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MEASUREMENT OF THE SLIP DEPENDENCE OF THE PARAMETERS AND EVALUATION OF THE STEADY STATE TORQUE-SPEED CURVE OF ASYNCHRONOUS MACHINES BASED ON A RUN-UP TEST

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Abstract

Traditional methods for measuring the slip dependence of the parameters and the torque of asynchronous motors in steady state condition give unreliable results due to the great changes of the winding resistances when the motor is operated along the range $0 \leq s \geq 1$. The steady state torque-speed curve at nominal voltage can be calculated on the base of the equivalent circuit, if the true slip ddependence of its elements is known.

Keywords: asynchronous machines, slip dependent parameters, run-up test.

1. Introduction

Traditional methods for measuring the slip dependence of the parameters and the torque of asynchronous motors in steady state condition give unreliable results due to the great changes of the winding resistances when the motor is operated along the range $0 \leq s \leq 1$. The torque-speed characteristic can be calculated on the base of the equivalent circuit, if the true slip dependence of its elements is known.

The aim of this paper is to present a method for parameter measurement which yields the components of the equivalent circuit in function of the slip. The method is founded on the sampled values of two line voltages and two line currents of the asynchronous motor during a run-up process. This must be fast enough to avoid significant temperature changes of the motor resistances and slow enough to admit that the successive electromagnetic states of the motor during acceleration can be regarded as a sequence of steady states with virtually constant slip. It is worth to emphasize that the measurement of the speed is not necessary. The stator phase resistance R_s must be known.

2. The Model Equations

If saturation, skin, m.m.f. harmonics and core losses are neglected, the transient behaviour of the asynchronous motor can be described by the voltage equations derived from *Fig. 1*, completed by the equation of mechanical motion [1].

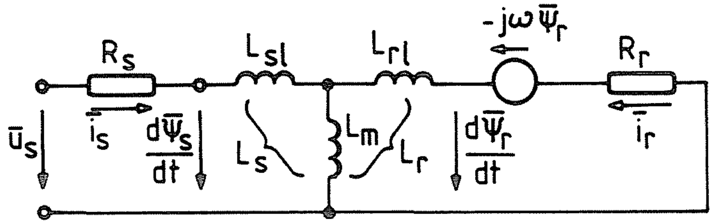

Fig. 1.

Applying the space-vector method these equations can be written as follows:

$$\overline{\overline{u}}_s = \overline{\overline{i}}_s R_s + \frac{d\overline{\overline{\psi}}_s}{dt}, \tag{1}$$

$$0 = \overline{\overline{i}}_r R_r + \frac{d\overline{\overline{\psi}}_r}{dt} - j\omega\overline{\overline{\psi}}_r, \tag{2}$$

$$m = \frac{3}{2}\mathrm{Im}[\overline{\overline{\psi}}_s^{*}\overline{\overline{i}}_s] = \theta\frac{d\omega}{dt}. \tag{3}$$

The fluxes and currents are related by the expressions:

$$\overline{\overline{\psi}}_s = L_s\overline{\overline{i}}_s + L_m\overline{\overline{i}}_r, \tag{4}$$

$$\overline{\overline{\psi}}_r = L_m\overline{\overline{i}}_s + L_r\overline{\overline{i}}_r. \tag{5}$$

If $\overline{\overline{u}}_s(t)$ and the machine parameters are known, the response functions $\overline{\overline{i}}_s(t)$ and $\omega(t)$ can be calculated from the *Eqs.* (1) to (5). The aim is now to perform an inverse operation; namely that one when the excitation $\overline{\overline{u}}_s(t)$ and the response $\overline{\overline{i}}_s(t)$ are given and the parameters have to be evaluated. The measurement of $\omega(t)$ with the proper accuracy is a very hard — if not unsolvable — problem. Fortunately, if R_s is known, instead of having an input file, $\omega(t)$ can be calculated, as it will be shown later.

