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THREE–DIMENSIONAL EDDY CURRENT ANALYSIS IN COLD CRUCIBLE USING AN INTEGRO-DIFFERENTIAL METHOD

H. TSUBOI*, T. MISAKI*, T. TAKE** and M. FUJITA**

* Department of Information Engineering
Fukuyama University
Gakuencho, Fukuyama 729–02
Japan
** Fuji Electric Corporate R & D, Ltd.
Ichihara, Chiba 290
Japan

Abstract

A current sheet approximation for eddy current distribution on the surface of the conducting bodies in cold crucibles is employed, and three-dimensional eddy current distributions in the cold crucible are calculated by an integro-differential method using electric vector potential. Furthermore, power loss, Lorentz force and levitation force of the molten metal are calculated from the obtained eddy current distributions.

Keywords: eddy current, cold crucible, integro-differential method.

1. Introduction

A cold crucible in which molten metal is levitated is a useful equipment for melting metal without contamination. Eddy currents are induced on the surface of the crucible and the molten metal by high frequency currents in the coils around the crucible. The molten metal is levitated by the Lorentz force of the eddy current. We have to know the distribution of eddy currents on the surfaces of the crucible and the molten metal for the design of cold crucible system.

Finite Element methods, integral equation methods and boundary element methods are used for the eddy current analysis. However, in the case of small penetration depth in comparison with the dimension of conducting bodies, large computer resources are required for these methods because of the fine mesh of volume or boundary elements for the active parts. We employed a current sheet approximation and an integro-differential method using electric vector potential for the calculation of eddy current distributions [1], [2]. In the integro-differential method, the dimension of boundary elements approximating current sheet does not depend on the penetration depth. The boundary elements are arranged to approximate the eddy current distribution on the surfaces. Furthermore, the region to be analysed is reduced by rotational and reflective symmetry [3]. Therefore, the required computer resources can be reduced remarkably.

In this paper, we present the integro-differential method for eddy current analysis in cold crucibles and calculated results of eddy current distributions, power loss, Lorentz force and levitation force for some cold crucible models.

2. Integro-Differential Method

When an electric vector potential $\mathbf{T}$ is introduced, the current density $\mathbf{J}$ is given by

$$\mathbf{J} = \nabla \times \mathbf{T} . \tag{1}$$

The governing equation of $\mathbf{T}$ with sinusoidal time dependence is obtained from the Faraday's law as follows:

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{T} \right) = -j\omega \mathbf{B} , \tag{2}$$

where σ is the conductivity and $\mathbf{B}$ is the magnetic flux density [4].

For thin conducting plates, the current density $\mathbf{J}$ is expressed by only the normal component of $\mathbf{T}$

$$\mathbf{J} = \nabla \times (\mathbf{n}T) , \tag{3}$$

where T is the normal component of $\mathbf{T}$ and $\mathbf{n}$ is the unit normal vector on the boundary surface.

The integro-differential equation for the normal component of the electric vector potential is obtained from (2) and (3) as follows:

$$\frac{1}{\sigma} \nabla^2 T = \frac{j\omega\mu_0 h}{4\pi} \iint\limits_S \frac{\{\nabla \times (\mathbf{n}T)\} \times \mathbf{r} \cdot \mathbf{n}}{\mathbf{r}^3} ds + j\omega \mathbf{B}_s \cdot \mathbf{n} , \tag{4}$$

where $\mathbf{r}$ is the vector from the source point to the field point, $\mathbf{B}_s$ is the magnetic flux density by the external source, h is the thickness of the active parts and S is the surface of the conducting bodies. The value of the thickness h is chosen as the same value of the penetration depth but the distribution of eddy current does not depend on the value h.

The power loss (the Joule heat) W, the Lorentz force f and the levitation force F are calculated by

$$W = \frac{h}{\sigma} \iint_S |\nabla \times (\mathbf{n}T)|^2 ds \ , \tag{5}$$

$$f = \{\nabla \times (\mathbf{n}T)\} \times \mathbf{B}^* \ , \tag{6}$$

$$F = \left(h \iint_S f ds \right)_z \ , \tag{7}$$

where * denotes complex conjugation.

