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METHODS OF CALCULATION OF MSW STRUCTURES

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Abstract

The paper reviews the existing methods for the solution of structures supporting propagation of magnetostatic waves. Due to the fact that these are mostly multilayered structures the mostly used numerical techniques for their calculation are the method of the surface permeability, finite element method and the boundary element method. Because each of them is more or less suitable in special cases, the advantages of each are discussed and pointed out in the paper. The general magnetic anisotropy formulation has been introduced into boundary element method.

Keywords: magnetostatic wave, surface permeability, magnetic anisotropy, multilayered structures, finite element method, boundary element method

Introduction

Much attention has been paid recently to propagation of magnetostatic wave (MSW) modes in various complicated structures such as multilayered or inhomogeneously magnetized films. This has been due to the possibility of their use in signal processing devices directly at the microwave rather than at the RF frequencies. However, stringent control of the frequency dispersion is required what leads to the necessity to find the proper evaluation methods which would be able to handle the mentioned structures. These methods range from variational [1] through numerical [2], [3] to methods based on the TEM approximation [4]. Each of the mentioned methods is more or less suitable in the special cases. So e. g. the problem of arbitrary inhomogeneities cannot be easily attacked by the classical boundary value techniques. Consequently, the variational method has been introduced to analyze nonuniform geometries. On the other hand, however, this method is valid for solution of arbitrary magnetization profiles, great care is necessary in choosing the trial functions for fast convergence of the numerical solutions.

In this paper we review the existing methods for solution of structures supporting propagation of MSW.

Basic Equations

We consider a multilayered inhomogeneous waveguide for MSW as shown in *Fig. 1*. When the bias field $\mathbf{H}_0$ is applied in parallel with x, y and z directions magnetostatic forward volume (MSFVW), backward volumen (MSBVW) and surface (MSSW) magnetostatic modes propagate along the y direction respectively. In the magnetostatic limit the fields $\mathbf{B}$ and $\mathbf{H}$ are related to each other and to the magnetostatic potential ψ in the usual manner

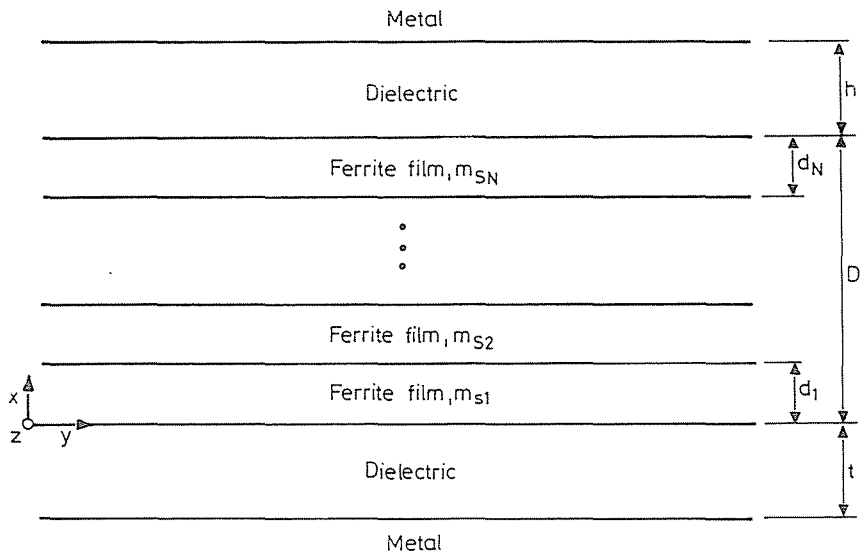

Fig. 1. Geometry of planar MSW waveguide

$$\begin{aligned} \mathbf{H} &= -\nabla\psi, \\ \mathbf{B} &= \mu_0 \bar{\bar{\mu}}.\mathbf{H} \quad \text{for ferrite,} \\ \mathbf{B} &= \mu_0 \mathbf{H} \quad \text{for dielectric,} \end{aligned} \tag{1}$$

