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THEORY AND PRACTICE OF LINEARLY TUNABLE LC OSCILLATORS

L. PAP

Department of Telecommunications
Technical University, H-1521 Budapest

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Abstract

This paper is concerned with the theory and design of linearly tunable LC oscillators using variable transductance elements and linear twoports. The new ideal model introduced in the paper can be considered as a generalization of the well-known and widely used admittance transistor circuits. The new ideal structure uses linearly tunable transductance elements and two linear twoports with Hilbert transform transfer function. The ideal model and its approximate non ideal versions are analyzed and the *f-V* characteristics is presented in each case. One of the approximate versions of the general ideal structure can easily be implemented by IC technology.

Keywords: tunable oscillators, LC oscillators, VCO-s, admittance transistor, Hilbert transform.

Introduction

PLL's are among the most important components in coherent transmission systems. All PLL's require at least one electronically tunable oscillator (VCO) (VITERBI, 1966), (LINDSEY, 1972), (LINDSEY, 1978). Transient behaviour of the PLL is influenced by a number of factors, e.g. phase detector characteristics, transfer characteristics of the loop filter, the tuning characteristics of VCO. It has long been an aim of practical PLL designers to achieve constant loop gain in the whole tuning range. This requires among others linear tuning characteristics of the VCO, that is, an oscillator must be constructed of which the frequency can be linearly altered with some electronically tunable parameter. Several solutions have been published in the literature and used in practice for this problem [(GREBENE, 1972a), (GREBENE, 1984), (YOUNG, 1981), (GREBENE, 1984), (EGAN, 1981)]. Among these circuits the relaxation type RC oscillators are of widest use (GREBENE, 1972b), (EXAR, 1981). These circuits can be tuned linearly over a large range by changing a bias current but their frequency is rather sensitive to changes in the environment, i.e. mainly to the temperature . This limits severely the applicability of these oscillators in narrowband PLL's especially when false synchronization is possible. Temperature dependence can be drastically decreased by utilization of crystals but there arise other problems: the tuning range of quartz oscillators is too small for most purposes, parasitic modes of oscillation are often present and secondary parameters of crystals show a large variance in production. Thus an intermediate solution between RC and quartz circuits seems to be the right one for our purposes, namely the electronically tunable LC oscillator. Classical circuits (varicap tuning, admittance transistor) have strongly nonlinear tuning characteristics that can be compensated by any of the numerous known methods (making use of some memoryless nonlinear two- or fourpoles) but here again the problems of temperature dependence and production spread arise and direct our attention to some other constructions with inherent linearity. In the present paper a new oscillator model (PAP, 1990), (PAP, 1991) is introduced and analyzed in which voltage controlled transductance devices are used to realize a linear *f*-*V* characteristics. Between the above mentioned two extreme solutions the use of this LC based oscillator seems to be an appropriate compromise, namely the long term stability and the tuning range can be acceptable, and the linearity is fairly good.

1.Ideal Model of the New Linearly Tunable LC Oscillator (Linear Oscillation Criterion)

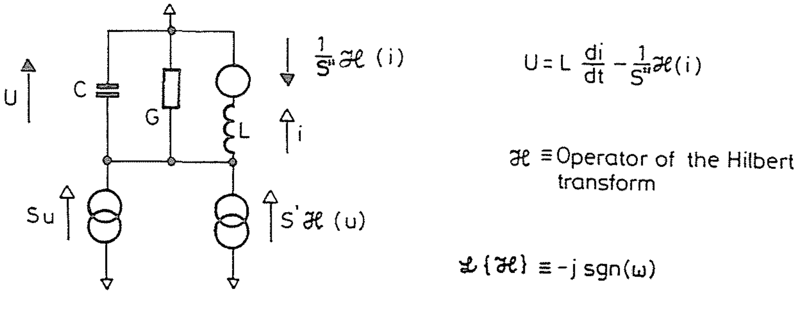

Fig. 1. Ideal (linear) model of the novel LC based linearly tunable VCO

A general theoretical model of the class of oscillator in question is shown in *Fig. 1.* The basic idea is to achieve tuning by linearly changing a gain or transfer admittance as this can be readily realized. The differential equation describing the network is the following:

$$i + CL\frac{d^2i}{dt^2} - \frac{C}{S''}\mathcal{H}\left(\frac{di}{dt}\right) - S'L\mathcal{H}\left(\frac{di}{dt}\right) +$$

$$+\frac{S'}{S''}\mathcal{H}(\mathcal{H}(i)) + \left(L\frac{di}{dt} - \frac{1}{S''}\mathcal{H}(i)\right)(G - S) = 0. \tag{1.1}$$

Here G is the loss of the LC resonant circuit, S is the transfer admittance of the active fourpole, S' and S'' are tunable parameters and $\mathcal{H}(\cdot)$ denotes Hilbert transformation. *Eq.* (1.1) can be solved by using a constant amplitude sinewave trial function. The following equation is produced:

$$1 - CL\omega^2 - \frac{C}{S''}\omega\, sgn(\omega) - S'L\omega sgn(\omega) -$$

$$-\frac{S'}{S''}[sgn(\omega)]^2 + \left(jL\omega + \frac{j}{S''}sgn(\omega)\right)(G - S) = 0 \tag{1.2}$$

that yields as the so-called linear oscillation criteria the equations:

$$1 - CL\omega^2 - \frac{C}{S''}\omega\, sgn(\omega) - S'L\omega\, sgn(\omega) - \frac{S'}{S''}[sgn(\omega)]^2 = 0,$$

$$\left(L\omega + \frac{1}{S''}sgn(\omega)\right)(G - S) = 0. \tag{1.3}$$

The second equation is called the amplitude condition and it yields the well-known formula

$$(G - S) = 0. \tag{1.4}$$

By choosing

$$S' = S^*C \quad \text{and} \quad S'' = \frac{1}{S^*L} \tag{1.5}$$

the frequency condition can be written in the form

$$\frac{1}{LC} = \omega^2 + 2\omega S^* sgn(\omega) + [S^* sgn(\omega)]^2 = [\omega + S^* sgn(\omega)]^2,$$

$$\omega_r = \frac{1}{\sqrt{LC}} - S^*. \tag{1.6}$$

It is clear from (1.6) that by linearly varying S^* the oscillation frequency will change linearly as well. S^* can take on positive and negative values.

In the ideal case linear tuning can be achieved by means of two real Hilbert transformer circuits, that cannot be realised (or, to be more precise, the circuits needed have a very high degree of complexity). Simple approximate solutions are sought in the following but first the nonlinear oscillation criteria will be analysed in the presence of a hard limiter with the aid of the simplest harmonic balance equations.

2. Nonlinear Oscillation Criteria of the LC VCO (First Order Harmonic Balance Equations)

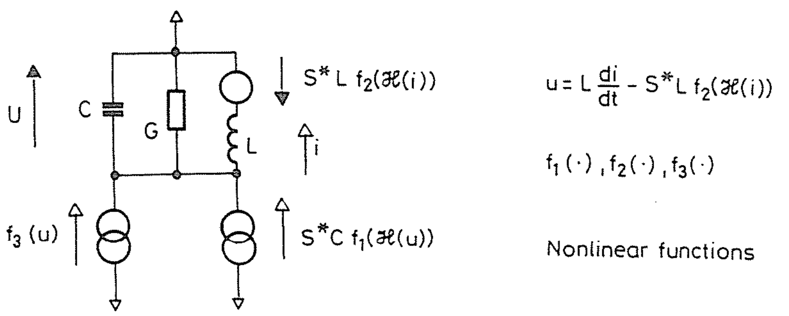

Fig. 2. Nonlinear model of the LC based VCO

Nonlinear model in question is shown in *Fig. 2.* The only difference between this circuit and the above treated one is that this one contains controlled generators with nonlinear characteristics. Nonlinearities are described by the functions $f_1(\cdot), f_2(\cdot)$ and $f_3(\cdot)$. General differential equation of the system can be formulated as