3. Calculation of the Parameters

The rotor current $\bar{\bar{i}}_r$ can be expressed from (4):

$$\bar{\bar{i}}_r = \frac{1}{L_m}(\bar{\bar{\psi}}_s - L_s\bar{\bar{i}}_s). \tag{6}$$

Its time derivative is:

$$\frac{d\bar{\bar{i}}_r}{dt} = \frac{1}{L_m}\left(\frac{d\bar{\bar{\psi}}_s}{dt} - L_s\frac{d\bar{\bar{i}}_s}{dt}\right). \tag{7}$$

Substituting (6) and (7) in *Eq.* (2), this takes the form:

$$\frac{d\bar{\bar{\psi}}_s}{dt} - j\omega\bar{\bar{\psi}}_s = \frac{L_sR_r}{L_r}\bar{\bar{i}}_s - \frac{R_r}{L_r}\bar{\bar{\psi}}_s + \left(L_s - \frac{L_m^2}{L_r}\right)\left(\frac{d\bar{\bar{i}}_s}{dt} - j\omega\bar{\bar{i}}_s\right). \tag{8}$$

$d\bar{\bar{\psi}}_s/dt$ can be expressed from (1):

$$\frac{d\bar{\bar{\psi}}_s}{dt} = \bar{\bar{u}}_s - R_s\bar{\bar{i}}_s. \tag{9}$$

From here:

$$\bar{\bar{\psi}}_s = \bar{\bar{\psi}}_s(t) = \int_0^t (\bar{\bar{u}}_s - R_s\bar{\bar{i}}_s)d\tau \tag{10}$$

or in real form:

$$\psi_x = \int_0^t (u_x - R_si_x)d\tau, \tag{10a}$$

$$\psi_y = \int_0^t (u_y - R_si_y)d\tau. \tag{10b}$$

Integrating *Eq.* (3) from $t = 0$ to a sufficiently great upper limit, the mechanical angular velocity reaches its steady state value. This is equal — with an approximation of a few tenth per cent — to the synchronous speed:

$$\frac{3}{2}\int_0^\infty \mathrm{Im}[\bar{\bar{\psi}}_s^*\bar{\bar{i}}_s]d\tau = \theta\omega_1. \tag{11}$$

The left side of (11) is the area under the moment versus time curve (i.e. the final value of the moment of momentum of the rotor). Denoting this with M_m, the inertia moment of the rotor can be expressed as

$$\theta = \frac{M_m}{2\pi f_1}. \tag{12}$$

This can be used for the calculation of the instantaneous values of the mechanical angular velocity:

$$\omega = \omega(t) = \frac{2\pi f_1}{M_m} \int_0^t \mathrm{Im}[\overline{\overline{\psi}}_s^* \bar{\bar{i}}_s] d\tau$$

or decomposing $\overline{\overline{\psi}}_s$ and $\bar{\bar{i}}_s$ in their real and imaginary parts:

$$\omega = \frac{2\pi f_1}{M_m} \int_0^t (\psi_x i_y - \psi_y i_x) d\tau. \tag{13}$$

In order to calculate the motor parameters from *Eq.* (8), $\bar{\bar{i}}_s$, $d\bar{\bar{i}}_s/dt$, $\overline{\overline{\psi}}_s$ and ω must be disposables as known entries.

The appearance of $d\bar{\bar{i}}_s/dt$ in (8) requires a strategic decision: it must be either calculated from the file of $\bar{\bar{i}}_s$ or eliminated by integration of (8).

Both operations can be carried out easily by software means. Nevertheless, derivation enhances the noises, therefore a filter must be applied. However, it is very difficult — if it is possible at all — to define the frequency range which may be eliminated from $d\bar{\bar{i}}_s/dt$ without distorting the measured information.

The integration suppresses the noises with zero mean value, but the offset errors inevitably present in the measured data cause accumulative errors in $\overline{\overline{\psi}}_s$ and ω. However, it is obvious that $\bar{\bar{u}}_s$ must not contain D.C. component at all, and not even $\bar{\bar{i}}_s$ and $\overline{\overline{\psi}}_s$ in their steady-state condition. These are physical evidences; therefore they authorize the supervision of the measured $\bar{\bar{u}}_s$, $\bar{\bar{i}}_s$ and calculated $\overline{\overline{\psi}}_s$ data in order to eliminate the offset errors. The successive mean values of the flux components $\psi_x(t)$ and $\psi_y(t)$ over the period $T = 1/f_1$ must have a strictly constant value after reaching steady state conditions.

