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COMPUTATION OF THREE-DIMENSIONAL ELECTRIC FIELD PROBLEMS BY A BOUNDARY INTEGRAL METHOD AND ITS APPLICATION TO INSULATION DESIGN

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Abstract

This paper describes a boundary integral method for computation of three-dimensional electric field distribution. In the boundary integral method, the surfaces of electrodes and insulators are divided into curved surface elements because the use of the curved surface elements provides a good approximation of the contours of the electrodes and insulators. Furthermore, the boundary integral method is applied to optimum design of electrode and insulator shapes.

Keywords: boundary integral method, curved surface elements, optimization.

1. Introduction

In the design of high-voltage equipment, the computation of electric field is one of the important technologies. The computation of three-dimensional electric field distribution is often performed by boundary integral methods [1] because of the high accurate computation and the less input data preparation. In this paper, curved surface elements which provide good approximation of the boundary surface are used in a boundary integral method. The surface shape of each element is approximated by quadratic, cubic or fifth-degree function of coordinates. The distribution of surface charge on each element is approximated by a linear function of coordinates and discretized to the values on the vertices (nodes). After computing the charge distribution, potential and electric field at each computation point are calculated. The computation results of electric field distribution are presented in the SF_6 gas insulated cable.

On the other hand, optimization of electrode and insulator shapes is performed to reduce the maximum electric field strength or to obtain the desired electric field distribution by using iteration methods. Any one or more of the surfaces are modified by the iteration methods in order to obtain desired electric field strengths at any electrode and/or insulator surfaces.

The stress-ratio method [2], [3] is one of the simple iteration methods and used to reduce the maximum electric field strength on the surface of the electrode. The modification of the electrode surface is determined in proportion to the stress.

When applying the nonlinear programming to the iteration methods, an object function is defined [3], [4]. The object function which is used for modification of the surfaces takes a form of least squares. Then, the object function is evaluated by using integrals over the surfaces on which desired electric field distributions are given, and it is minimized iteratively by the Newton method or other iteration methods. The modification of the surfaces is performed by the iteration methods: the steepest descent method, the conjugate gradient method, the quasi-Newton method and the Gauss–Newton method.

The application of the stress-ratio method to insulation design is illustrated by a practical model.

2. Boundary Integral Method

The boundary integral method for electric field analysis is known as surface charge simulation method [1]. In the boundary integral method, the surfaces of electrodes and insulators are divided into numbers of boundary elements on which the distribution of appeared surface charge density are defined. And the discretized surface charge densities are solved by simultaneous equations.

2.1 Basic Equations

In the boundary integral method, the potential V_i and the electric field $\mathbf{E}_i$ induced at the computation point i are given by

$$V_i = \frac{1}{\varepsilon_0} \iint_S \sigma \phi ds \ , \tag{1}$$

$$\mathbf{E}_i = -\frac{1}{\varepsilon_0} \iint_S \sigma \nabla \phi ds \ , \tag{2}$$

where ϕ is the fundamental solution of the Laplace's equation, S is the total area of the boundary surface, σ is the surface charge density and ε_0 is the permittivity of free space.

The electric field on the boundary surface is given by

$$\mathbf{E}_i = \pm\frac{\mathbf{n}_i\sigma_i}{2\varepsilon_0} - \frac{1}{\varepsilon_0}\iint\limits_S \sigma\nabla\phi ds \qquad \text{(on the insulator surface)} \tag{3}$$

$$\mathbf{E}_i = \frac{\mathbf{n}_i\sigma_i}{2\varepsilon_0} \qquad \text{(on the conductor surface)} \tag{4}$$

where $\mathbf{n}_i$ is the unit normal vector at i.

The flux continuity condition at i on the insulator surface is given by

$$(\varepsilon_1\mathbf{E}_{1i} - \varepsilon_2\mathbf{E}_{2i}) \cdot \mathbf{n}_i = 0 , \tag{5}$$

where $\mathbf{E}_{1i}$ and $\mathbf{E}_{2i}$ are electric fields in the dielectric 1 and 2, respectively, ε_1 and ε_2 are the permittivities of dielectric.

