PERIODICA POLYTECHNICA SER. EL. ENG VOL. 37, NO. 3, PP. 211-221 (1993)

DYNAMIC ANALYSIS OF POWER CONVERTERS

István SZABÓ

Microwave Telecommunication Department
Technical University of Budapest
H-1111 Budapest, Goldmann 3. Hungary
Phone, fax: +36 1 1-869-231
email: D-TAYLOR@NOV.MHT.BME.HU

Received: June 12, 1993

Abstract

In recent years many papers have been published on analysis and simulation of DC/DC converters. The development of improved modeling methods for the steady state and transient analysis of power electronic circuits and systems has constantly been of interest for circuit design, for control purposes or to enhance the stability and performance of the system. This paper introduces a new circuit analyzer program supporting the design and development of some DC/DC converter types.

Keywords: DC/DC converters, injected and absorbed currents, characteristic coefficients, feedback-loop design.

1. Main Features of the Program

In the last decade, there have been significant advances in developing good modeling methods. However, these methods may be subject to one or more of the following limitations:

- Many methods are circuit dependent or applied to a specific circuit family
- The method can be based on approximations and simplifying assumptions that are not sufficiently general
- The approach may not be computationally efficient

The new DYANA program can solve these problems and more. The calculations are performed for open or closed voltage regulating loops. Including the input and the output post-filters and the voltage-error amplifier, DYANA uses the characteristic coefficients of the injected and absorbed currents to describe the converter cell [1]. The reference book gives theoretical derivations of injected-absorbed current analysis method.

The cell model based on the characteristic coefficients is completely general. DYANA combines the general model with the particular coefficients of the cell to be analyzed. Therefore it can be easily updated for new converter topologies and control methods.

The program specifies the characteristic coefficients of the switching cell by specifying component values or by entering the coefficients of the polynomials in the form of complex variables. The parameters of the error amplifier, the input and output post-filters are defined similarly.

The program is particularly useful for predicting the dynamic behaviour of the regulator and for designing the feedback-control loop. Using the software, we can optimize the compensation of the error amplifier or determine the effect of the input/output filter. DYANA is able to plot the frequency-domain transfer functions and time-domain transient responses of the error amplifier, input/output filters, magnitude and phase of all characteristic coefficients, circuit impedances versus frequency.

DYANA performs the following calculation functions:

- Analyzes buck, boost and buck-boost converters in voltage-mode control
- Computes and plots the frequency-domain magnitude and phase and the time-domain step and impulse responses of all of the functions listed below.

  Open loop functions:
    - Transfer function of control to output voltage
    - Transfer function of source voltage to output voltage
    - Output impedance
    - Loop gain

  Closed loop functions:
    - Transfer function of reference to output voltage
    - Transfer function of source voltage to output voltage
    - Output impedance
    - Transfer function of source voltage to input voltage
    - Input impedance of the regulator without input filter
    - Input impedance of the regulator with input filter

  Auxiliary functions:
    - Transfer function of the error amplifier
    - Transfer function and output impedances of the input filter and output post-filter
- Computes and plots the magnitude and phase of all of the following parameters vs. frequency
- Coefficient of controlled quantity-to-injected current
- Coefficient of output voltage-to-injected current
- Coefficient of input voltage-to-injected current
- Coefficient of controlled quantity-to-absorbed current
- Coefficient of output voltage-to-absorbed current
- Coefficient of input voltage-to-absorbed current
- Transfer function of the error signal-to-controlled quantity

- Impedance of the output capacitor
- Series impedance of the output post-filter
- Parallel impedance of the output post-filter
- Feedback impedance of the error amplifier
- Series input impedance of the error amplifier
- Series impedance of the input filter
- Parallel impedance of the input filter
- High-frequency extension function (for constant-frequency current-mode control)

Other useful functions provided by the software:

- Reading and writing disk files
- Displaying the computed results in graphs
- Allowing the user to specify numerical limits for the circuit-parameter inputs, etc.

The program is menu-driven and very easy to use.

2. Theoretical Description of the Computing Method

The switching cell is a combination of the power converter and its duty-ratio/frequency modulator, but not including the output filter capacitor. The model of the complete regulator includes the following (*Fig. 1*):

- the switching cell
- a general input filter in the form of a voltage divider (Z_{is}, Z_{ip})
- the impedance of the output filter cap (Z_{cf})
- a general output post-filter in the form of a voltage divider (Z_{os}, Z_{op})
- a load comprising a resistor R_L and a current sink (I_L)
- an error amplifier comprising an ideal operational amplifier, a feedback impedance Z_{ef}, a series input impedance Z_{es}, and a bias resistor R_b, and
- the modulator N which converts the output voltage of the voltage-error amplifier into the controlled quantity X.

