the beginning of the calculation

we get the voltage ( $U(kj\omega)$ ) and the current ( $I(kj\omega)$ ) spectrums (Fig. 6) from the  $U(t)$ ,  $I(t)$  signals. The current density was calculated by the following way

$$\mathbf{J} = \mathbf{J}_f - \sigma \frac{\partial \mathbf{A}}{\partial t}. \quad (7)$$

The total current can be calculated by the integration for the whole surface of the railway

$$i(t) = \int_{\Omega_1} (\mathbf{J}_f - \sigma \frac{\partial \mathbf{A}}{\partial t}) d\Omega. \quad (8)$$

The voltage can be determined by the help of Eqs (9–11).

$$\oint_{l_0} E dl = -\frac{\partial \varphi}{\partial t} = -\int_s \frac{\partial \mathbf{B}}{\partial t} ds = -\oint_{l_0} \frac{\partial \mathbf{A}}{\partial t} dl, \quad (9)$$

$$l \left( \frac{\mathbf{J}_f}{\sigma} - \frac{\partial \mathbf{A}}{\partial t} (p_3) \right) - u(t) = -l \frac{\partial \mathbf{A}}{\partial t} (p_3), \quad (10)$$

$$u(t) = l \left( \frac{\mathbf{J}_f}{\sigma} - \frac{\partial \mathbf{A}}{\partial t} (p_3) \right) + l \frac{\partial \mathbf{A}}{\partial t} (p_3) = l \frac{\mathbf{J}_f}{\sigma}. \quad (11)$$

Dividing the voltage and the current the impedance can be determined

$$Z(kj\omega) = R_k + jX_k = \frac{U(kj\omega)}{I(kj\omega)}. \quad (12)$$

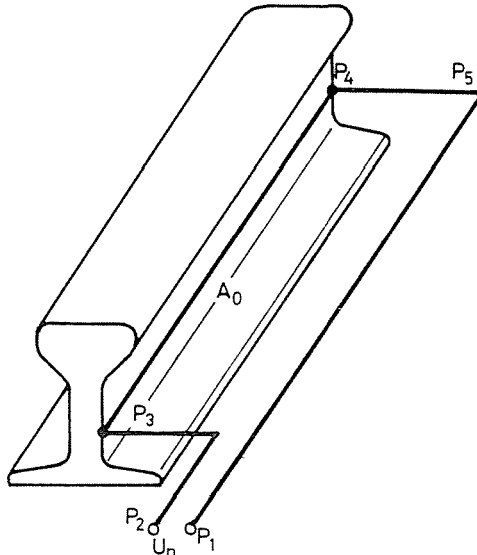


Fig. 5. The arrangement of the calculation

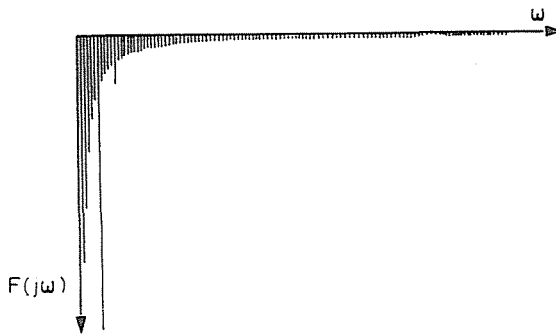


Fig. 6. The Fourier spectrum

## Results

There was applied sinusoidal excitation current and determined the field. The effects of the different parameters, the frequency, the permeability, the value of the excitation current and the effects of this parameters together can be followed during the simulation. The hysteresis energy losses and the Joule losses were determined in the case of sinusoidal  $n$  for a period.

The simulation was controlled by the help of a coaxial cable, which can be calculated manually and the results was compared to the determined one. The comparison shows no error in this case.

Comparing the calculated resistance to the measured data in case of the railway there was a difference at a low excitation current. The reason is that in case of very low field intensity values inside the iron the discretised Preisach model works with a great error.

Comparing the inductivity there was a difference at a low excitation frequency, because the finite element mesh was not fine enough.

### Conclusion

The Preisach model in the framework of the finite element method leads to several problems. First of all, a vector Preisach model is necessary because all of the known composition method cannot simulate well a real material. Secondly, for the more accurate parameter calculation a continuous rather than a discretised Preisach model is necessary.

The accuracy of the impedance depends on the fineness of the finite element mesh. It would be very convenient if a twice or three times greater mesh could be used, but the computer used could not calculate with it.

The simulation process is good for determining the impedance of any shape of a 'two and a half dimension' arrangement. The results can be determined in a very short time, due to the fast solver process.

### References

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