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COMPUTER AIDED DESIGN OF STEPPED IMPEDANCE TRANSFORMERS REALIZED IN COAXIAL TRANSMISSION LINE

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Abstract

This paper describes computer programs that have been developed to design quarter-wave and short-step impedance transformers. In the literature dealing with these transformers usually there are tables to make designing easier, but in these the fringing capacitances occurring at the steps between lines of different impedances are not taken into consideration. The computer programs can calculate these capacitances and compensate for their effects. For short-step impedance transformers they can design dielectric supports as well. These programs are able to design transformers up to ten sections in a structure where the outer conductor has constant diameter and the inner conductor is alternating. At the end of this publication the different matching structures are compared on some examples.

1. Introduction

Quarter-wave transformers are widely used to obtain an impedance match within a specified tolerance between two lines of different characteristic impedances over a specified frequency band.

The computer programs are able to design Chebyshev and maximally flat quarter-wave transformers consisting of up to ten sections. The methods used by the programs were developed by FELDSTEJN [1]. The quarter-wave transformers are useful at microwave frequencies, but for applications involving frequencies much below 1000 MHz the size of step transformers can become impracticably large.

In this case we can use short-step impedance transformers. The computer program designs such transformers in two structures. One structure is of the form of a conventional low-pass filter structure. Such procedures are approximate but can give very good results if the design is worked out carefully.

The other structure is designed directly from the distributed-element impedance transformers and treated on an exact basis.

The short-step structures are realized in coaxial form, and they consist of short lengths of line sections having various impedances, where relatively high impedance transmission lines alternate with relatively low impedance lines. The line-section lengths are $l = \lambda_0/16$ or $\lambda_0/32$. The computer program is useful for synthesizing and analyzing these quarter-wave and short-step transformers.

2. Quarter-wave Transformers

2.1. Design of Quarter-wave Transformers

The formulas to design Chebyshev and maximally flat quarter-wave transformers are obtained from FELDSTEJN [1]. The transducer attenuation of Chebyshev transformers is given by

$$L = 1 + h^2 {T_n}^2(x) = 1 + h^2 {T_n}^2\left(\frac{\cos\Theta}{S}\right), \tag{1}$$

where $T_n(x)$ is the Chebyshev polynomial, h is a parameter depending on the maximum reflection coefficient in the passband, S is a scale factor, Θ is the electrical length of the section, and for maximally flat transformers

$$L = 1 + Q^2 \cos^{2n}\Theta, \tag{2}$$

where Q is a factor that sets the scale of attenuation.

The formulas given by FELDSTEJN [1] are not exact but they are comparatively simple, easily programmable and give very good results by means of mutual compensation of approximations. Such transformers are useful about and over 1000 MHz frequency bands because in these cases the total length of transformers will be of favourable size. At lower RF frequencies the total size of a transformer can be several meters or even more, thus they are too long for practical realization and the short-step impedance transformers described in the next chapter give acceptable size. Since transmission lines and impedance transformers are designed for the dominant mode, higher frequencies are limited due to the appearance of higher-order modes. Usually the use of Chebyshev transformers is more practical; we can realize them with fewer sections and shorter sizes. The maximally flat transformers are favourable concerning phase and group delay characteristics.

2.2. Corrections for Fringing Capacitances

At the steps between lines with different impedances due to these discontinuities the electric and the magnetic fields become deformed and higher-order modes occur (*Fig. 1*).

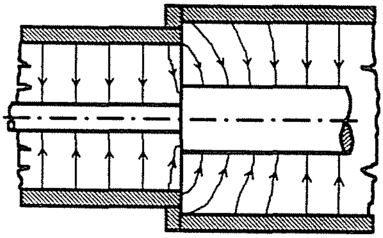

Fig. 1.

The changes in the section are accounted for by a lumped discontinuity admittance shunting the lines at the junction (shunt capacitance) [2]. Using these shunt capacitances for the discontinuities of stepped transformers, the equivalent circuit in *Fig. 3* will be obtained.

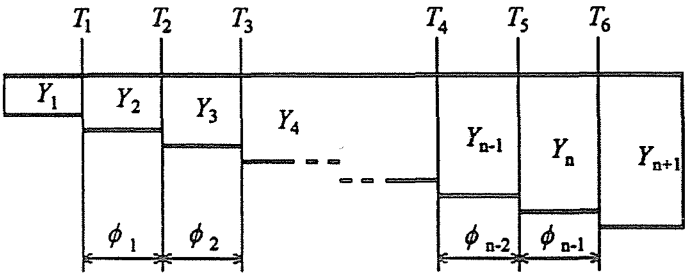

Fig. 2.

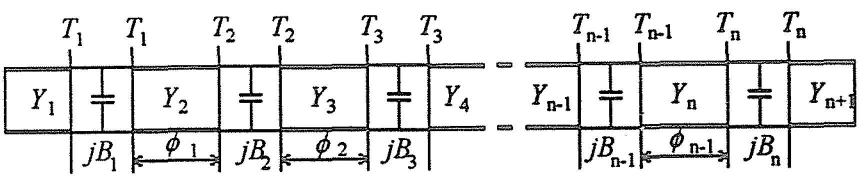

Fig. 3.