2.1. Model of the Switching Cell Based on Injected and Absorbed Currents

DYANA uses the characteristic coefficients of the injected and absorbed currents to describe the converter cell. Six characteristic coefficients are necessary to describe the switching cell. Those coefficients represent how the averaged injected current (i_c, the currrent injected to the load circuit from the switching cell) and the averaged absorbed current (i_e, the current absorbed by the switching cell from the source circuit) depend on the input

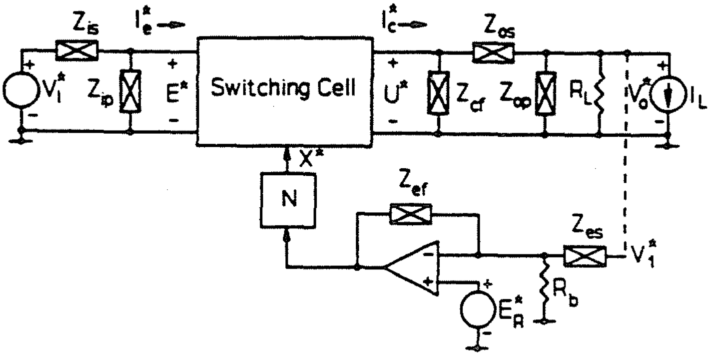

Fig. 1. Model of the switching regulator

voltage (e), the output voltage (u), and the controlled quantity (x) (e.g. the switch duty-ratio). *Fig. 2* shows the block diagram of the switching cell, and the quantities i_c, i_e, e, u and x.

The source circuit includes the primary energy source and the optional input filter. The load circuit includes the output filter capacitor with its ESR, any other optional output filter, and the load.

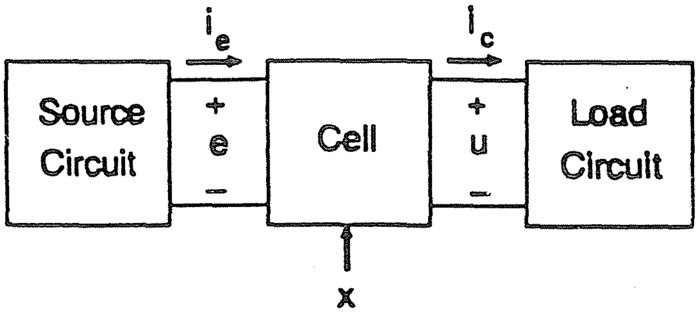

Fig. 2. Block diagram for injected-absorbed current model

The original injected and absorbed current method contains the following steps [2]:

1. **Express the injected (or absorbed) average current i_c (or i_e) as a function of the x, u, e, i_L quantities**

$$\begin{aligned} i_c &= i_c(x, u, e, i_L), \\ i_e &= i_e(x, u, e, i_L). \end{aligned} \tag{1}$$

The quantities are averaged over a period of the switching frequency [1].

2. Develop the total differential of the injected (or absorbed) current

$$di_c = \frac{\partial i_c}{\partial x}dx + \frac{\partial i_c}{\partial u}du + \frac{\partial i_c}{\partial e}de + \frac{\partial i_c}{\partial i_L}di_L. \tag{2}$$

3. Apply Laplace transformation to the total differential

$$I_c^* = \frac{\partial i_c}{\partial x}X^* + \frac{\partial i_c}{\partial u}U^* + \frac{\partial i_c}{\partial e}E^* + \frac{\partial i_c}{\partial i_L}I_L^*. \tag{3}$$

4. Determine the time derivative di_L/dt of the inductor current

$$di_L/dt = (i_{L_{n+1}} - i_{L_n})/T = f(x, u, e, i_L),$$

or

$$di_L/dt = u_L/L = f(x, u, e, i_L). \tag{4}$$

5. Develop the total differential of $f = di_L/dt$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial u}du + \frac{\partial f}{\partial e}de + \frac{\partial f}{\partial i_L}di_L. \tag{5}$$

6. Apply Laplace transformation to the total differential of $f = di_L/dt$

$$sI_L^* = \frac{\partial f}{\partial x}X^* + \frac{\partial f}{\partial u}U^* + \frac{\partial f}{\partial e}E^* + \frac{\partial f}{\partial i_L}I_L^*. \tag{6}$$