with the form of the relative permeability tensor $\bar{\bar{\mu}}$ depending on the orientation of the bias magnetic field $\mathbf{H}_0$ with the diagonal

$$\mu_k = \frac{\omega_i(\omega_i + \omega_{mk}) - \omega^2}{\omega_i^2 - \omega^2} \tag{2a}$$

and nondiagonal component

$$\mu_{ak} = \frac{\omega_{mk}\omega}{\omega_i^2 - \omega^2} \tag{2b}$$

Here $\omega_i = \gamma\mu_0 H_i$, $\omega_{mk} = \gamma\mu_0 M_{sk}$, γ is the gyromagnetic ratio, H_i the internal magnetic field in the ferrite and M_{sk} the saturation magnetization of the k-th ferrite layer. Using (1) and the Maxwell equations $\mathrm{div}\mathbf{B} = 0$ one obtains the equation for the magnetic potential in the form

$$\mathrm{div}(\bar{\bar{\mu}}.\mathrm{grad}\psi) = 0 \quad \text{for ferrite,} \tag{3a}$$

$$\text{and} \quad \Delta\psi = 0 \quad \text{for dielectric.} \tag{3b}$$

Further we will assume in all the cases that the time dependence is $\exp(j\omega t)$ and that MSW propagates in the y direction with the $\exp(-js\beta y)$ functional dependence. Here β is the propagation constant in the y direction and $s = \pm 1$ is a directional parameter. In the case when the structure extends into infinity in the z direction are $\partial/\partial z = 0$. Otherwise the width of the MSW waveguide w has to be introduced.

Methods of Solution

Surface Permeability Approach

This approach is especially suitable in the case of obtaining dispersion equations in the multilayered structures. These structures can be also viewed as special cases of an arbitrary thickness variation of the magnetization M_s. Following this approach we introduce the surface permeability as

$$\mu_s = -jb_x/h_y. \tag{4}$$

This quantity is artificial, but because b_x and h_y are due to boundary conditions continuous at the boundary, μ_s is also continuous. Solving the equation (3a) for the given direction of the applied magnetic field $\mathbf{H}_0$ for the ferrite slab of the thickness d, and assuming that the surface permeabilities from one and the other side of the ferrite slab are μ_{s0} and μ_{s1} one obtains the expression

$$\mu_{s0} = s\frac{ks\mu_{s1}\mu_{22} + (\mu_{11}\mu_{22} - \mu_{s1}\mu_{12} - \mu_{12}^2)\mathrm{th}(k\beta d)}{k\mu_{22} + s(\mu_{12} + \mu_{s1})\mathrm{th}(k\beta d)} \tag{5}$$

which connects the permeabilities from one and the other side of the ferrite slab. Here μ_{11}, μ_{22} equal μ or 1 and μ_{12} equals μ_a or 0 depending on the orientation of the applied field $\mathbf{H}_0$ and $k=\sqrt{\mu_{11}/\mu_{22}}$. Introducing into (5) $\mu_{11}=\mu_{22}=1$ and $\mu_{12}=0$ one obtains the relation between the surface permeabilities on both sides of the dielectric slab. On the metal $\mu_s=0$. Using the relation (5) for different sheets of ferrite and dielectric and introducing boundary conditions one gets the dispersion relation for the investigated MSW structure. This procedure can be done automatically in the computer giving the possibility to analyze any multilayer structure.

Finite Element Solution (FEM)

Fig. 2a shows a MSW waveguide to be solved by the FEM method [2]. The boundaries Γ_1 and Γ_2 are assumed to be perfect electric conductors or perfect magnetic conductors and Γ_3 and Γ_4 are assumed to be again perfect electric conductors. Γ is the boundary of the ferrite. For MSW's propagating along the y direction the fundamental equations can be written in the form