$$CL\tfrac{d^2i}{dt^2} - S^*LC\tfrac{d}{dt}\left[f_2(\mathcal{H}(i))\right] - S^*Cf_1\left(\mathcal{H}\left(L\tfrac{di}{dt} - S^*Lf_2(\mathcal{H}(i))\right)\right) + \\ +i + GL\tfrac{di}{dt} - GS^*Lf_2(\mathcal{H}(i)) - f_3\left(L\tfrac{di}{dt} - S^*Lf_2(\mathcal{H}(i))\right) = 0. \quad (2.1)$$

In the following the functions are supposed to be of the form

$$f(x) = A\frac{\Pi}{4}sgn(x) \quad \text{or} \quad f(x) = x. \quad (2.2)$$

(2.1) will be solved by applying first order harmonic balance equations. If

$$i(t) = I\cos(\omega t), \tag{2.3}$$

then

$$\frac{di}{dt} = -I\omega\sin(\omega t), \quad \frac{d^2i}{dt^2} = -I\omega^2\cos(\omega t), \quad \mathcal{H}(i(t)) = I\sin(\omega t). \tag{2.4}$$

a. VCO with Linear Frequency Determining Components

Let us suppose that the frequency determining elements are linear, that is $f_1(x) = f_2(x) = x$ and

$$f_3(u) = I_0\frac{\Pi}{4}sgn(u). \tag{2.5}$$

In this case the first order harmonic balance equation concerning the main harmonic components takes on the form

$$-CLI\omega^2\cos(\omega t) - S^*CLI\omega\cos(\omega t) - S^*CLI(\omega + S^*)\cos(\omega t) + I\cos(\omega t) - \\ - GLI\omega\sin(\omega t) - GS^*LI\sin(\omega t) + I_0\sin(\omega t) = 0, \tag{2.6}$$

that yields

$$\omega_r = \frac{1}{\sqrt{LC}} - S^*, \\ I_r = I_0\frac{1}{G}\sqrt{\frac{C}{L}} = I_0Q_0\,. \tag{2.7}$$

This means that the amplitude of oscillation is constant and its frequency is linearly dependent on S^*.

b. VCO with Nonlinear Frequency Determining Components

Let $f_3(u)$ be given by (2.5) and let the other two functions be nonlinear as well:

$$f_1(x) = U_s\frac{\Pi}{4}sgn(x) \quad \text{and} \quad f_2(x) = I_s\frac{\Pi}{4}sgn(x). \tag{2.8}$$

The harmonic balance equation now is of the form

$$-CLI\omega^2\cos(\omega t) - S^*CLI_s\omega\cos(\omega t) - S^*CU_s\cos(\omega t) + I\cos(\omega t) - \\ - GLI\omega\sin(\omega t) - GS^*LI_s\sin(\omega t) + I_0\sin(\omega t) = 0. \qquad (2.9)$$

Simple transformations yield

$$I_0 = GL[\omega I + S^*I_s]\,, \\ CLI\omega^2 + CLI_sS^*\omega + S^*CU_s = I. \qquad (2.10)$$

Oscillation frequency can be computed from

$$[1 - CL\omega^2] = G\frac{CL\omega S^*[\omega LI_s + U_s]}{I_0 - GLS^*I_s}. \qquad (2.11)$$

After introducing the following notations:

$$\omega_r = \frac{1+x}{\sqrt{LC}} \quad ; \quad x = \omega_r\sqrt{LC} - 1 \quad ; \quad y = S^*\sqrt{LC} \qquad (2.12)$$

and performing some transformations we obtained the tuning characteristics:

$$y = \frac{1-(1+x)^2}{(1+x)\frac{U_sG}{I_0} + \frac{I_s}{I_0Q_0}}. \qquad (2.13)$$

Notice that if

$$\frac{U_sG}{I_0} = \frac{I_s}{I_0Q_0} \qquad (2.14)$$

the above equation is equivalent to (2.7), that is

$$y = -x\frac{I_0}{U_sG}. \qquad (2.15)$$

The (2.13) tuning characteristics is shown normalized in *Fig. 3* with different values of U_sG and I_s/Q_0. If $b/a = 0$ or $b/a = \infty$ one of the tuning controlled generators in *Fig. 2* is not needed.