7. By combining the two Laplace-transformed functions, eliminate the inductor current from expressions i_c and i_e

$$\begin{aligned} I_c^* = {} & \left[\frac{\partial i_c}{\partial x} + \frac{\partial f}{\partial x} \cdot \frac{\frac{\partial i_c}{\partial i_L}}{s - \frac{\partial f}{\partial i_L}}\right] X^* + \left[\frac{\partial i_c}{\partial u} + \frac{\partial f}{\partial u} \cdot \frac{\frac{\partial i_c}{\partial i_L}}{s - \frac{\partial f}{\partial i_L}}\right] U^* + \\ & + \left[\frac{\partial i_c}{\partial e} + \frac{\partial f}{\partial e} \cdot \frac{\frac{\partial i_c}{\partial i_L}}{s - \frac{\partial f}{\partial i_L}}\right] E^*. \end{aligned} \tag{7}$$

Expression (7) already clearly shows the characteristic coefficients of the cell

$$I_c^* = A_c(s)X^* - B_c(s)U^* + C_c(s)E^*, \tag{8}$$

and

$$I_e^* = A_e(s)X^* - B_e(s)U^* + C_e(s)E^*. \tag{9}$$

In general case, nine coefficients provide complete characterization of a non-reciprocal and non-symmetrical three-port network. In this case I_x is usually negligible (the control power is approximately zero) therefore six coefficients suffice

$$\begin{bmatrix} I_c^* \\ I_e^* \\ I_x^* \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} X^* \\ U^* \\ E^* \end{bmatrix} \Leftrightarrow \begin{bmatrix} I_c^* \\ I_e^* \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} A_c & -B_c & C_c \\ A_e & -B_e & C_e \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X^* \\ U^* \\ E^* \end{bmatrix}.$$

From the matrix equations

$$A_c = y_{11}, \quad B_c = -y_{12}, \quad C_c = y_{13},$$

$$A_e = y_{21}, \quad B_e = -y_{22}, \quad C_e = y_{23}.$$

Eqs. (8) and (9) lead to the equivalent circuit of the switching cell (*Fig. 3*).

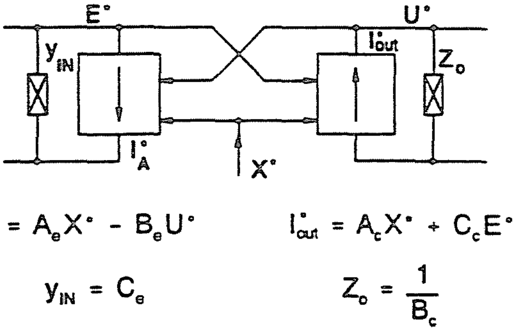

Fig. 3. Universal equivalent circuit of a switching cell

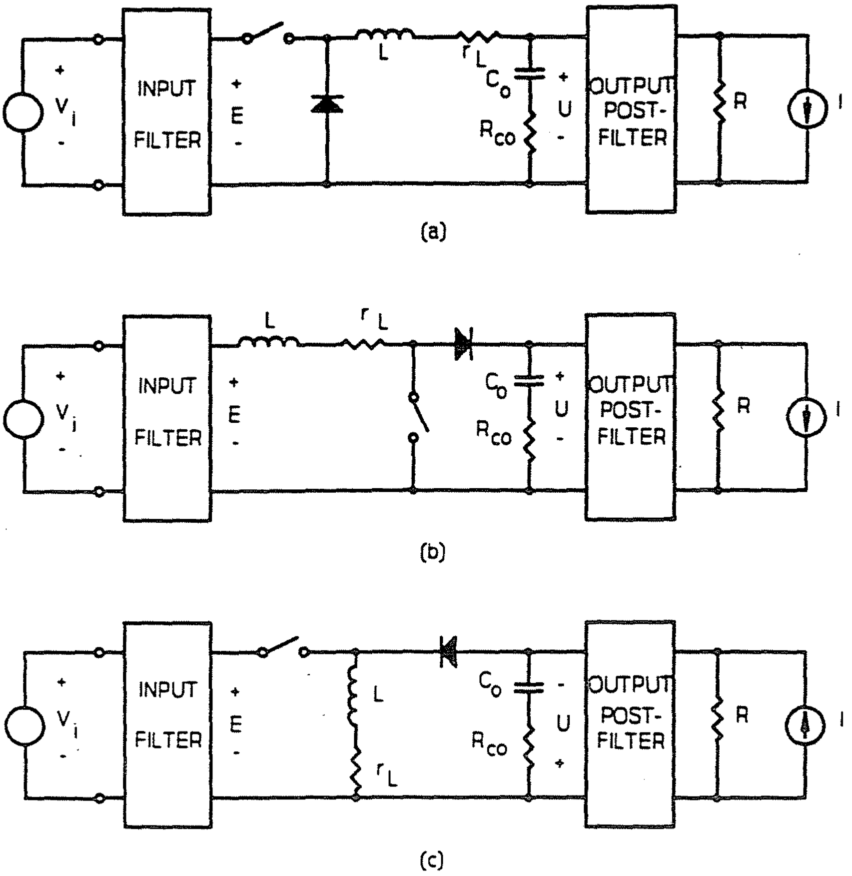

Fig. 4. Built-in converter topologies: (a) buck, (b) boost, (c) buck-boost converters