Although the changes in the section in case of quarter-wave transformers are smaller than in short-step transformers, we can see from the results of analysis for the design examples, at the end of this article, that it is important to compensate their effect. Reference [3] gives corresponding procedure for compensating the fringing capacitances. The compensation technique obtained by using scattering matrix analysis of the equivalent circuit gave good results in computing input reflection coefficient. This procedure compensates for these capacitances by making small adjustments in the physical lengths of the various line sections of the impedance transformers.

2.3. Analysis

The computer program uses some approach formula for the synthesis procedure and takes advantage of mutual compensations. Corrections for fringing capacitances take only phase modification effects into consideration. After having transformers designed, there is a possibility to analyse them and compare the results of analysis with the specifications. The analysis is free from approaches, takes the fringing capacitances into consideration so it can give correct results. The program calculates step by step the value of input admittances at different planes of equivalent circuit, and from this, it determines the input reflection coefficient function.

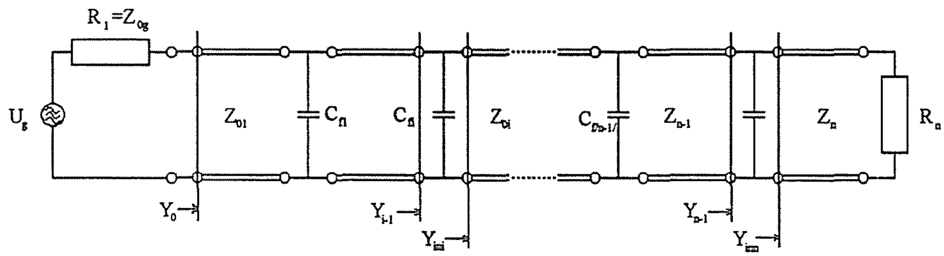

Fig. 4.

$$Y_{ini}(\omega) = Y_{0i}Y_i(\omega) + jY_{0i}\tan\beta l, \tag{3}$$

$$Y_{i-1}(\omega) = Y_{ini} + j\omega C_i, \tag{4}$$

$$\frac{Y_0 i = 1}{Z_{0i},} \tag{5}$$

where

$$\Gamma = \frac{Y_{0g} - Y_0}{Y_{0g} + Y_0} \tag{6}$$

and

$$\frac{Y_{0g} = 1}{Z_{0g}.} \tag{7}$$

The results of analysis can be found at the end of this article.

3. Short-step Transformers

3.1. Design of Short-step Transformers

It is often necessary to design impedance transformers for frequencies below 1000 MHz. In this frequency band the length of quarter-wave transformer is unfavourably long. The length of this transformer with many sections may be several meters. For this case the practical solution is the use of a short- step impedance transformer (*Fig. 5*).

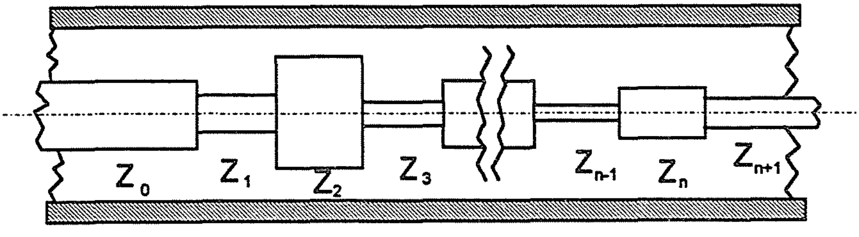

Fig. 5.

The computer program is able to design these transformers with two kinds of procedure, namely from lumped element low-pass filter form and in an exact way using the theory of distributed element networks.

The first procedure uses an approximate equivalent circuit of the transmission line, and the capacitances of lumped element low-pass-filter are replaced by transmission lines of small characteristic impedances and the inductances by transmission lines of large characteristic impedance. In this procedure the tangent functions describing the transmission lines are approximated by their arguments, because the length of the lines is small. In case of $l = \lambda/16$ the results are only approximate.

The other procedure to design short-step transformers is treated on an exact basis. It makes directly the synthesis of distributed-element network. This procedure has another advantage, namely the impedance ratio at the steps between lines of different impedance is smaller and easier to realize. In order to eliminate the periodic functions of distributed-element impedance transformers, and to simplify the synthesis problem, Richards makes use of the mapping functions

$$p = \tan h \frac{as}{2}, \tag{8}$$

where $s = u + j\Omega$ is the complex-frequency variable for the transmission-line circuit and $p = \sigma + j\omega$ is the frequency variable of the mapped transfer function. The parameter a is defined by

$$\frac{a}{2} = \frac{l}{\nu} = \frac{\pi}{2\Omega_{1/4}}, \tag{9}$$

where ν is the velocity of propagation and $\Omega_{1/4}$ is the radian frequency for which $l = \lambda_m/4$.

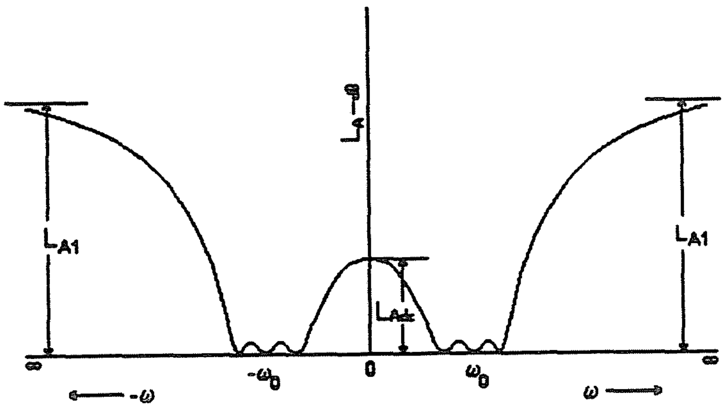

Fig. 6.