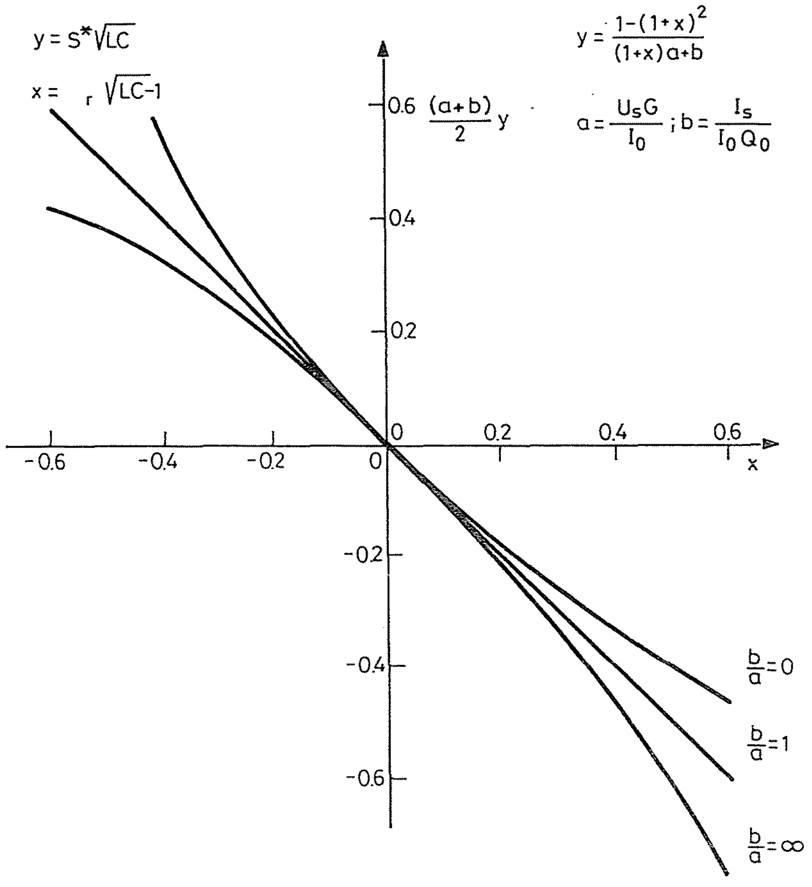

Fig. 3. Tuning characteristics of the VCO shown in *Fig. 2*

3. Approximate Solutions of the Hilbert transformation

Realizability of the circuit shown in *Fig. 1* is basically limited by the presence of the Hilbert transformers. In the following the typical approximate solutions will be introduced that fulfil a vital requirement in oscillator circuits: that of simplicity. These are (i) the ideal delay line, (ii) the derivating circuit and (iii) the first order all-pass fourpole.

a. Computation of the Characteristics in the Case of Linear Oscillation Conditions

If the Hilbert transformers in *Fig. 1* are replaced by a general linear fourpole with operator $\mathcal{A}$ then the following differential equation will describe the system:

$$i + CL\frac{d^2i}{dt^2} - 2S^*CL\mathcal{A}(\frac{di}{dt}) + (S^*)^2CL\mathcal{A}(\mathcal{A}(i)) + \left(L\frac{di}{dt} - S^*L\mathcal{A}(i)\right)(G - S) = 0. \tag{3.1}$$