3. Built-In Topologies

Buck, boost and buck-boost converter topologies are built into DYANA. *Fig. 4* shows them.

Fig. 5 shows the topologies of the built-in error amplifier, input and output post-filter. The control method built into the basic version of DYANA is constant-frequency duty-ratio control (in other form: voltage-mode PWM control). An optional program package is available for cons-

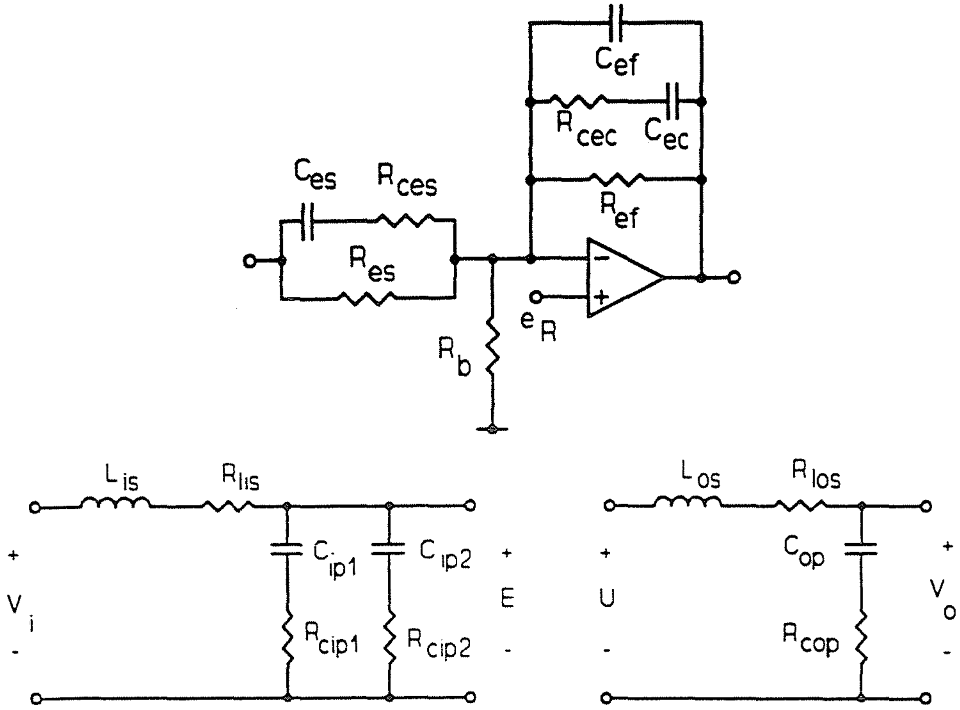

Fig. 5. Built-in error amplifier, input filter and output post-filter

tant-frequency peak-current-commanding control and constant off-time current-mode control.

4. Example. How to Use DYANA for Feedback-Loop Design?

The purpose of the feedback-loop design is to select the error amplifier components such that the loop gain of the system reaches zero dB at a given 'crossover' frequency f_0. The phase margin M_p is the difference between the phase shift of the loop and –360 ° at the crossover frequency . The gain margin M_g is the difference between 0 dB and the gain of the loop at the frequency where the phase is –360 °. The phase margin M_p and the gain margin M_g of the loop gain must be sufficient to provide the required transient response and stability. Instability may be caused by varying the input voltage or load, component tolerances, or extra capacitance added to the load by the user. In most cases the designer chooses the crossover frequency f_0. It must be high enough for good dynamic behaviour and low enough for avoiding subharmonic instability and noise amplification ($f_0 = 0.02 \div 0.2\ f_{sw}$). Usually the phase margin M_p and the gain margin M_g are also chosen by the designer. The gain margin should be large enough to accommodate gain variations. The phase margin should be large enough (between 30 ° and 90 °) to provide well-damped transient response and safety against unforeseen excess phase shift.

