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REED-SOLOMON CODED FREQUENCY-HOPPED PACKET RADIO NETWORKS WITH RECEIVER MEMORY, THROUGHPUT-DELAY ANALYSIS

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Abstract

This paper investigates the performance of frequency hopped packet radio networks which employ a memory at the receiver. The main feature of the memory is that all successive transmissions are utilized for packet reconstruction. Two schemes based on RS codes are investigated. System performance is analyzed in both slotted and unslotted channels. Fixed and adaptive packet lengths are considered. It is demonstrated that it is possible to achieve high throughput even in dense jamming environments.

Keywords: frequency hopping, packet radio, ARQ systems, multiple-access communications.

1. Introduction

Packet radio networks (PRNs) which employ Aloha random-access scheme and use spread-spectrum signaling are currently a topic of intensive research in the open literature (IEEE, 1987; IEEE, 1990 and IEEE, 1991). Spread-spectrum signaling technique is attractive in these networks because of its good signal capture, multiple-access, anti-multipath and narrow band interference rejection capabilities. The two most common forms of spread — spectrum are frequency — hopped (FH) and direct-sequence (DS) spread-spectrum communications. This paper is concerned with the FH-PRNs.

When many users try to transmit bursty packets in an uncoordinated fashion, it is possible for packets to encounter traffic dependent collision. In FH-PRNs, collision results in a random number of symbol errors within each packet involved in the collision. In conventional systems forward error correcting codes (FEC) are combined with the ARQ retransmission protocol to form the so called *type-I hybrid ARQ* system. In these systems, if the errors in a received packet exceed the error (erasure) capability of the FEC code, an additional transmission is requested, either by the absence of an acknowledgement (ACK) or by the transmission of a negative acknowledgement (NACK).

The extra redundant symbols added to the packet increase the multiple access interference to the other users, and hence there is an optimum code rate which must be used for optimizing the network performance. For Reed-Solomon coded FH-PRNs an optimum rate of about 1/3 has been recognized and reported in the literature (PURSLEY, 1985; KIM, 1989; LAM, 1990; MADHOW, 1990). However, this optimum code rate depends on the network conditions, and hence the coding rate must be adopted to match any change in network conditions due to change in traffic requirements or channel conditions (due to jamming or fading).

In this paper we consider schemes which do not ignore the whole erroneous packets completely. The receiver identifies and stores successful symbols from these packets, and the original information packet can be reconstructed from these symbols, which may be collected from different retransmissions of the same packet. There are several methods and techniques for generating side informations, which enables the receiver to identify error free symbols (PURSLEY, 1987; KWON, 1990).

We investigate two schemes based on Reed-Solomon codes with erasure only decoders. The (n, k) singly extended RScode can correct any number ε of symbol erasures in a received word, provided that $\varepsilon \leq n-k$.

Scheme A

The transmitter encodes every newly arrived data packet, of length 'k' symbols, by using (n, k) Reed-Solomon code with large n. In the first transmission, the transmitter sends L symbols, $L \geq k$ from the codeword. The receiver identifies erasures in these L symbols, and it simply inserts erasures in the remaining $n-L$ positions to give a word of length n. The full sequence of n symbols is then decoded as if it were a sequence received from the transmission of the entire codeword. The receiver can decode this word correctly if the number of symbols erased in the L received symbols does not exceed $L-k$. If the word decodes correctly, the receiver sends an ACK to the transmitter, but if a decoding failure occurs, the receiver sends a NACK.

Upon receiving a NACK, the transmitter sends L different symbols from the original codeword. The receiver attempts a decoding, using all successful symbols in the first and second words, and inserting erasures in the remaining $n-2L$ symbols. The decoder is able to decode successfully if the total number of erasures in the two words does not exceed $2L-k$. The transmitter continues sending increments of redundancy in this manner until either a successful decoding results or all n symbols have been transmitted in which case it is assumed that a high level protocol decides whether to discard the transmission of the data packet or repeat the process. This scheme was first introduced, for point-to-point links, by MANDELBAUM (1974). We refer to this scheme as type II-hybrid ARQ. Simulation results of the performance of FH-PRNs with this transmission protocol were presented in (PURSLEY, 1989).