Solving this equation by applying a constant amplitude trial function we obtain the general characteristic equation

$$1 - CL\omega^2 - 2S^*CLj\omega A(j\omega) + (S^*)^2CLA^2(j\omega) + (j\omega L - S^*LA(j\omega))(G - S) = 0. \tag{3.2}$$

Here $A(j\omega)$ denotes transfer function corresponding to the linear operator $\mathcal{A}$. The conditions for oscillation are

$$1 - CL\omega^2 + 2S^*CL\omega Im(A(j\omega)) + (S^*)^2CLRe(A^2(j\omega)) - S^*LRe(A(j\omega))(G - S) = 0,$$
$$-2S^*CL\omega Re(A(j\omega) + (S^*)^2CLIm(A^2(j\omega)) + (\omega L - S^*LIm(A(j\omega)))(G - S) = 0. \tag{3.3}$$

After some simple but tedious computations the following tuning characteristics are produced:

— Ideal delay line, $\tau = \frac{\Pi}{2}\sqrt{LC}$:

$$A(j\omega) = \exp(-j\omega\tau)$$
$$[1 - (1 + x^2)]\left[(1 + x) + y\cos\left(\tfrac{\Pi}{2}x\right)\right] - 2(1 + x)y\left[y + (1 + x)\cos\left(\tfrac{\Pi}{2}x\right)\right] - -y^2\left[(1 + x)\cos(\Pi x) + y\cos\left(\tfrac{\Pi}{2}x\right)\right] = 0\,. \tag{3.4}$$

— Derivator, $\mathcal{A} \equiv -\sqrt{LC}\frac{d}{dt}$:

$$A(j\omega) = -\sqrt{LC} j\omega \,,$$
$$y = -\tfrac{x}{1+x} \,. \qquad (3.5)$$

— First order all-pass fourpole:

$$A(j\omega) = \frac{1 - j\omega\sqrt{LC}}{1 + j\omega\sqrt{LC}} \,,$$

$$\begin{aligned} &\left(1-(1+x)^2\right)^3 + y^2\left(1-(1+x)^2\right)^2 + 4(1+x)^2\left(1-(1+x)^2\right) + \\ &+2y\left(1-(1+x)^2\right)\left(1+(1+x)^2\right) - 4(1+x)^2 y\left(1+(1+x)^2\right) - \\ &-2y^2\left(1+(1+x)^2\right)^2 - 4y^2(1+x)^2 - 2y^3\left(1+(1+x)^2\right) = 0 \,. \qquad (3.6) \end{aligned}$$

The notations

$$y = S^*\sqrt{LC} \quad ; \quad \omega_r = \frac{1+x}{\sqrt{LC}} \qquad (3.7)$$

are the same as in the above treatments. In the case of multiple roots the oscillation frequency can be obtained from the main branch of the function associated with the trivial ($y = 0$, $x = 0$) solution of the implicit equations.

The practical circuit can be further simplified if the controlled voltage generator of *Fig. 1* is deleted and only the effective capacitance is controlled via the reactance elements.

b. Linear Oscillation Conditions for the Capacitively Tuned One-sided System

The linear differential equations for the above mentioned system that contains no controlled voltage generator can be put as

$$i + CL\frac{d^2 i}{dt^2} - S^* CLA\left(\frac{di}{dt}\right) + L\frac{di}{dt}(G - S) = 0. \qquad (3.8)$$

The linear oscillation conditions are

$$
\begin{aligned}
1 - CL\omega^2 + S^* CL\omega Im\,(A(j\omega)) = 0\,, \\
-S^* CL\omega Re(A(j\omega)) + \omega L(G - S) = 0.
\end{aligned} \tag{3.9}
$$

The tuning characteristics in the above analysed cases will be the following:
— Ideal delay line, $\tau = \frac{\Pi}{2}\sqrt{LC}$:

$$
\begin{aligned}
A(j\omega) &= \exp(-j\omega\tau)\,, \\
y &= \frac{1-(1+x)^2}{(1+x)\cos\left(\frac{\Pi}{2}x\right)}\,.
\end{aligned} \tag{3.10}
$$