Opting for the alternative of integration and introducing for the unknown machine parameters the notations

$$p_1 = L_s R_r / L_r = L_s / T_{ro}; \quad p_2 = R_r / L_r = 1/T_{ro};$$
$$p_3 = L_s - L_m^2 / L_r = \sigma L_s \tag{14}$$

Eq. (8) can be rearranged as follows:

$$\overline{\overline{\psi}}_s(t_2) - \overline{\overline{\psi}}_s(t_1) - j\int_{t_1}^{t_2} \omega\overline{\overline{\psi}}_s d\tau = p_1\int_{t_1}^{t_2} \overline{\overline{i}}_s d\tau - p_2\int_{t_1}^{t_2} \overline{\overline{\psi}}_s d\tau +$$
$$+ p_3\left[\overline{\overline{i}}_s(t_2) - \overline{\overline{i}}_s(t_1) - j\int_{t_1}^{t_2} \omega\overline{\overline{i}}_s d\tau\right]. \tag{15}$$

This equation can be decomposed in its real and imaginary part:

$$\psi_x(t_2) - \psi_x(t_1) + \int_{t_1}^{t_2} \omega\psi_y d\tau = p_1\int_{t_1}^{t_2} i_x d\tau - p_2\int_{t_1}^{t_2} \psi_x d\tau +$$
$$+ p_3\left[i_x(t_2) - i_x(t_1) - \int_{t_1}^{t_2} \omega i_y d\tau\right], \tag{16a}$$

$$\psi_y(t_2) - \psi_y(t_1) - \int_{t_1}^{t_2} \omega\psi_x d\tau = p_1\int_{t_1}^{t_2} i_y d\tau - p_2\int_{t_1}^{t_2} \psi_y d\tau +$$
$$+ p_3\left[i_y(t_2) - i_y(t_1) - \int_{t_1}^{t_2} \omega i_x d\tau\right]. \tag{16b}$$

Eqs. (16a) and (16b), as well the auxiliary *Eqs.* (10a), (10b) and (13) represent a continuous time model. However, this can be considered as a discrete one, replacing t by

$$t_i = (i-1)/f_s; \quad 1 \le i \le N, \tag{17}$$

where f_s is the sampling frequency and N is the number of samples, generally not less than 4096. Due to the free choice of the lower and upper limit of the integrals in (16a) and (16b), the number of equations disposable for parameter calculation is practically unlimited. The great redundancy permits the computation of the roots by the least squares method, and in addition, in function of the time or the slip.

Computational experiments proved that a set of ten equations were sufficient to calculate with good accuracy a triplet of roots belonging to a selected moment t_i. Let rewrite (16a) and (16b) in the more general form:

$$C_1(t_i) = A_{11}(t_i)p_1(t_i) + A_{12}(t_i)p_2(t_i) + A_{13}(t_i)p_3(t_i), \tag{18a}$$
$$C_2(t_i) = A_{21}(t_i)p_1(t_i) + A_{22}(t_i)p_2(t_i) + A_{23}(t_i)p_3(t_i). \tag{18b}$$

For convenience the limits will be chosen symmetrically to t_i:

$$A_{11}(t_i) = \int_{t_i-\Delta \mathrm{T}}^{t_i+\Delta \mathrm{T}} i_x d\tau, \tag{19}$$

$$A_{12}(t_i) = -\int_{t_i-\Delta \mathrm{T}}^{t_i+\Delta \mathrm{T}} \psi_x d\tau. \tag{20}$$

$$A_{13}(t_i) = i_x(t+\Delta T) - i_x(t-\Delta T) + \int_{t_i-\Delta T}^{t_i+\Delta T} \omega i_y d\tau, \tag{21}$$

$$A_{21}(t_i) = \int_{t_i-\Delta T}^{t_i+\Delta T} i_y d\tau, \tag{22}$$

$$A_{22}(t_i) = -\int_{t_i-\Delta T}^{t_i+\Delta T} \psi_y d\tau. \tag{23}$$

$$A_{23}(t_i) = i_y(t_i+\Delta T) - i_y(t_i-\Delta T) - \int_{t_i-\Delta T}^{t_i+\Delta T} \omega i_x d\tau, \tag{24}$$