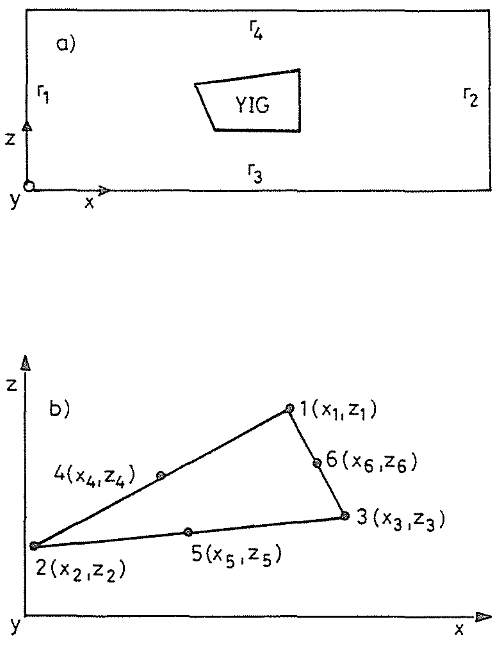

Fig. 2. a/ MSW waveguide b/ Second-order triangular element

$$\partial B_x/\partial x+\partial B_z/\partial z-js\beta B_y=0 \tag{6a}$$

and

$$H_x = -\partial\psi/\partial x; \quad H_y = js\beta\psi; \quad H_z = \partial\psi/\partial z. \tag{6b}$$

Dividing the region enclosed by the boundaries Γ_1 to Γ_4 into a number of second order line elements for the configuration in *Fig. 1* or second order triangular elements as shown in *Fig. 2b* for the configuration in the *Fig. 2a*, the magnetic potential ψ within each element is defined in terms of the magnetic potential ψ_k at the nodal point $k\,(=1, 2, \ldots, 6)$ in the form

$$\psi = [N]^T[\psi]_e \exp(-js\beta y), \tag{7}$$

where

$$[\psi]_e = [\psi_1, \psi_2, \ldots, \psi_6]^T, \tag{8a}$$

$$[N]_T = [N_1, N_2, \ldots, N_6]^T, \tag{8b}$$

where N_i are shape functions given by the area coordinates L_i in a known manner [5]. The area coordinates are related with the Cartesian coordinates by the expression

$$\begin{bmatrix} x \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ z_1 & z_2 & z_3 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}, \tag{9}$$

where (x_j, z_j) are the Cartesian coordinates of the vertex j of the triangle. Also the components of the permeability tensor within each element are approximated by the relations similar to (7)

$$\mu = [N]^T[\mu]_e, \tag{10a}$$

$$\mu_a = [N]^T[\mu_a]_e, \tag{10b}$$

with μ and μ_a taken at the nodal points. Using the Galerkin procedure on (6a) one obtains

$$\int_e [N](\partial B_x/\partial x + \partial B_z/\partial z - js\beta B_y)d\Omega = [0], \tag{11}$$

where the integration is carried out over the element subdomain Ω_e. Integrating by part (11)

$$\int_e [B_x[N_x] + [N_z]B_z + js\beta[N]B_y]d\Omega - \int_\Gamma [N]B_n d\Gamma = [0], \tag{12}$$

where

$$B_n = B_x n_x + B_y n_y, \quad [N_x] = \partial[N]/\partial x, \quad [N_z]\partial[N]/\partial z$$

and the second integration is carried out over the contour Γ_e of the region Ω_e. Taking into account the boundary conditions together with (1), (6b), (7) the equation (12) represents the FEM solution of the problem because it can be transformed into the global matrix equation. The concrete form of the matrix equation depends on the geometry, boundary conditions and the orientation of the applied external magnetic field. It is to be mentioned that in this approach the nonphysical spurious solutions do not appear.