— Derivator, $\mathcal{A} \equiv -\sqrt{LC}\frac{d}{dt}$:

$$
\begin{aligned}
A(j\omega) &= -\sqrt{LC}j\omega\,, \\
y &= \frac{1-(1+x)^2}{(1+x)^2}\,.
\end{aligned} \tag{3.11}
$$

— First order all-pass fourpole:

$$
\begin{aligned}
A(j\omega) &= \frac{1 - j\omega\sqrt{LC}}{1 + j\omega\sqrt{LC}}\,, \\
y &= \frac{1-(1+x)^4}{2(1+x)^2}\,.
\end{aligned} \tag{3.12}
$$

These three characteristics can be compared in *Fig. 4*. It is clear from the figure that the all-pass circuit produces the widest linearity range.

It is worth to note that the characteristics are not influenced by the nonlinearity of the function $f_3(\cdot)$ but in the case of a non ideally 90 degrees phase shift $\mathcal{A}$ operator the nonlinearity of $f_1(\cdot)$ and $f_2(\cdot)$ will drastically reduce the linearity range as shown in the next chapter.

c. Nonlinear Oscillation Conditions for the Capacitively Tuned One-sided Systems

Let us investigate the oscillation conditions of the VCO of *Fig. 5* when (2.5) and (2.8) apply for the functions $f_3(\cdot)$ and $f_1(\cdot)$ and only the basic harmonic

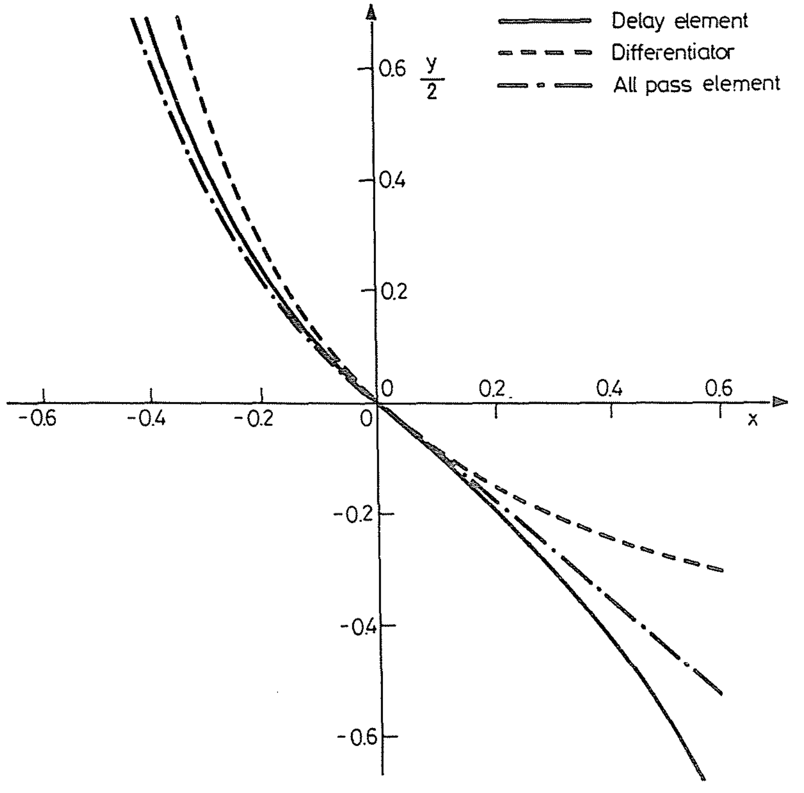

Fig. 4. Comparison of the VCO circuits described in 3.b.