$$C_1(t_i) = \psi_x(t_i+\Delta T) - i_y(t_i-\Delta T) + \int_{t_i-\Delta T}^{t_i+\Delta T} \omega \psi_y d\tau, \tag{25}$$

$$C_2(t_i) = \psi_y(t_i+\Delta T) - \psi_y(t_i-\Delta T) - \int_{t_i-\Delta T}^{t_i+\Delta T} \omega \psi_x d\tau. \tag{26}$$

$$\Delta T = T/4 = 1/4f_1 \ . \tag{27}$$

In order to construct further equations, additional coefficients have to be calculated. It seems reasonable to select 4 equidistant points located symmetrically around t_i:

$$\begin{aligned} t_i^I &= t_i - 2T, \\ t_i^{II} &= t_i - T, \\ t_i^{III} &= t_i + T, \\ t_i^{IV} &= t_i + 2T, \\ T &= 1/f_1. \end{aligned} \tag{28}$$

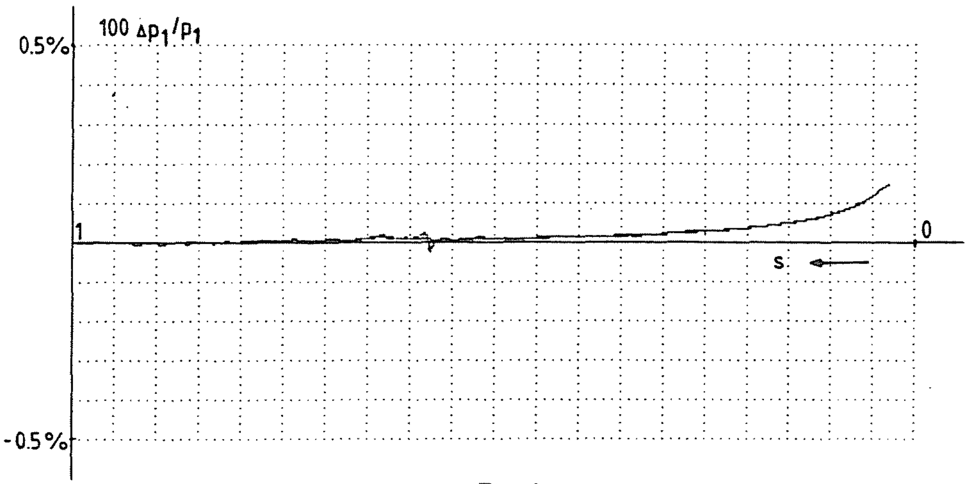

Fig. 2.

Replacing t_i as suggested by (28), 4 new sets of coefficients (19)-(26) are available. The system of equations for the roots pertaining to t_i written in matrix form will be as follows:

$$K(t_i)p(t_i) = C(t_i). \tag{29}$$

$K(t_i)$ is a 10 by 3 and $C(t_i)$ is a 10 by 1 matrix. The symbols $K(t_i)$ and $C(t_i)$ have to bring into prominence that both matrices are associated with the roots belonging to t_i although they contain also elements calculated with the base points defined by (28).

The parameter matrix is defined as

$$p(t_i) = [p_1(t_i)\ p_2(t_i)\ p_3(t_i)]^T. \tag{30}$$

The solution of (29) can be written in the form:

$$p(t_i) = [K^T(t_i) \cdot K(t_i)]^{-1} K^T(t_i) \cdot C(t_i). \tag{31}$$

The reliability of the above proposed algorithm has been tested with single type input data calculated by the 4th order R.K. method with balanced 3 phase input voltage and known machine parameters. The errors of the calculated parameters against their true values are shown in *Figs. 2, 3, 4* in function of the slip. It has to be taken into consideration that the parameters are the outputs of a rather intricate computational process which involves a certain amount of accumulated errors. In the light of this the *Figs. 2, 3* and *4* may be regarded as the proof of consistency and reliability of the algorithm.