The attenuation characteristic in *Fig. 6* can be obtained by mapping the attenuation characteristic of a conventional Chebyshev low-pass filter (which has an equal-ripple pass band from $\omega'' = 0$ to $\omega'' = \omega_1''$) by use of the mapping function

$$\omega' = \omega'_x A\Big(\frac{\omega^2 - \omega_0^2}{\omega^2 + 1}\Big), \tag{10}$$

. where ω' is the sinusoidal frequency variable for the conventional low-pass filter, ω'_x is the cut-off frequency of the conventional filter, and ω is the frequency variable for the circuit of the form in *Fig. 6*. The parameter ω_0 is as defined in *Fig. 6*:

$$\omega_0^2 = \frac{\omega_a^2 + \omega_b^2}{2} \tag{11}$$

, where ω_a and ω_b are limit frequency of pass band. In this way the desired reflection coefficient function is obtained by using two mapping functions. The voltage reflection coefficient function is

$$\Gamma(p) = \frac{K(p - j\omega_a)(p + j\omega_\alpha)(p - j\omega_\beta)(p + j\omega_\beta)\dots}{(p - p_1)(p - p_2)(p - p_3)(p - p_4)\dots} \tag{12}$$

where K is a constant. *The design steps are*: Specification data is set: frequency band, maximum of reflection coefficient absolut value in the passband and parameters of transmission lines connecting to the ends of the transformers. First the degree of transfer function is obtained (the degree is equal to the number of sections), then the reflection coefficient function of conventional Chebyshev low-pass filter from FANO's paper [4] is calculated. Next the poles and zeros of this function are mapped to obtain the reflection coefficient function for the desired form of network, from which the input impedance function is formed. Finally the circuit is synthesized by removing one unit element at a time by successive application of RICHARDs method [5]. In this manner the impedance values Z_1,Z_2, etc. are obtained, and these impedance values are also the characteristic impedances of the line sections.

3.2. Dielectric Supports

At the realization of short-step impedance transformers the diameter of the inner conductor of coaxial lines with the large characteristic impedances may be such small that it is a problem to hold it in the axis of the outer conductor. Usually the transformers consist of air-filled coaxial transmission lines for the sake of transmitting large power. In this case dielectric supports are used to hold the inner conductor coaxially. There are dielectric supports in several forms. The form used in this computer program and its equivalent circuit are shown in *Fig. 8*.

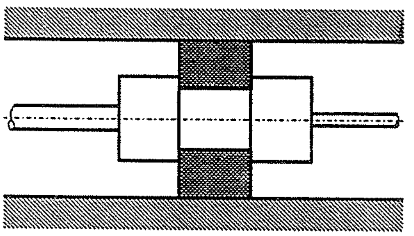

Fig. 7.

Dielectric support modifies some properties of the transmission line, decreases its characteristic impedance due to changing the dielectric constant and increasing the electrical length of the line sections.

$$Z_{0i} = \frac{60}{\sqrt{\epsilon_{ri}}} \ln \frac{b_i}{a_i} \tag{13}$$

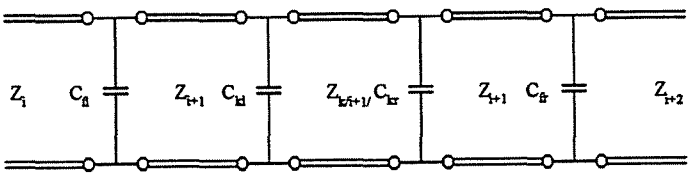

Fig. 8.

$$\Phi_i = \beta l_i \sqrt{\epsilon_{ri}}, \tag{14}$$

where: ϵ_{ri} is the dielectric constant of the section of the transmission line, Φ_i is the electrical length of the section, l_k is the physical length of section, a_i is the inner diameter of transmission line, b_i is the outer diameter of transmission line.

We can compensate for the change in the characteristic impedance by decreasing the inner conductor diameter of these sections. At the decrease of inner conductor's diameter fringing capacitances occur in both ends of support. These capacitances and the increase in the electrical length caused by support are compensated by decreasing the physical length of transformer section.

3.3. Corrections for Fringing Capacitances

It is very important to compensate for the effects of fringing capacitances in short-step impedance transformers. The compensation of transformers designed from lumped element form is comparatively simple. Every even-numbered line section with lower characteristic impedance was obtained by replacing capacitances of lumped element form of low-pass-filter. So we can subtract the fringing capacitances occurring at both ends of these sections of transformers from these capacitances, and we will obtain the reduced capacitance value from which we can calculate the compensated lengths of the line sections.

In the other case, when the synthesis is achieved directly on the distributed-element networks, we use the procedure mentioned at compensation of quarter-wave transformers. We can add the capacitances generated by the dielectric supports to the fringing capacitances occurring at the steps between lines of different impedances and compensate them together. We can take the coaxial-line step discontinuities into consideration as lumped elements only if the distance of two discontinuities is at least as large as the diameter of the outer conductor.