Scheme B

This scheme is similar to the conventional type-I hybrid ARQ. An (n,k) RS code is used to encode the k-length data packet. The same copy of the packet, of length $L=n$, is retransmitted until the receiver is able to decode the packet. The receiver identifies and stores all successful symbols from the different received copies of the same data packet. The original data packet can be decoded successfully if the total accumulated successful symbols is are least k symbols. We refer to this transmission protocol as type-I hybrid ARQ protocol with memory. (DALLOS and GYÖRFI, 1982) have investigated the performance of the pure uncoded-unspread-Aloha with a similar protocol.

This paper is organized as follows. Section 2 provides description of the system and its basic parameters. In section 3 the average number of retransmissions required for every new packet is calculated for a slotted system. In section 4 we study the performance of the unslotted systems, where we investigate the improvement which can be gained by using a kind of adaptive packet length systems. Numerical results and their interpretations together with our conclusions are the subjects of section 5.

2. System Model

The FH-PRN under consideration is organized in a paired-off topology, see *Fig. 1,* where M transmitter-receiver pairs share a common wideband channel that is divided into q frequency bins — subbands. Every pair share a common unique FH/SS code which specify the pattern of frequency bins used for communication.

The different FH patterns are assumed to be independent and identically distributed. Each frequency bin is uniformly chosen from the set of the q frequency bins. Slow frequency hopping (SFH) is assumed in this paper, where the hop interval, T_h is equal to a single data symbol. Time is normalized such that $T_h=1$.

Data are transmitted in packets of length 'L'. Each packet contains a header part, for addressing and synchronization purposes, and an information part. The length of the header is assumed to be zero.

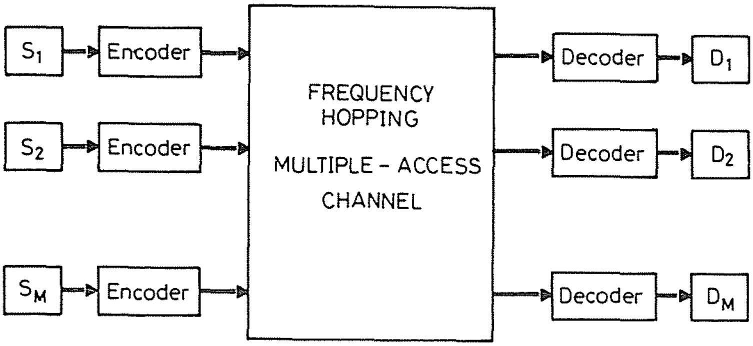

Fig. 1. FH multiple access channel

It is assumed that each receiver is able to acquire each packet addressed to him, and dehop it with his own hopping pattern. No receiver tries to dehop packets which are not addressed to him.

Both time-slotted and time-unslotted modes are considered. In the slotted mode, each transmission must begin on a network synchronization boundary, where the slot time is at least enough to transmit a packet. In the unslotted mode there is no such synchronization. In either case, we assume that network synchronization is not feasible at the symbol level.

The number of active (ready) users at any time is a random variable 'm'. We assume that $M \to \infty$ and m is distributed according to a Poisson random variable with a rate Λ packets/hop. Note that Λ is the rate of the aggregate traffic which consists of the new and the retransmitted packets.

2.1. Hit Probabilities and Erasure Rates

If two or more active transmitters choose the same frequency bin during overlapping symbols transmissions, then a 'hit' occurs. It is assumed that all hits are detected (perfect side information), and that all symbols involved in a hit are erased by the receiver, even though they might be only partially overlapped. The probability that two users hop to the same frequency is

$$p_h = 1 - \left(1 - \frac{1}{q}\right)^2 = \frac{2}{q} - \frac{1}{q^2}, \tag{1}$$

where q is the number of frequencies.