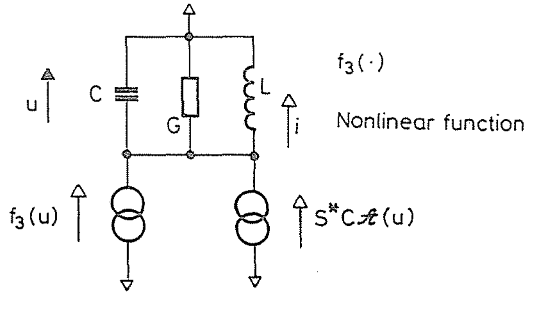

Fig. 5. Nonlinear model of the capacitively tuned VCO

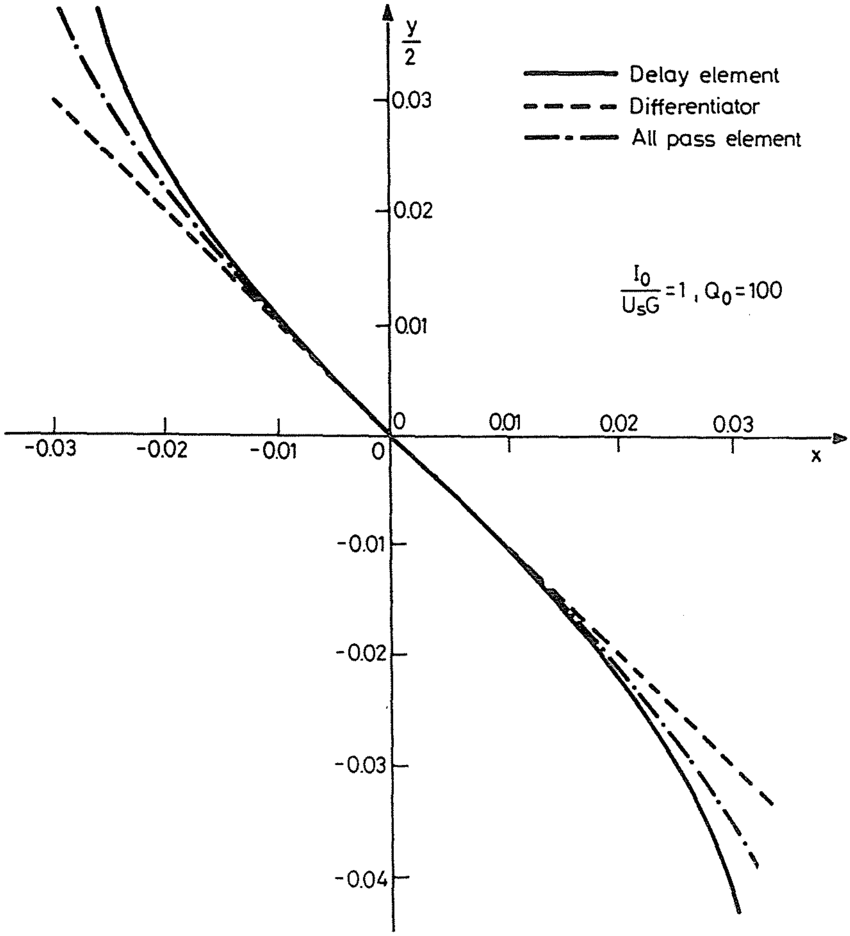

Fig. 6. Comparison of the VCO circuits described in 3.c.

component is considered in the first order harmonic balance equations. The system is described by the differential equation

$$i + CL\frac{d^2i}{dt^2} - S^*Cf_1\left(\mathcal{A}\left(L\frac{di}{dt}\right)\right) + GL\frac{di}{dt} - f_3\left(L\frac{di}{dt}\right) = 0. \quad (3.13)$$

The transfer function corresponding to the linear operator $\mathcal{A}$ is

$$A(j\omega) = a(\omega)\exp(j\varphi(\omega)). \quad (3.14)$$

We obtain for the oscillation conditions:

$$I(1-\omega^2 LC) + S^* C U_s \sin(\varphi(\omega)) = 0$$
$$-GLI\omega + I_0 + S^* C U_s \cos(\varphi(\omega)) = 0. \tag{3.15}$$