3.4. Analysis

This part of the program is similar to the part used for quarter-wave transformers because in this case transmission line sections with different impedances alternate too, but the sections containing dielectric supports will consist of three transmission line sections.

4. Comparison and Evaluation of the Results

Three different computer programs were made to design transformers of different types.

All of them are able to design transformers built from coaxial air-filled, lossless transmission line sections.

After entering the specifications, the programs will check that data. If the discontinuities are very close to each other, or higher-order modes can occur because the mean diameter is large enough, brings it to the user's attention and will ask for new data.

Next the program calculates the data of desired transformers and displays them on the computer screen in the format that will be shown in the examples. If the user wishes, it can perform the analysis of the transformers. If we have a look at the examples, we can see that the compensation for fringing capacitances is less important in case of quarter-wave transformers, than in case of short-step transformers.

The first two examples present quarter-wave Chebyshev (*Figs. 9–10*) and maximally flat (*Figs. 11–12*) transformers with and without compensation. We can see, that the number of sections for the Chebyshev transformer is smaller than for the maximally flat.

The next example (*Figs. 13–14*) presents a short-step transformer, which was designed directly by the help of synthesis of distributed-element network because this procedure gave a better solution.

Measurements were achieved on a short-step transformer built some years ago at the Department of Microwave Telecommunications. The results are shown in *Fig. 15.* The calculated results by the computer program using synthesis and analysis are presented for uncompensated (*Fig. 13.*) and compensated (*Fig. 14.*) transformers.

The results obtained by calculations and measurements show good agreement. The small differences mainly originate from inaccuracies in geometrical sizes of manufacturing. Further investigation will be carried out to obtain answer for the tolerance sensitivity of short-step transformers.

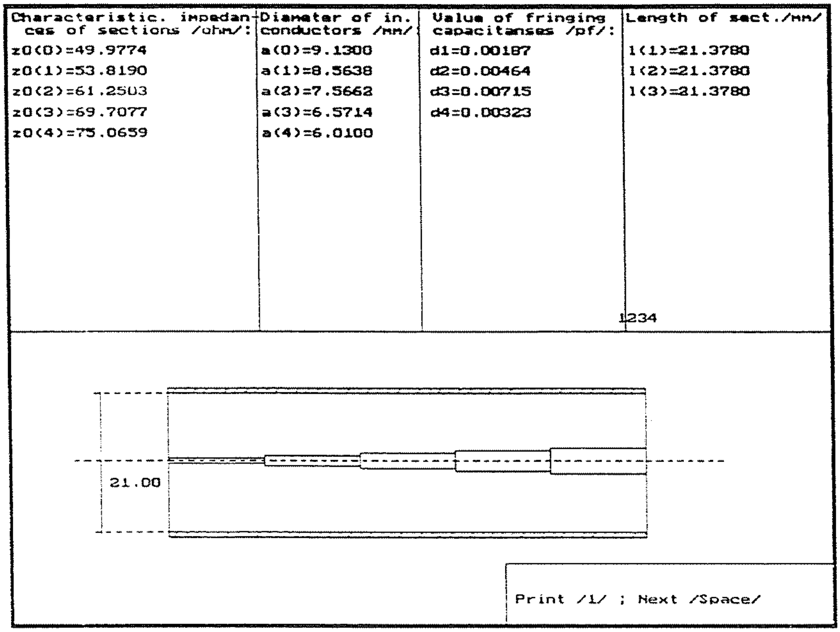

Fig. 9a.