When applying the potential condition given by *Eq.* (1) to the node on the electrode surface and the flux continuity condition given by *Eq.* (2) to the node on the insulator surface, the final simultaneous equations for the surface charge densities discretized at the nodes on the boundary surface are obtained by

$$[C]\{\sigma\} = \{\nu\} , \tag{6}$$

where $[C]$ is the coefficient matrix having N rows and N columns and $\{\sigma\}$ is the vector of unknown surface charge densities. And $\{\nu\}$ is given by

$$\{\nu\} = \{V_1V_2 \ldots V_i \ldots V_{Nc}0 \ldots 0\}^T , \tag{7}$$

where N_c is the number of nodes on the electrode surfaces.

2.2 Computation Accuracy

The computation accuracy of the electric field by the integral equation method depends on the following accuracies:

(*a*) approximation of boundary surfaces by boundary elements,
(*b*) approximation of surface charge density distribution,
(*c*) numerical integrations for setting up coefficient matrix of the final simultaneous equations.

In order to approximate curved surfaces of electrodes and insulators in high voltage equipment, curved surface elements which provide good approximation of the boundary surfaces are used. The surface shape of each element in triangular type case is approximated by a quadratic function or a fifth-degree function. In rectangular type case, it is replaced by a

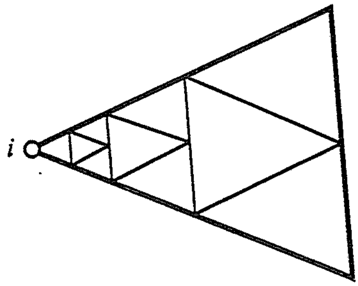


Fig. 1. Subdivision of a triangular element

cylindrical surface. Furthermore, in two-dimensional and axi-symmetric problems, the shape of line element is approximated by a cubic function.

The distribution of surface charge density which is assumed on each element is approximated by a linear function and discretized to the values on the vertices (nodes) of the elements.

When using the curved surface triangular elements, the integrals appeared in *Eqs.* (1) and (2) can be evaluated by the use of the Gaussian quadrature formula. However, in the case of the integrals including a singular point, the numerical integration formulae for polynomial approximation are not sufficient for the accuracy because of the singular behaviour of the kernel function. Therefore, an adaptive integration method is introduced and applied to reduce computation time and to obtain the high accuracy of the integration [5], [6]. For example, the triangular element shown in *Fig. 1* is subdivided into four small congruent triangles until the value of integration for a small triangle with a singular point is regarded as zero, and the Gaussian quadrature formula with seven sampling points is applied to each small triangle.

3. Optimization Methods

The optimization of electrode and insulator shapes is performed by iteration methods. In order to obtain the distribution of the desired electric field strength, any one or more of the surfaces are modified by using the iteration methods.

3.1 Stress-Ratio Method [2]

One of the simple iteration methods is the stress-ratio method. The displacement of electrode surface is determined in proportion to the stress (force). Therefore, the displacement vector $\mathbf{d}_i$ at the node i is given by

$$\mathbf{d}_i = c\mathbf{f}_i = -c\frac{\varepsilon}{2}E_i^2\mathbf{n}_i \ , \tag{8}$$

where c is the constant, f_i is the stress and $\mathbf{E}_i$ is the electric field strength.

The stress-ratio method is effective to reduce the maximum electric field strength. However, the desired electric field distribution cannot be obtained by the method. Furthermore, it is difficult to fix the nodes which are located at the end of the area to be modified.

3.2 Newton Method [3], [4]

When the Newton method is applied to optimization, an object function is introduced. The object function is evaluated by integrals over the surfaces on which desired electric field distributions are given, and it is minimized by iteration methods. The object function W is given by the form of the method of least squares as

$$W = \sum_{i=1}^{L}(E_i - E_{i0})^2 \ , \tag{9}$$

where E_i is the calculated electric field strength, E_{i0} is the desired electric field strength and L is the number of nodes at which desired electric field strength is given.

The design variables are displacements of the nodes on the electrode and insulator surfaces to be modified. In this method, the direction of the modification is fixed to that of the normal vector in order to reduce the number of the design variables. The design variable vector $\{x\}$ is given by

$$\{x\} = \{x_1 x_2 \ldots x_j \ldots x_M\}^T \ , \tag{10}$$

where x_j is the displacement of the node j and M is the number of the nodes to be modified.