The conditional probability of a symbol erasure is equal to the probability that at least one of m active users hops to the same frequency as one user

$$\varepsilon(m) = 1 - (1 - p_h)^m \,. \tag{2}$$

We assume that the hits in one packet are conditionally independent, although they are neither independent nor Markovian (FRANK, 1990; GEORGIOPOULOS, 1988; HEGDE, 1990, a, b). The error due to this assumption, as pointed out in the above references, is negligible.

For the slotted system, the interference level is fixed during the whole packet duration. The probability of receiving 'i' unerased symbols in a packet of length L, denoted by $\beta(L,i)$ is given by the binomial distribution:

$$\beta(L,i) = \mathcal{E}_m \binom{L}{i} (1 - \varepsilon(m))^i \, \varepsilon(m)^{L-i} \,, \tag{3}$$

where the expectation is with respect to 'm', the number of transmissions which may start at the beginning of a slot,

$$Pr\{m = k\} = \frac{(\Lambda L)^k}{k!} \exp(-\Lambda L) \,.$$

To avoid the infinite sum involved in the average of (3), one can write (3) in the following form

$$\beta(L,i) = \binom{L}{i} \exp(-\Lambda L) \sum_{j=0}^{L-i} \binom{L-i}{j} (-1)^j \exp\left(\Lambda L (1 - p_h)^{(i+j)}\right) \,.$$

In the limit as q and $\Lambda \to \infty$ such that $\Lambda / q \to \lambda$ it is known that the symbol erasure probabilities become independent (PURSLEY, 1986), and $\beta(L,i)$ becomes

$$\beta(L,i) = \binom{L}{i} (1 - \varepsilon)^i (\varepsilon)^{L-i} \,, \tag{4}$$

where $\varepsilon = \mathcal{E}_m \left[\varepsilon(m)\right] = 1 - \exp(-2\lambda L)$

For unslotted systems, the interference level at different symbols, in a tagged packet, varies with time and is not independent. The system evolves as an $M/D/\infty$ queuing system ($M/G/\infty$ for variable packet length case). The number of transmissions at any instant is a Poisson random variable with intensity $G = \Lambda L$ ($G = \Lambda \overline{L}$ for variable packet length case, where $\overline{L}$ is the average packet length in the channel). The exact analysis requires the characterization of the high order statistics of the number of users in the $M/G/\infty$ system which requires an enumeration procedure that counts all possible states. Different approximations and bounds appeared in the literature (TARR, 1990; YIN, 1988; DAIGLE, 1987; PURSLEY, 1986; MADHOW, 1990). In this paper we assume that symbol erasures are independent. This model is true for $q \to \infty$ and it is valid for both fixed and variable packet lengths (MOHAMED, 1992; MADHOW, 1990). For finite q, it is conjectured in (PURSLEY, 1986; CLARE, 1989) that the true throughput falls between the independent and the slotted model.

In conventional type-I hybrid ARQ FH-PRNs, the probability of receiving a correct packet is

$$P_s = \sum_{i=k}^{L} \beta(L, i)\,, \tag{5}$$

and if the packet is uncorrectable, then it is completely ignored and the receiver waits for another repeated packet. The normalized throughput is given by (PURSLEY, 1986),

$$S = \lambda k P_s\,, \tag{6}$$

and since different retransmissions of the same packet have the same chance of success, then the average number of access attempts required to deliver the packet successfully is

$$T = \frac{1}{P_s} = \frac{\lambda k}{S}\,.$$

2.2. Jammer Characteristics

We assume that there is a single jammer and it affects all receivers equally. The jamming alternates between ON and OFF. The duration of time when the jammer is ON is taken to be larger than the packet transmission time, so the probability of changing the jammer state during the packet transmission time is negligible. When the jammer is ON, a contiguous subset of the q frequency bins is jammed. Each new jam pulse selects a different partial band from the preceding one. The jammer is characterized by the following parameters:

$\rho_t =$ proportion of time that the jammer is active, which is equal to the probability that a packet is jammed,

$\rho_f =$ proportion of the q frequencies that is jammed when the jammer is active.