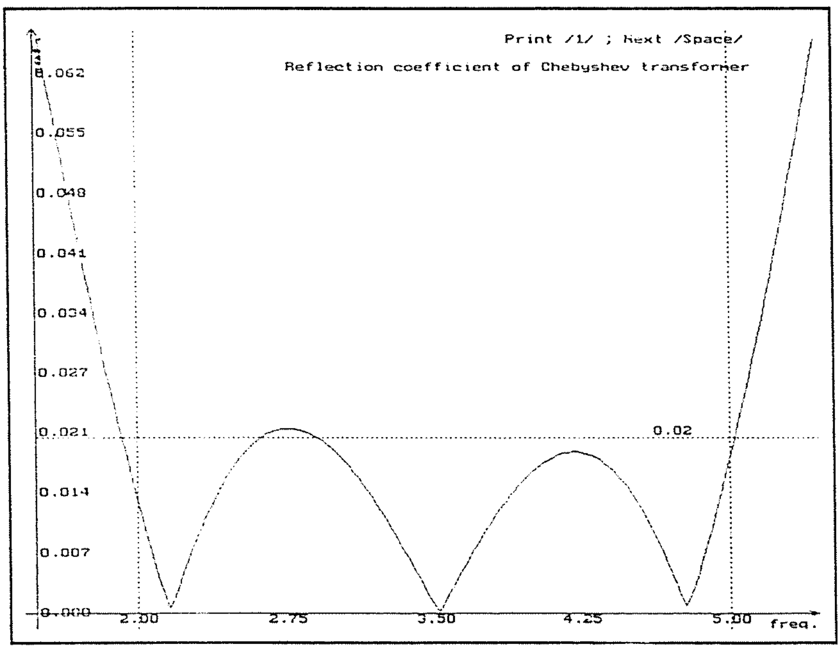

Fig. 9b. Quarter-wave Chebyshev transformer without compensation

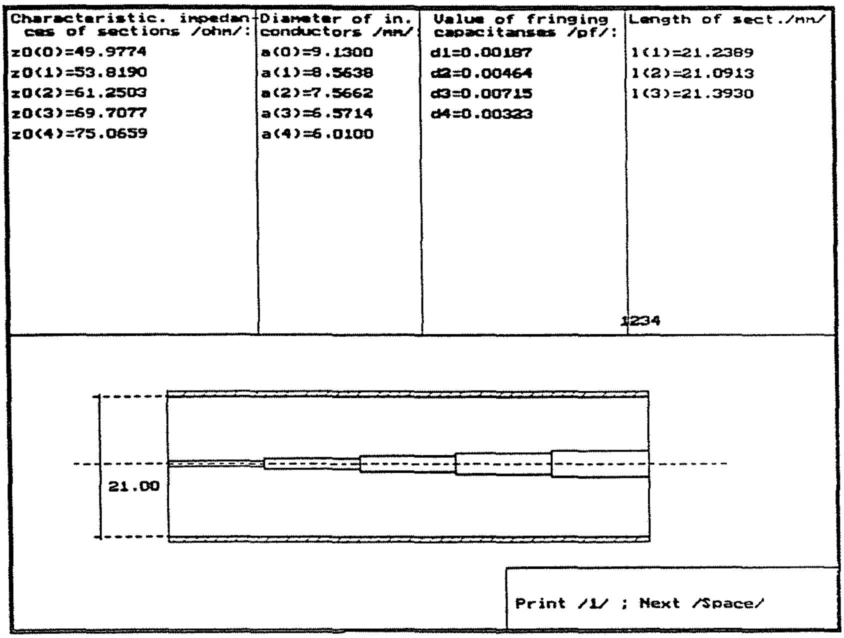

Fig. 10. a.

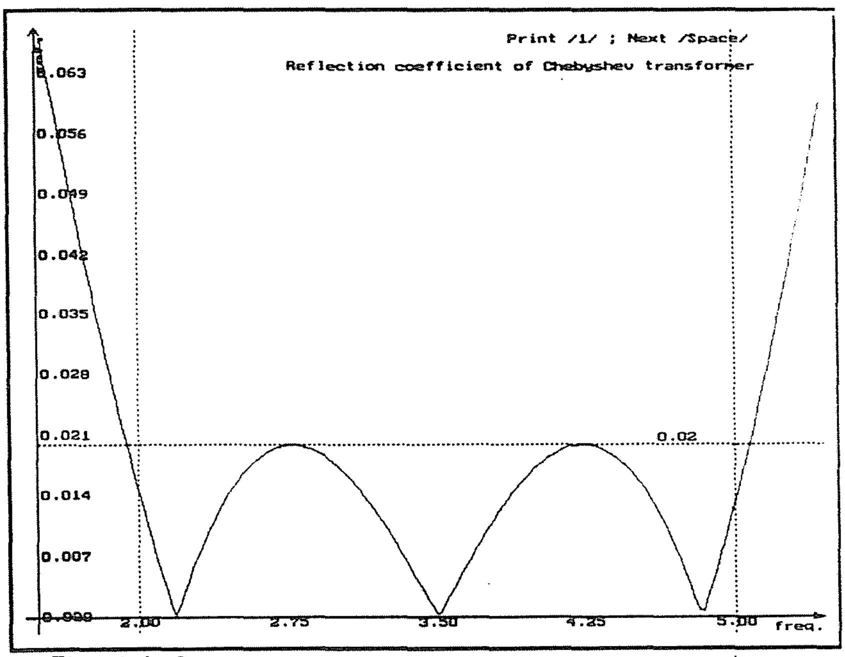

Fig. 10. b. Quarter-wave Chebyshev transformer with compensation

Characteristic. impedances of sections /ohm/:	Diameter of in. conductors /mm/	Value of fringing capacitanses /pf/:	Length of sect./mm/
z0(0)=49.9774	a(0)=9.1300	d1=0.00000	l(1)=21.4286
z0(1)=50.6168	a(1)=9.1300	d2=0.00197	l(2)=21.4286
z0(2)=53.9385	a(2)=8.5467	d3=0.00450	l(3)=21.4286
z0(3)=61.2503	a(3)=7.5662	d4=0.00738	l(4)=21.4286
z0(4)=69.5532	a(4)=6.5883	d5=0.00109	l(5)=21.4286
z0(5)=74.1176	a(5)=6.1057	d6=0.00243	
z0(6)=75.0659	a(6)=6.0100		
			123456

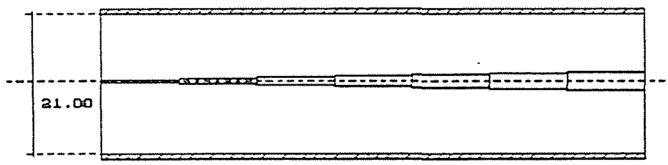

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Fig. 11. a.