We consider a problem to find $\{x\}$ which provides the minimum value of the object function. The design variable vector $\{x\}$ is modified by the following equation

$$\{x\}^{k+1} = \{x\}^k + \alpha^k\{d\}^k \ , \tag{11}$$

where k is the iteration step, α is determined by a linear-search method and $\{d\}$ is the modification of the design variable vector which is determined by the iteration methods: the steepest descent method, the conjugate gradient method, the quasi-Newton method and the Gauss–Newton method.

The iteration for the optimization is terminated by the convergence of the object function. The convergence is decided by the normalized value of the object function as follows:

$$\left| \frac{\sqrt{W^k/L} - \sqrt{W^{k-1}/L}}{(E_{i0})_{\max}} \right| < \delta \,, \tag{12}$$

where $(E_{i0})_{\max}$ is the maximum value of desired electric field strength. For the value of δ, 0.01 is normally chosen.

4. Computation Results

4.1 Electric Field Analysis of SF_6 Gas Insulated Cable [1]

Fig. 2 shows a single-core SF_6 gas insulated cable model. The relative permittivities of SF_6 gas and epoxy-resin spacer are 1 and 5, respectively. The arrangement of curved surface elements in the gas insulated cable

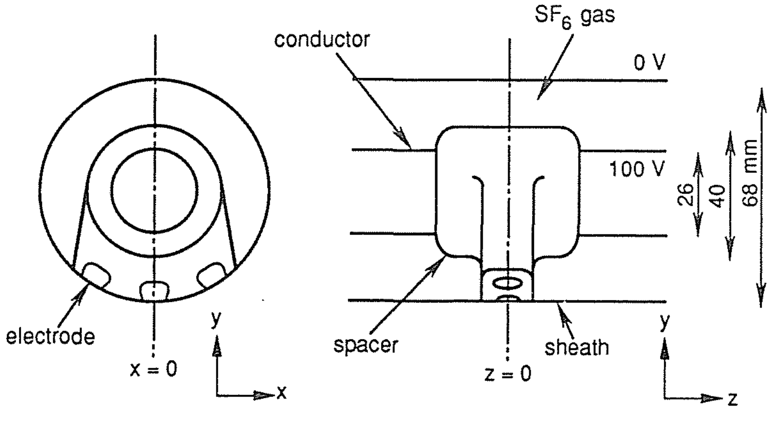


Fig. 2. Single-core SF_6 gas insulated cable model

model is shown in *Fig. 3*. Equipotential lines and distributions of the electric field on the conductor and insulator surfaces are shown in *Figs. 4* and *5*, respectively. The maximum electric field strength occurs at the point M in SF_6 gas.

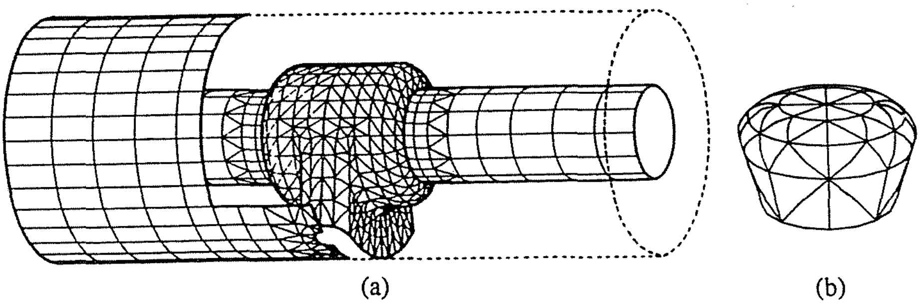


Fig. 3. Arrangement of curved surface element in the gas insulated cable, (*a*) conductor, spacer and sheath, (*b*) electrode