Boundary Element Method (BEM)

In this section we will present the basic approach, the BEM method to the magnetostatic wave problems [3]. The geometry of the problem is shown in the *Fig. 2a*. Using Green's formulas the magnetic potential ψ, at the arbitrary point inside the region given by the boundary Γ, is described as

$$\psi(P_i) = \mu_0 \int_\Gamma \psi_i^*(\bar{\bar{\mu}}.\nabla\psi).\mathbf{n}\, d\Gamma - \mu_0 \int_\Gamma \psi(\bar{\bar{\mu}}.\nabla\psi_i^*).\mathbf{n}\, d\Gamma, \qquad (13)$$

where $\mathbf{n}$ is unit vector to Γ and ψ_i^* is a fundamental solution to the equation (3). If the point P_i is on the boundary, (13) leads to singular integral equation and the calculation of the contribution to the potential in the point on the boundary we express through the coefficient C^i. On the boundary (13) will be

$$C^i\psi(P_i) = \mu_0 \int_\Gamma \psi_i^*(\bar{\bar{\mu}}.\nabla\psi).\mathbf{n}\, d\Gamma - \mu_0 \int_\Gamma \psi(\bar{\bar{\mu}}.\nabla\psi_i^*).\mathbf{n}\, d\Gamma. \qquad (14)$$

The term $(\bar{\bar{\mu}}.\nabla\psi).\mathbf{n}$ represents the flux density through the boundary. Introducing $q=(\bar{\bar{\mu}}.\nabla\psi).\mathbf{n}$ the equation (14) is

$$C^i\psi(P_i) = \int_\Gamma \psi_i^* q d\Gamma - \int_\Gamma \psi q^* d\Gamma, \qquad (14a)$$

the well-known equation of the direct formulation of the boundary problem. The boundary conditions can be easily introduced through the values of ψ and q along the boundary contour. The next step is to discretize the equation (14a). The details can be found e. g. in the original paper [3]. In the case of more regions the system of equations has to be derived for each region. The systems are then interconnected by the boundary conditions between the regions.

There is also another possibility, the so-called indirect formulation of the boundary problem, when the potential in the point P_i inside of the region can be expressed in the form [6]

$$\psi(P_i) = \int_{\Gamma} \sigma \psi_i^* d\Gamma, \tag{15}$$

where σ is the unknown initial distribution density of ψ_i^* on the boundary Γ. The equation (15) represents again a singular integral equation. For the solution of this equation for the two dimensional problems the properties of complex functions and integrals can be successfully used exploiting the Sochotski – Plemel formulas [7]. The properties of the regions are then introduced through the boundary conditions. Just for the illustration the electric field intensity in the point t_i on the boundary as shown in the *Fig. 3* can be expressed as [8]

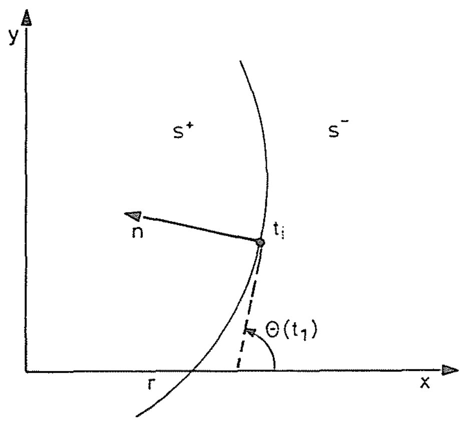

Fig. 3. The geometry for the electric field intensity on the boundary

$$E_x^{\pm}(t_i) = \pm\pi\sigma(t_i)\sin\Theta(t_i) + Re_j[\int_{\Gamma} \frac{\sigma(t)ds}{t - t_i} + \sum_k \frac{A_k}{z_k - t_i}], \tag{16}$$

$$E_y^{\pm}(t_i) = \pm\pi\sigma(t_i)\cos\Theta(t_i) + Im_j[\int_{\Gamma} \frac{\sigma(t)ds}{t - t_i} + \sum_k \frac{A_k}{z_k - t_i}].$$

Here A_k is an unknown complex constant and z_k an arbitrary point from inside of the metallic areas. The further procedure is again straightforward and can be found e. g. in [8].

Conclusions

In the present paper we have reviewed some of the possible approaches which can be used for the solution of the multilayered structures containing anisotropic and isotropic media. It has been shown that in principle any of the known and used methods can be used. The choice of the method depends on the authors' possibilities, their experience and of course on the problem which has to be solved. Recently the work on numerical methods for solution of nonuniformly magnetized MSW structures is in progress.

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