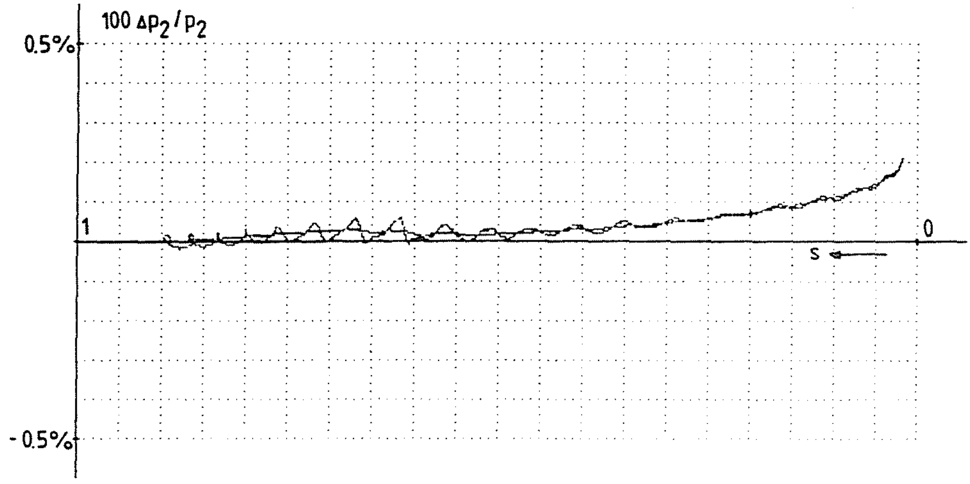

Fig. 3.

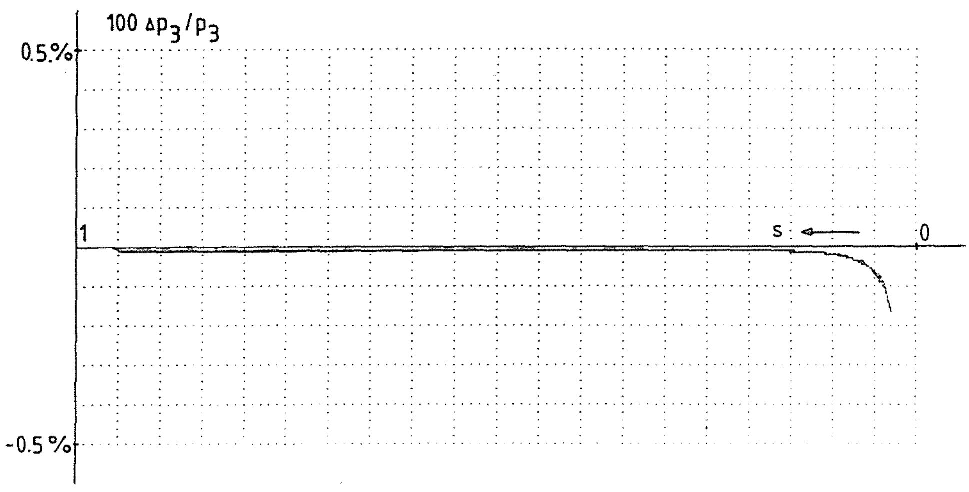

Fig. 4.

4. Switching the Motor to the Network for Data Measurement

Two types of D.C. currents arise in asynchronous motors during transient operation. The first one is needed to build up the magnetic energy stored in the leakage field, the role of the second one is the same with respect to the main field. The first D.C. component vanishes very quickly, but the time constant of the second is much greater, therefore it vanishes slowly. The magnetization of the core becomes asymmetrical during its existence and the peak values of the flux density reach much higher values then in steady state operation. The inductances of the machine diminish, consequently measured data containing slow D.C. component must not be used for parameter identification, when steady state parameters are requested.

In order to overcome this problem a specific switching on process can be applied [1, p. 66]. Experience has proved that the best results concerning the elimination of the D.C. components can be achieved by reversing the phase sequence on the terminals. A sensor is needed in order to detect the instant $\omega = 0$ and trigger the sampling, or to appoint the first set of data to be taken into consideration.

5. Notes on the Parameters Calculated from Measured Data

The parameters p_1, p_2, p_3 do not fit with those of the equivalent circuit, but they are in perfect accordance with the per phase input impedance of the asynchronous motor:

$$Z = R_s + j\omega_1 L_s \frac{1 + j\sigma s\omega_1 T_{ro}}{1 + js\omega_1 T_{ro}}. \tag{32}$$

Replacing the p_i-s from (14) the input impedance takes the form:

$$Z = R_s + j\omega_1 \frac{p_1 + js\omega_1 p_3}{p_2 + js\omega_1}. \tag{33}$$

Strictly speaking, the expression (33) represents a serial impedance with the components $\mathrm{Re}[Z(s)]$ and $\mathrm{Im}[Z(s)]$ and although describes perfectly the relation between $\overline{\overline{u}}_s$ and $\bar{\bar{i}}_s$, does not allow the construction of a unique T-shaped (equivalent) circuit.