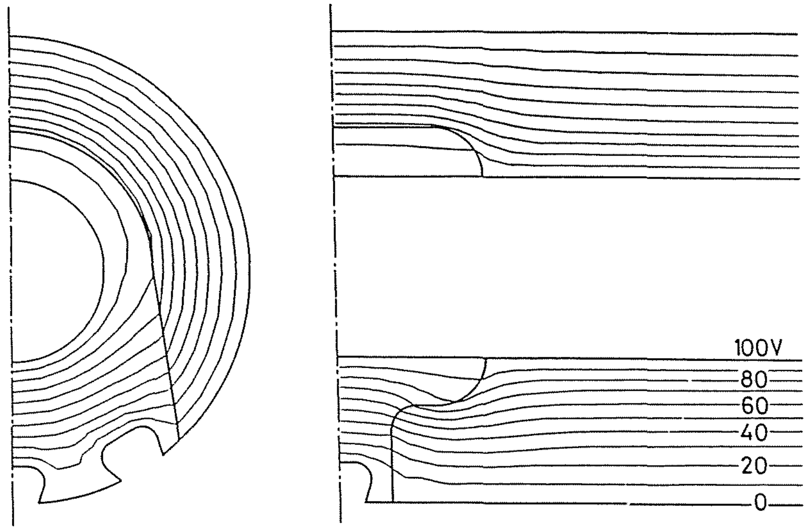


Fig. 4. Equipotential lines in the gas insulated cable model

4.2 Optimization of Sleeve Electrode [2]

Fig. 6 shows a sleeve electrode model. The contour between the point A and C is optimized by the stress-ratio method to reduce the maximum

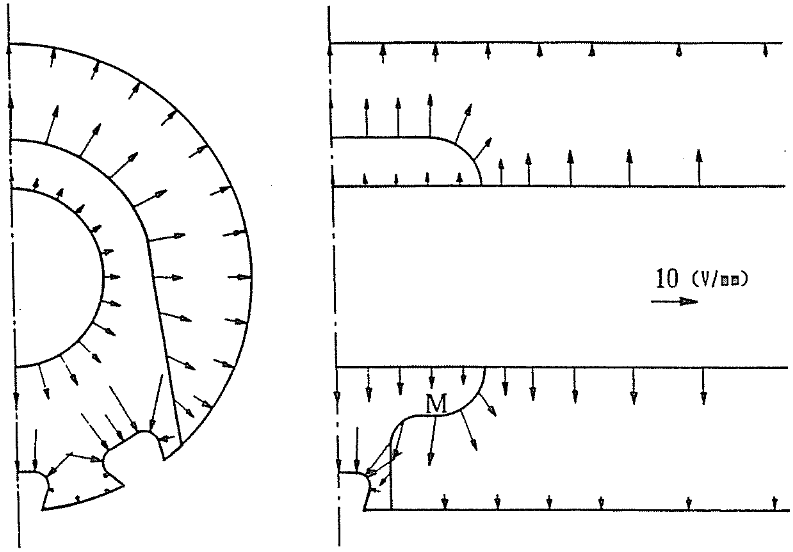


Fig. 5. Distribution of electric field on the conductor and insulator surfaces

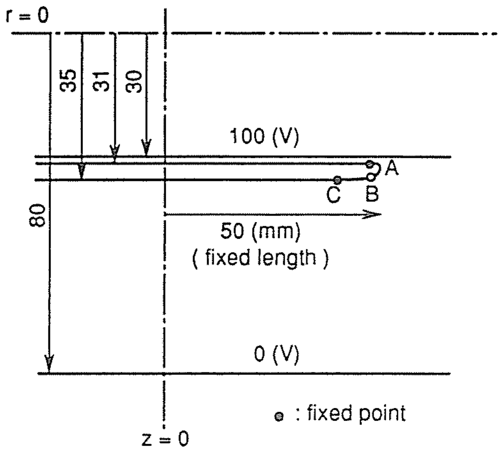


Fig. 6. Sleeve electrode model

electric field strength. The initial and optimized contours are shown in *Fig. 7*. The maximum electric field strength at the point M is reduced by 38 %. The changes of the electric field distribution on the contour between A and C and the changes of the maximum electric field are shown in *Figs. 8* and *9*, respectively. Using the samples in *Fig. 10* the results of