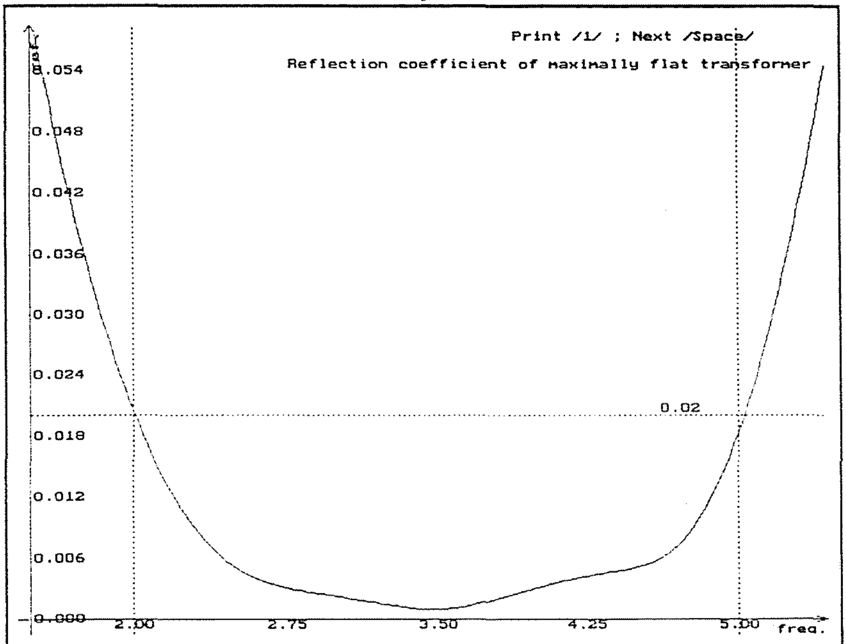

Fig. 11. b. Quarter-wave maximally flat transformer without compensation

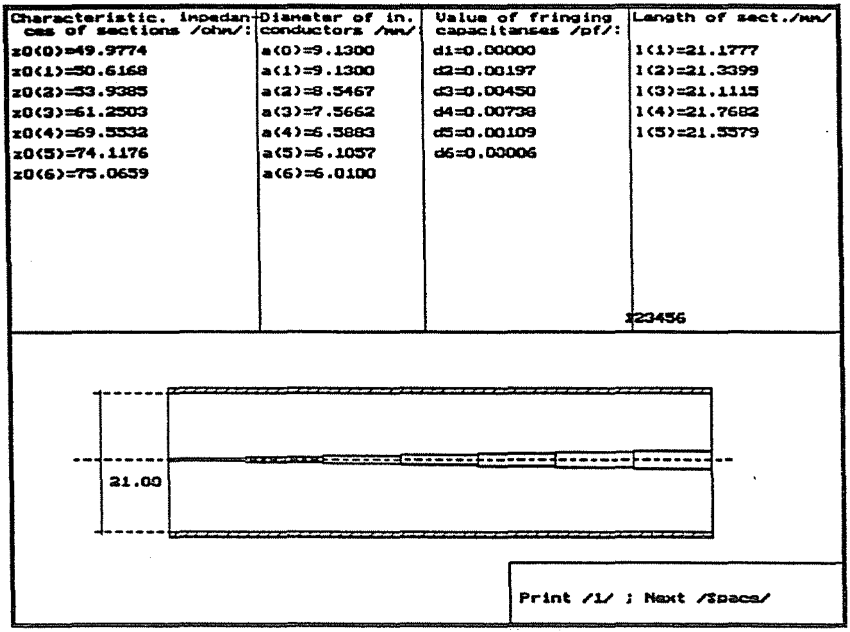

Fig. 12. a.