In order to do this an additional and arbitrary condition has to be introduced. This may be done prescribing the stator and reduced rotor inductances to be equal:

$$L_s = L_r = L. \tag{34}$$

Substituting (34) in (14) the expressions of the parameters become:

$$p_1(s) = R_r(s); \quad p_2(s) = R_r(s)/L(s); \quad p_3 = \sigma(s)L(s). \tag{35}$$

The parameters $p_i(s)$ have to be calculated from measured voltage and current data files. They allow the computation of the main and leakage filed reactances $X_m(s)$ and $X_l(s)$ of the commonly used equivalent circuit (*Fig. 5*).

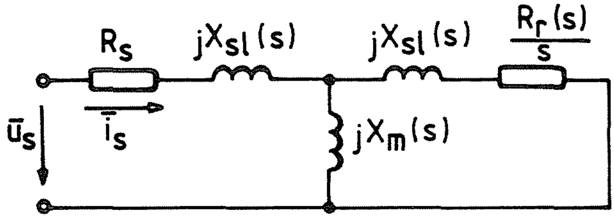

Fig. 5.

6. Test of the Method by Measured Data

The measurements have been carried out with the sampling frequency $f_s = 100 f_1$ and using 12 bit A/D converters, consequently the input data for parameter calculation have been integers in the range of $-2047 \leq x \leq 2048$. This denotes a significant loss of information in comparison to the single type input data; nevertheless the calculated values of the parameters do not display any remarkable local dispersion, or in other words, the curves on the *Figs. 6, 7* and *8* are reasonably smooth.

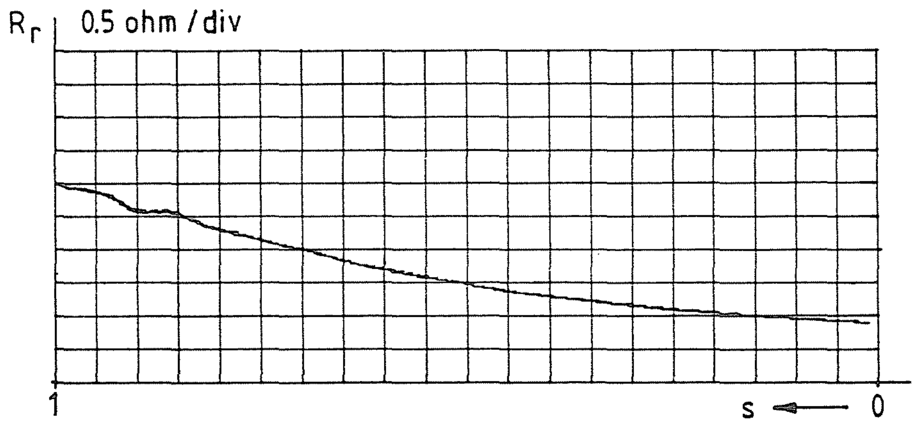

Fig. 6.

In opinion of the authors, the reliability of the proposed method is founded upon the excellent accuracy pointed out in Chapter 3. *Fig. 9* shows the steady state torque-slip curve, calculated using the slip dependent parameters.

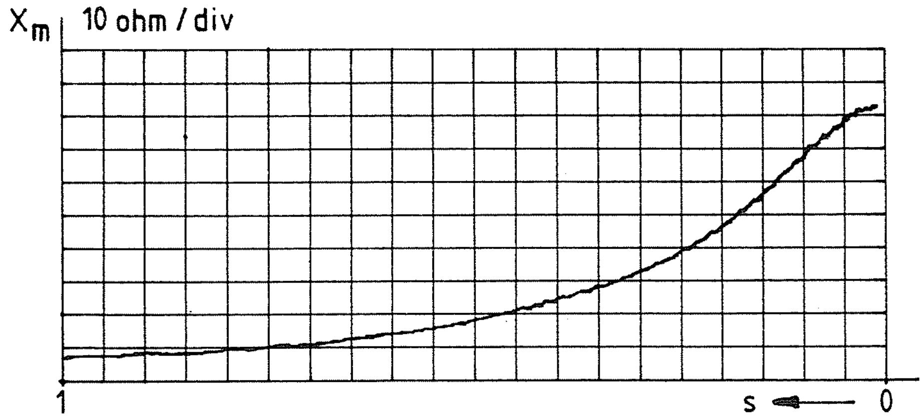

Fig. 7.