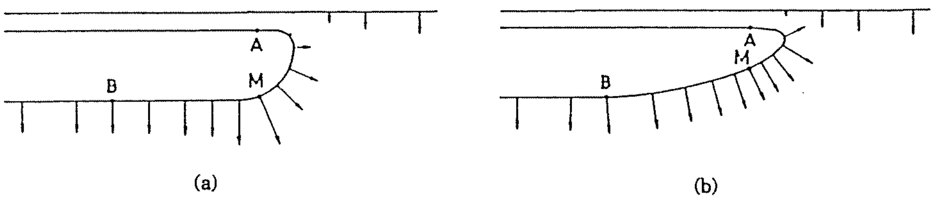


Fig. 7. Initial and optimized contours of the sleeve electrode model and electric field distributions, (*a*) initial contour, (*b*) optimized contour

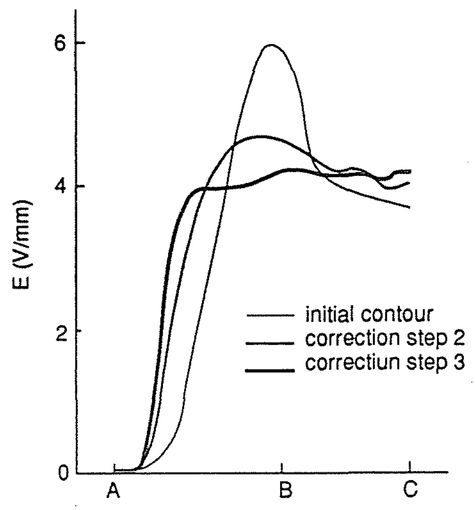


Fig. 8. Changes of the electric field distribution

AC flashover tests are shown in *Fig. 11.* The mean flashover voltage of the sample with the optimized shape is 13 % higher than that of the sample with the initial shape. The fact that the increase of the flashover voltage (13 %) is relatively low compared with the reduction (38 %) of maximum field strength brought by the optimization seems to be caused by the the approximation error in the production process and the area effect (the shape after optimization has extended the region of almost uniform field strength).

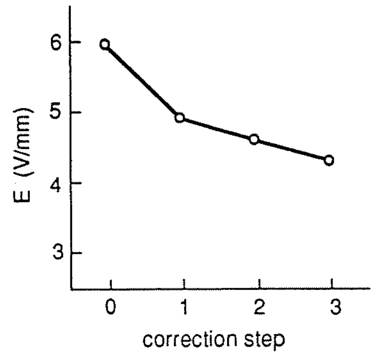


Fig. 9. Changes of the maximum electric field strength

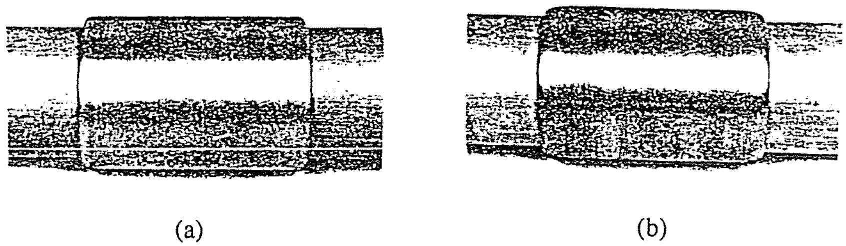


Fig. 10. Sleeve electrode for experiments, (*a*) initial, (*b*) optimized

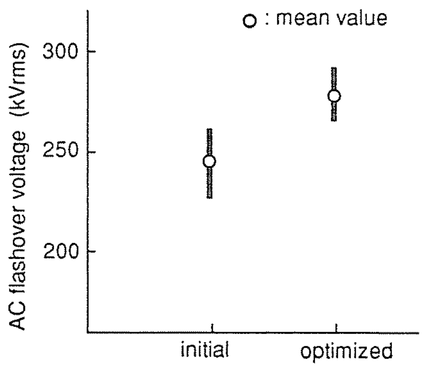
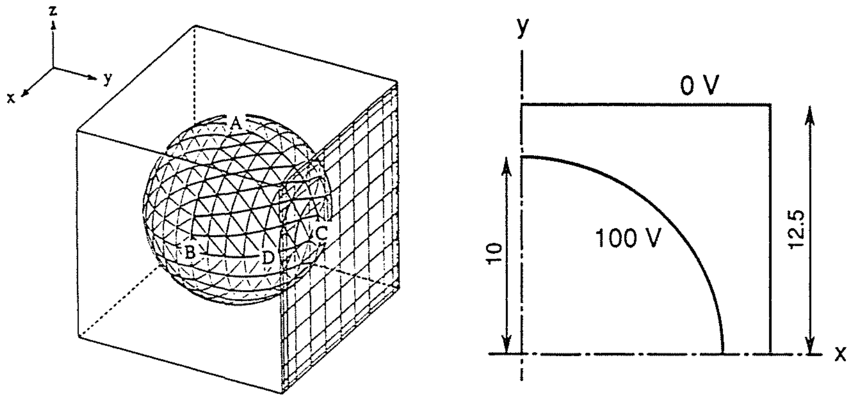