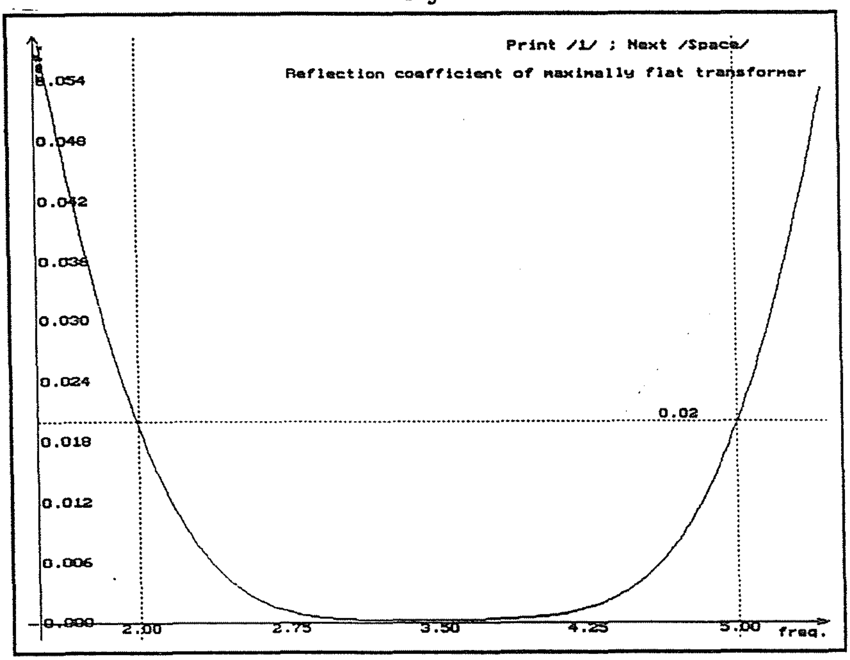

Fig. 12. b. Quarter-wave maximally flat transformer with compensation

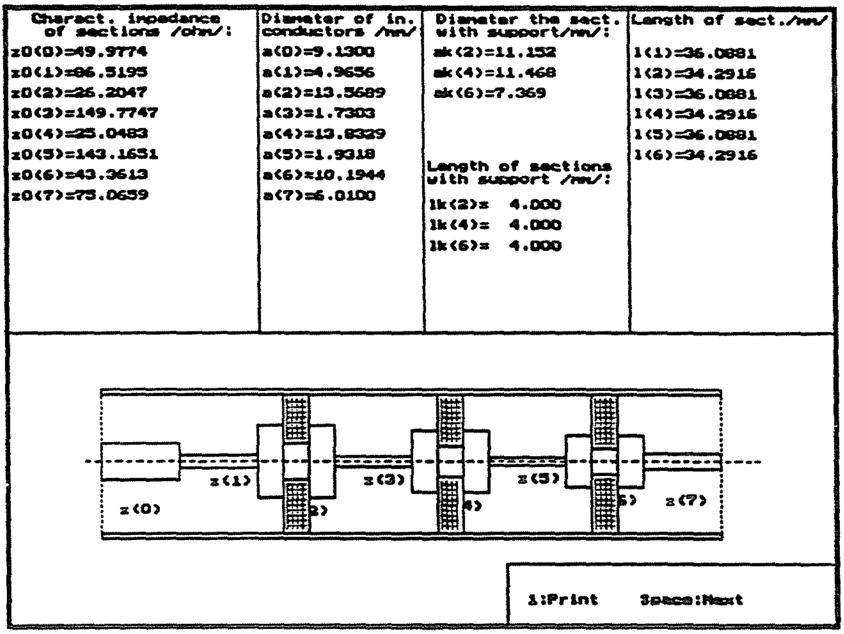

Fig. 13. a.

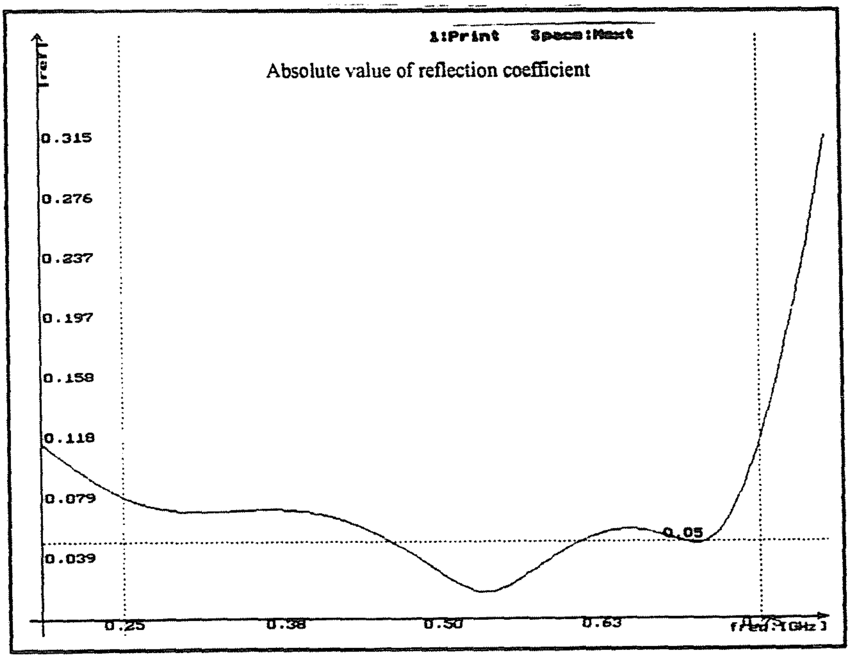

Fig. 13. b. Short-step transformer without compensation

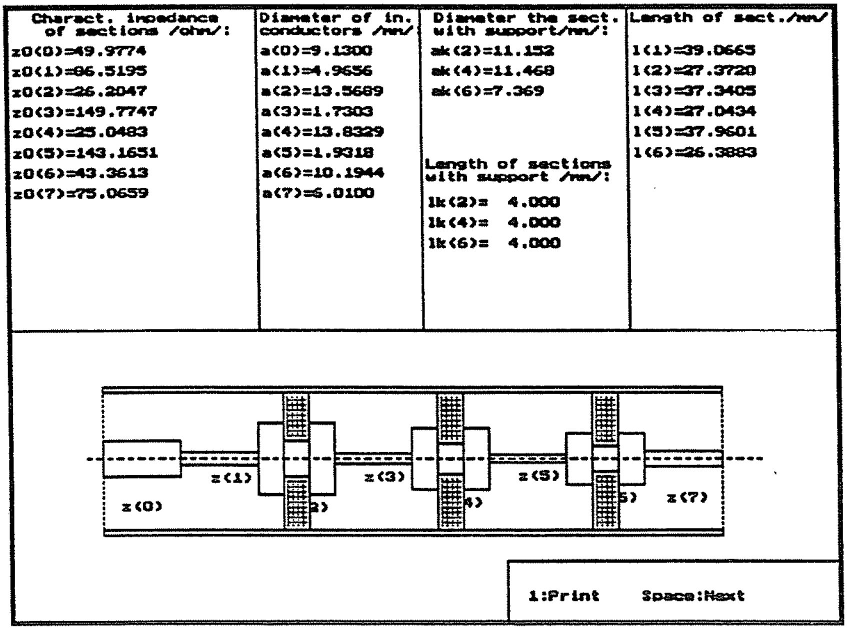

Fig. 14. a.