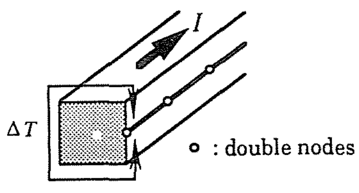

Fig. 1. Source current given by the potential difference using double nodes

When the total source current of the coil I is given, the source current can be introduced to the integro-differential method as an external source by the difference of the electric vector potential ΔT as follows:

$$\Delta T = \frac{I}{h} \ , \tag{8}$$

where the value of the total source current is constant. The distribution of the current density is determined by the potential difference and the induced eddy current. In the case of the coil having any cross-section, the potential difference is given by using double nodes as shown in *Fig. 1.*

3. Computation Results

A cold crucible model [4], [5] is shown in *Fig. 2.* The molten metal is approximated by a sphere whose conductivity is 2×10^7 S/m. The crucible, whose conductivity is 5×10^7 S/m, is divided into N segments. The frequency of the coil current is 3 kHz. The arrangement of triangular elements ($N = 24$) is shown in *Fig. 3*, where the number of triangular elements is 4512. The numbers of triangular elements for a segment of the crucible and the sphere are 144 and 1056, respectively. The region to be analysed can be reduced to one forty-eighth by rotational symmetry and reflective symmetry [3]. The number of unknowns is 71 and the computation time and memory storage are about 50 minutes and 800 kbytes, respectively, using SONY NEWS Workstation with R3000, 20MHz (17 MIPS).

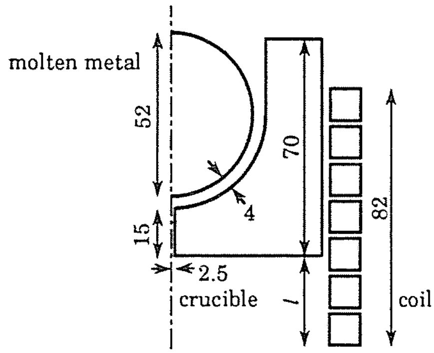

Fig. 2. Cold crucible model

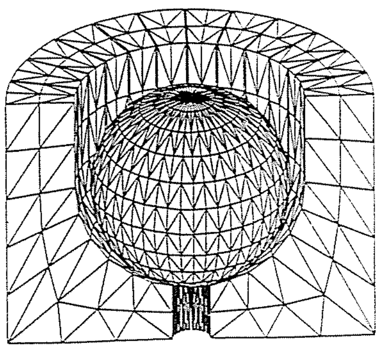

Fig. 3. Arrangement of triangular elements for the cold crucible model

Variations of the levitation force of the crucible model ($N = 24$) for the coil positions are shown in *Fig. 4*, where l is the distance between the coil bottom and the crucible bottom. The computed results agree with the experimental results. The influence of the number of segments on the

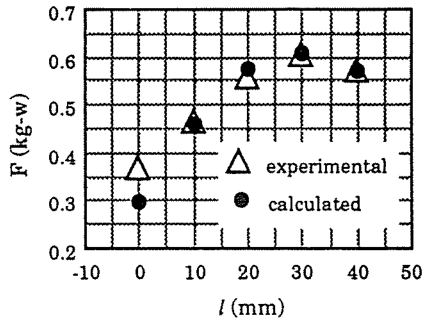

Fig. 4. Variation of the levitation force for the coil positions ($N = 24$)

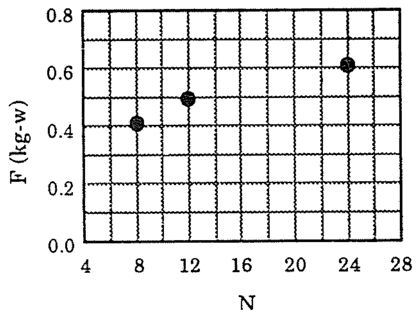

Fig. 5. Influence of the number of the crucible

levitation force ($N = 24$) is shown in *Fig. 5.* The levitation force increases with the number of segments N.

Fig. 6 shows the arrangement of the triangular elements for a practical cold crucible model ($N = 24$). The frequency of the currents applied to upper and lower coils is different. The input power and the frequency of each coil are controlled individually to stabilize the motion of the molten metal. The levitation force on the bottom surface of the molten metal is caused by the lower coil and the pressure on the upper and side surfaces is caused by the upper coil. The electromagnetic force and the power loss are calculated by superposing the forces and the power loss by the upper coil and the lower coil, respectively.