Oscillation frequency can be easily computed as

$$(1-\omega^2 LC)\frac{I_0}{GL\omega} = -\frac{S^*}{GL\omega} C U_s \cos(\varphi(\omega))(1-\omega^2 LC) - S^* C U_s \sin(\varphi(\omega)). \tag{3.16}$$

(3.16) can be evaluated as before:

— Ideal delay line, $\tau = \frac{\Pi}{2}\sqrt{LC}$:

$$A(j\omega) = \exp(-j\omega\tau),$$

$$y = \frac{\frac{1-(1+x)^2}{(1+x)}\frac{I_0}{U_s G}}{\frac{1-(1+x)^2}{(1+x)} Q_0 \sin\left(\frac{\Pi}{2}x\right) + \cos\left(\frac{\Pi}{2}x\right)}. \tag{3.17}$$

— Derivator, $\mathcal{A} \equiv -\sqrt{LC}\frac{d}{dt}$:

$$A(j\omega) = -\sqrt{LC} j\omega,$$

$$y = \frac{1-(1+x)^2}{(1+x)}\frac{I_0}{U_s G}. \tag{3.18}$$

— First order all-pass fourpole:

$$A(j\omega) = \frac{1-j\omega\sqrt{LC}}{1+j\omega\sqrt{LC}} \quad ; \quad \varphi(\omega) = -2\arctan\left(\omega\sqrt{LC}\right),$$

$$y = \frac{\frac{1-(1+x)^2}{(1+x)}\frac{I_0}{U_s G}}{-\frac{\left(1-(1+x)^2\right)^2}{(1+x)\left(1+(1+x)^2\right)} Q_0 + 2\frac{(1+x)}{\left(1+(1+x)^2\right)}}. \tag{3.19}$$

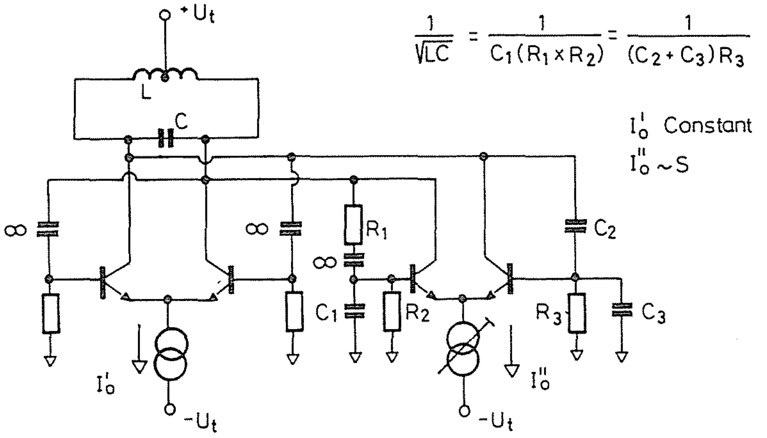

Fig. 7. Simplified diagram of a possible realisation of the VCO described in 2 and 3.

The results are illustrated in *Fig. 6.* It can be seen that in the case of a high Q_0 the characteristics are strongly distorted as compared to *Fig. 5* so it seems to be sensitive to choose the function $f_1(\cdot)$ and $f_2(\cdot)$ to be linear. *Fig. 7* shows a sketch of a circuit that can be realized in integrated form with only a few tuning elements.

4. Conclusions

In the present contribution a new oscillator model is introduced and analyzed, in which voltage controlled transductance devices are used to realize a linear $f - V$ characteristics. This LC oscillator seems to be an appropriate compromise between the relaxation type RC circuits with wide tuning range and high linearity but poor long term stability and quartz oscillators with high stability and poor tunable features. The paper deals with the analysis of the new ideal structure and its approximate versions and the author compares the performances of different voltage controlled LC structures. In contrast with earlier studies it is obvious that the new method is able to enhance the linearity of the circuit. Concluding the paper a practical electronic circuit is shown which can easily be implemented by IC technology.

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Address:

László PAP
Department of Telecommunications
Technical University
H-1521 Budapest, Hungary