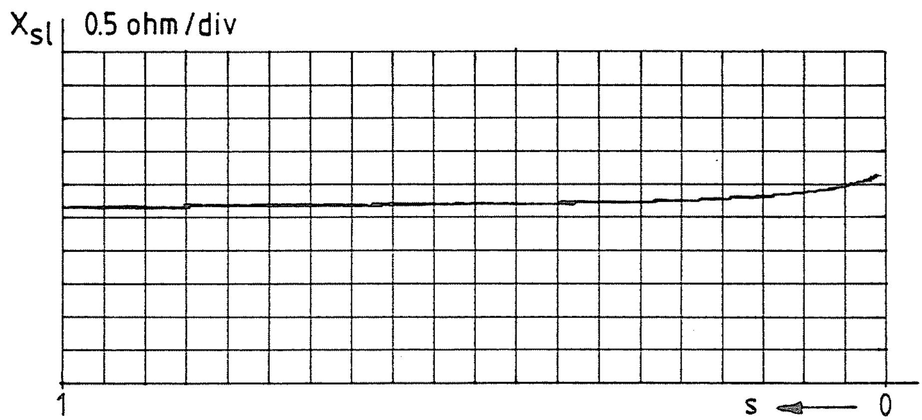

Fig. 8.

7. Summary

A relatively simple method for the calculation of the slip dependent parameters of the asynchronous motor is proposed. The measured data are the sampled values of two line voltages and two line currents under slightly slowed-down transient run-up conditions. The model of the machine is the one used by KOVÁCS and RÁCZ [1]. If the stator resistance is known, the

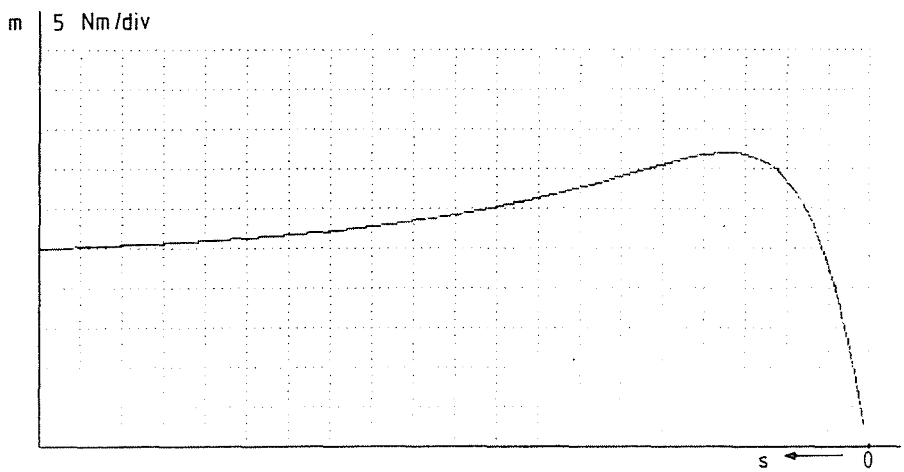

Fig. 9.

stator flux as well as the angular velocity of the shaft and the inertia moment of the rotor can be calculated from the measured data.

The elimination of the rotor current from the voltage equations written in terms of space phasors yields a relationship between four complex quantities depending on $\bar{\bar{i}}_s$, $\bar{\bar{\psi}}_s$, ω and three real quantities depending on the machine parameters σL_s, $1/T_{ro}$ and L_s/T_{ro} . By means of an adequate algorithm the supplementary condition $L_s = L_r$ this can be used for the calculation of the slip dependent values of the parameters and the steady-state torque-speed characteristic.

Acknowledgement

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INDEX