Figs. 7 and *8* show the equipotential lines of T and the equi-power-loss lines. The conditions of the calculations and the computation results of power loss are shown in *Table 1.* The conductivity of the crucible, coil and molten metal are 5.00×10^7, 5.00×10^7 and 1.37×10^7 S/m, respectively. Comparison between the calculated results and experimental

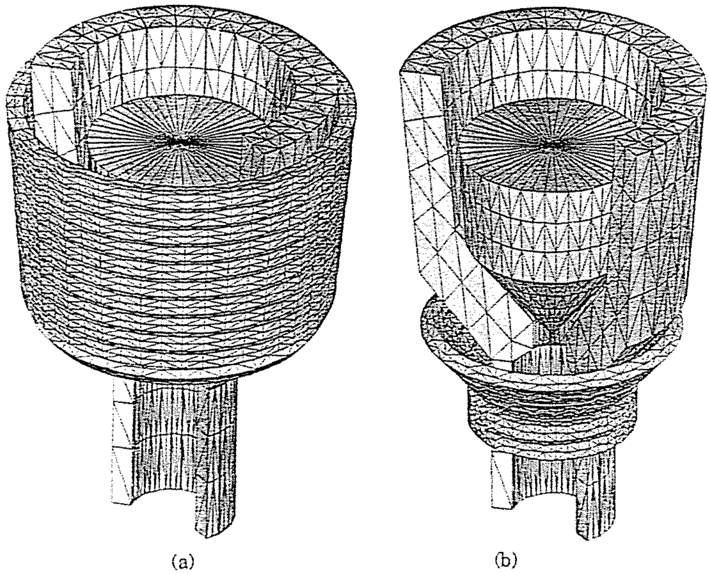

Fig. 6. Arrangement of triangular elements for a practical cold crucible model ($N = 24$), (a) for upper coil, (b) for lower coil

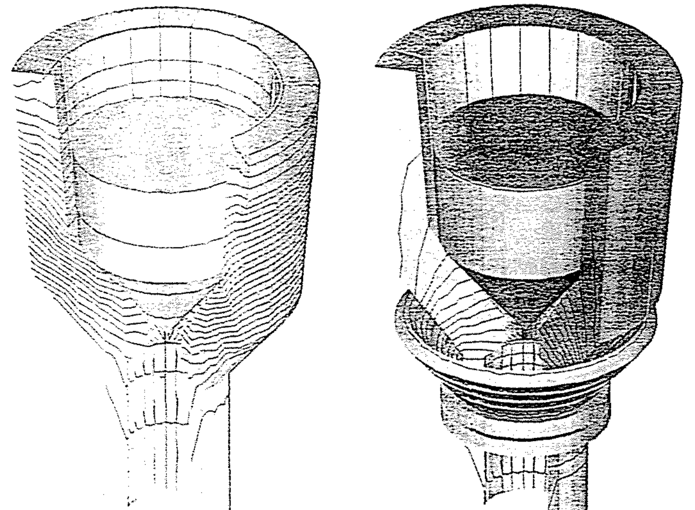

Fig. 7. Equipotential lines of T (real part), (a) by the upper coil, (b) by the lower coil

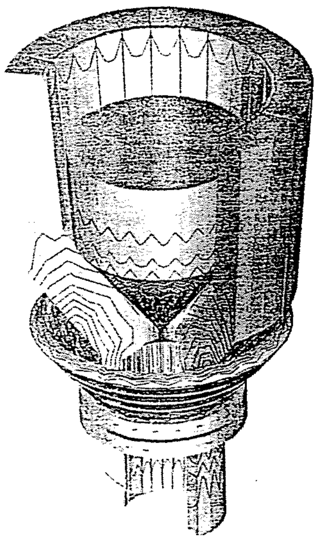

Fig. 8. Equi-power-loss-density lines by the lower coil

Table 1
Conditions and results of the calculations

	Upper coil	Lower coil
frequency (kHz)	50	3
coil current (AT)	1601	4494
number of elements	13104	9264
number of nodes	7730	5178
number of unknowns	269	179
computation time (minutes)	100	60
power loss (kW):		
crucible	63	50
molten metal	13	7
coil	9	12

results of power loss is shown in *Table 2.* The calculations were done using SPARC station 10.

Table 2
Comparison between calculated and experimental results

	Calculated	Experimental
power loss (kW):		
crucible and molten metal	133	117
coil	21	37
total	154	154

4. Conclusion

Three-dimensional eddy current distributions in cold crucibles were calculated by an integro-differential method using current sheet approximation for eddy current. Furthermore, the levitation force of the molten metal and the power loss were calculated and the calculated results agree with the experimental results.

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