Fig. 11. Experimental results of flashover voltages

4.3 Optimization of Sphere Electrode in Cube [3], [4]

Fig. 12 shows a sphere electrode model. The shape of the sphere electrode is optimized by the Newton method to obtain uniform electric field distri-



(a) (b)

Fig. 12. Sphere electrode model, (a) boundary elements, (b) cross section on the $x-y$ plane

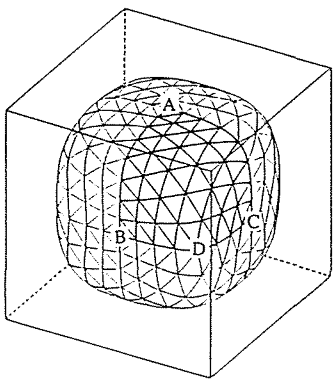


Fig. 13. Optimized sphere electrode

bution on the sphere electrode surface. The value of the desired electric field strength is 40 (= 100/2.5) (V/unit length). In one eighth part of the sphere electrode model, the triangular area A–B–C without nodes A, B and C is modified by the Gauss–Newton iteration so that the electric field strength on the triangular area A–B–C becomes equal to the desired electric field strength. The optimized shape of the sphere electrode obtained after three iteration steps is shown in *Fig. 13*, and the electric field distributions on the electrode surfaces are shown in *Fig. 14*. The Newton method provided good convergence characteristic for the sphere electrode model.

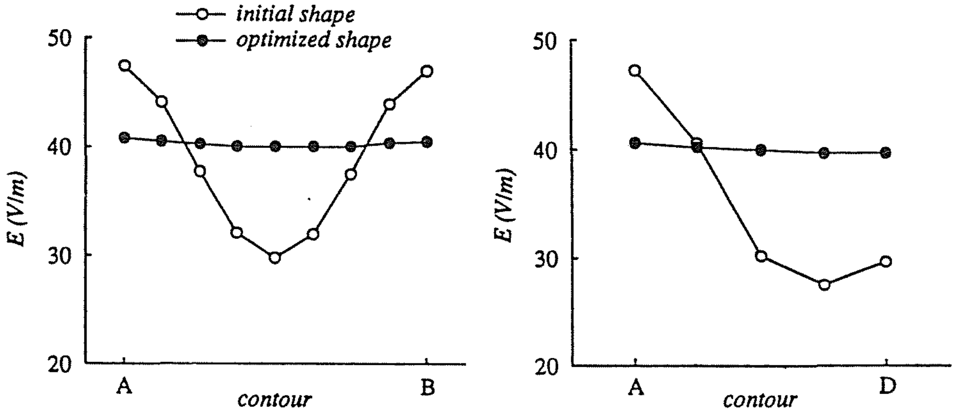


Fig. 14. Distributions of electric field on the sphere electrode surfaces

5. Conclusion

A boundary integral method for computation of the three-dimensional electric field distribution was described. The use of curved surface elements provided a good approximation of the electrode and insulator surfaces, and the accurate numerical integrations were performed by an adaptive integration method.

Optimization methods of electrode and insulator shapes in insulation design were presented. The stress-ratio method was effective to reduce the maximum electric field strength on the surface of the electrode. The desired electric field distribution was obtained by the Newton method using an object function.

The authors think that the boundary integral method and the optimization methods are applicable to the design of insulation in high voltage equipments.

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