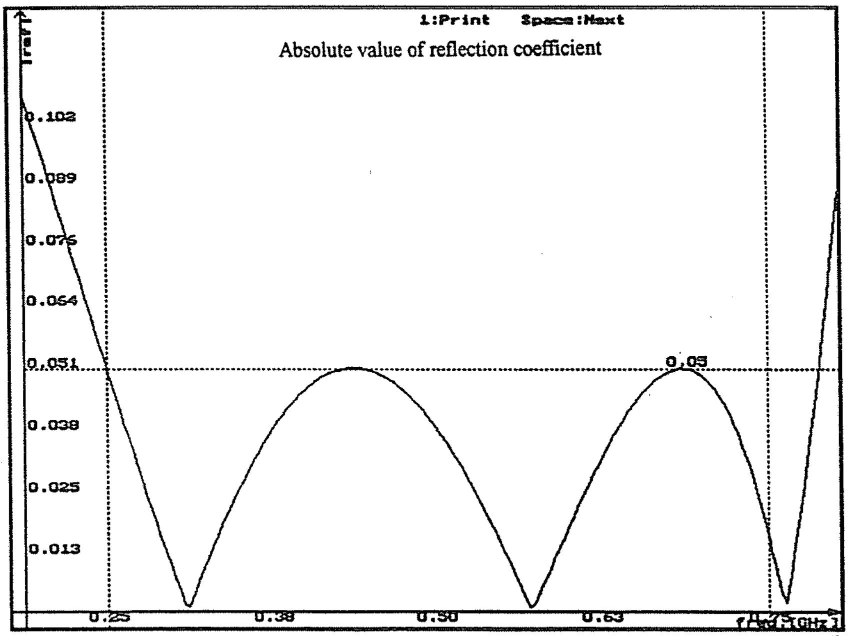

Fig. 14. b. Short-step transformer with compensation

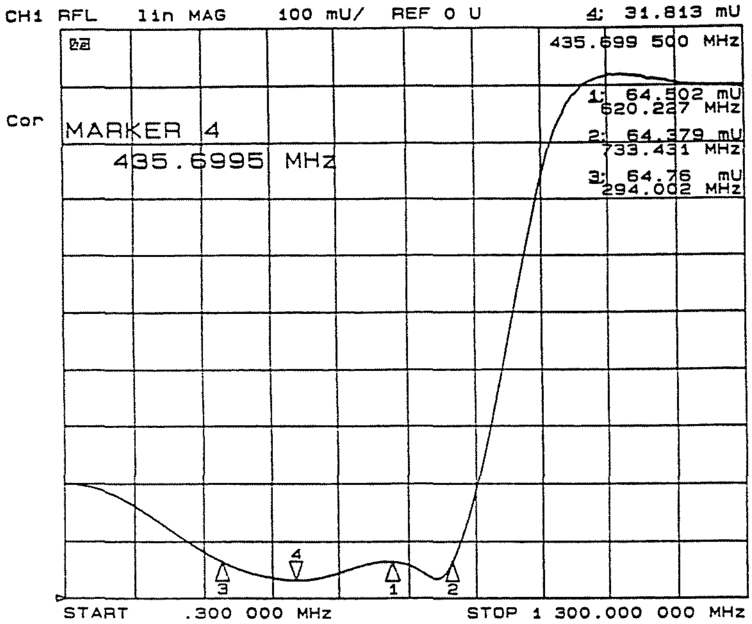

Fig. 15. a.

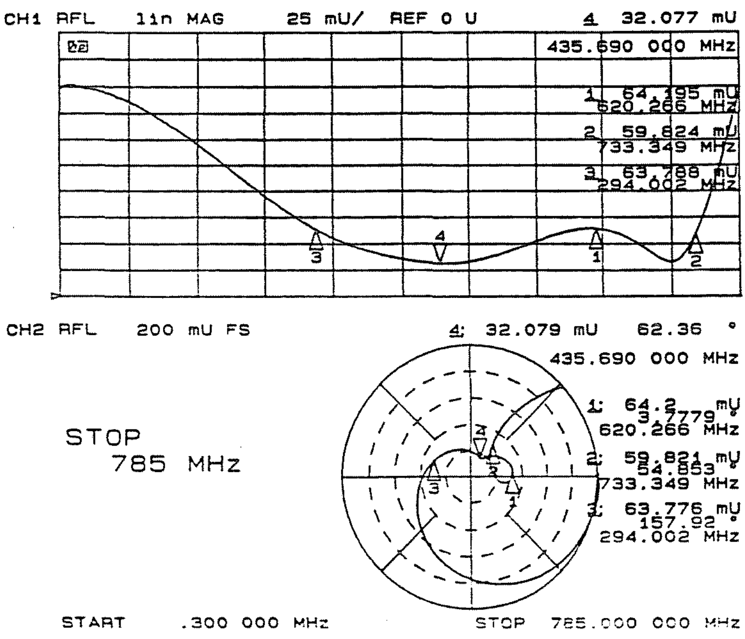

Fig. 15. b.

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