For the design, factor K is an auxiliary variable. K measures the zero-pole separation of the frequency response in the error amplifier. The step-by-step design procedure is as follows.

1. As a first step in the design of the feedback-loop we must determine the Bode plots of the magnitude and phase of the actual converter's control-to-output voltage transfer function. The Bode plots are obtained by using DYANA's AC analysis capability.

2. The second step is to select the crossover frequency f_0.

3. The next step is to determine the gain and phase shift of the cell (G_{cell}, P_{cell}) at the f_0.

4. In the fourth step we must select a phase margin (M_p) between $30 \div 90$ °.

5. The following step is to calculate the phase boost (B) to be provided by the error amplifier

$$B = M_p - P_{cell} - 90^\circ.$$

6. In the sixth step we must determine the gain of the error amplifier at f_0.

$$G_{amp} = 1/G_{cell}.$$

7. We must calculate the K factor for the actual type of error amplifier.

$$\text{Type 2. amplifier: } K = \tan(B/2 + 45^\circ)$$
$$\text{Type 3. amplifier: } K = \tan^2(B/4 + 45^\circ)$$

Fig. 6 shows the usual error amplifier types, *Table 1* gives expressions to calculate the elements of the error amplifiers.

With voltage-mode control in continuous inductor-current mode, the best dynamic performance can be achieved by using a Type 3. amplifier.

8. The last step in the design is to calculate the components of the amplifier from the expressions given in *Table 1*. The upper resistor of the feedback divider (R_{es}) and the reference voltage (E_R) can be freely selected.

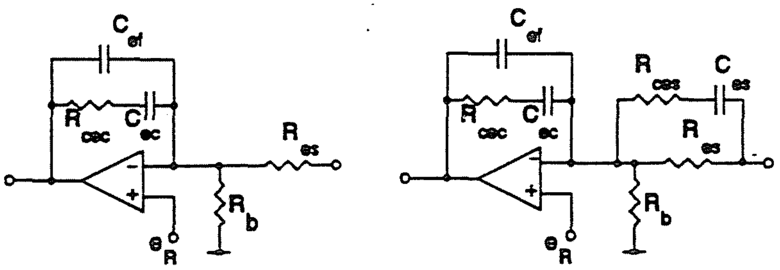

Fig. 6. Error amplifiers: (a) integrator with additional zero-pole pair (Type 2. amp.), (b) integrator with two coinciding pairs of zeros and poles (Type 3.)

Table 1
Expressions to calculate the components of the error amplifiers

	Type 2	Type 3
R_{es}	user-selected	
R_{cec}	$\frac{K^2}{K^2-1}G_{amp}R_{es}$	$\frac{\sqrt{K}}{K-1}G_{amp}R_{es}$
R_{ces}	not used	$\frac{R_{es}}{K-1}$
C_{ec}	$\frac{K^2-1}{K}\frac{1}{2\pi f_0 G_{amp}R_{es}}$	$\frac{K-1}{2\pi f_0 G_{amp}R_{es}}$
C_{ef}	$\frac{1}{K}\frac{1}{2\pi f_0 G_{amp}R_{es}}$	$\frac{1}{2\pi f_0 G_{amp}R_{es}}$
C_{es}	not used	$\frac{K-1}{\sqrt{K}}\frac{1}{2\pi f_0 R_{es}}$

After calculating all the elements of the analyzed model we can easily obtain the Bode plots of the resulting loop gain by entering the component values and performing AC loop gain analysis.

It is informative to observe the transient response of the regulator to step changes in the load current or input voltage. Ringing in those waveforms indicates the lack of sufficient phase margin (M_p). Also, by determining the peak-to-peak deviation of the output voltage, we can get quantitative information on the dynamic regulation. The transient analysis capability of DYANA makes it very easy to obtain the transient responses.

5. Summary

An efficient method to model and analyse DC/DC converters has been presented. The advantage of using DYANA is that we can optimize basic types of DC/DC converters without breadboarding a model circuit. The applied theory of operation is the injected and absorbed current method. It is a completely general tool, and it is sufficient for describing all of well-known types of DC/DC converters. The improved versions of DYANA will contain other types of converter topologies as well.

References

1. KISLOVSKI, A. — REDL, R. — SOKAL, N. O. (1991): Dynamic Analysis of Switching-Mode DC/DC Converters, Van Nostrand Reinhold, New York, ISBN 0-442-23916-5.
2. REDL, R.: Small-Signal High-Frequency Analysis of the Free-Running Current-Mode-Controlled Converter, PESC '91 Record (IEEE Catalog no. 91CH3008-0), pp. 897–906.