For a jammed packet the symbol erasure probability (2) becomes

$$\varepsilon_J(m) = 1 - (1-\rho_f)(1-p_h)^m , \tag{7}$$

and $\beta(L,i)$ becomes.

$$\beta(L,i) = \mathcal{E}_m\Big[\rho_t \binom{L}{i} (1-\varepsilon_J(m))^i \varepsilon_J(m)^{L-i} + + (1-\rho_t)\binom{L}{i}(1-\varepsilon(m))^i \varepsilon(m)^{L-i}\Big] . \tag{8}$$

3. Throughput-delay Analysis

In this section we present the throughput-delay analysis for both schemes in the case of slotted channel. We measure the average packet delay in terms of the average number of access attempts required for successful delivery of an arbitrary data packet. To simplify the analysis of the type-II hybrid ARQ, we assume that $n \to \infty$.

Consider the sequence of time epochs, $t \in \{1, 2, \ldots\}$, which define the end of every transmission attempt from an arbitrary user, and define the following random variables:

$m(t)$ number of interfering packets which is seen by the tagged packet at the t'th access slot,
$Y(t)$ number of successful symbols in the t'th transmission.

During the t'th access slot, the transmitter is sending an arbitrary packet, which might be a new or a retransmission from an arbitrary data packet. Let $X(t)$ be the accumulated number of successful symbols yet received by the receiver from that particular packet under transmissions at slot t. It is clear that $X(t) \in \{0, 1, \ldots, k-1, k, \ldots, k+L-1\}$. We embed the following state variable at the end of the t'th transmission epoch

$$\mathcal{X}(t) = \begin{cases} X(t), & X(t) < k \\ k, & X(t) \geq k . \end{cases}$$

Notice that $\mathcal{X}(t) = k$ implies that a data packet leaves the system at the end of slot 't' and at the slot '$t+1$' the transmitter starts transmission of a new data packet.

We assume that the average time between different transmission attempts is made large, this makes the number of interferers at different access attempts independent, which makes $Y(t)$ also independent, and hence $\mathcal{X}(t)$ is a Markov chain with finite state $\{0, 1, \ldots, k\}$.

For scheme A, the transition probabilities defined as:

$$p_{ij} = Pr\{\mathcal{X}(t+1) = j | \mathcal{X}(t) = i\},$$

is given by:

$$p_{ij} = \begin{cases} \beta(L, j-i), & j = i, \ldots, k-1 \\ \sum_{c=k}^{i+L} \beta(L, c-i), & j = k \\ 0, & \text{otherwise.} \end{cases} , \qquad i < k \tag{9}$$

and

$$p_{ki} = p_{0i}, \qquad \text{for all} \qquad i = 0, \ldots, k.$$

Since for scheme B the same packet is retransmitted, the probability of receiving new *different* $(j-i)$ symbols is the same as choosing $(j-i)$ successful symbols from $(L-i)$ which is given by

$$p_{ij} = \begin{cases} \beta(L-i, j-i), & j = i, \ldots, k-1 \\ \sum_{c=k}^{L} \beta(L-i, c-i), & j = k \\ 0, & \text{otherwise.} \end{cases} , \qquad i < k. \tag{10}$$

The steady state probabilities $\Pi = [\pi(0), \pi(1), \ldots, \pi(k)]$ can be found by solving the system of linear equations,

$$\pi(j) = \sum_i \pi(i) p_{ij}$$

with the normalization condition:

$$\sum_i \pi(i) = 1.$$

The number of access attempts required for transmitting an arbitrary data packet from an arbitrary user is called a user cycle, see *Fig. 2,* i. e. during a cycle, the arbitrary user is dealing only with one data packet. We assume that different users cycles are independent. The length of cycle 'τ' is formally defined as

$$\tau = \min\{r; \mathcal{X}(t+r) = k | \mathcal{X}(t) = k\}.$$

Since τ is the recurrence time to state 'k' then cycles from one user form a discrete time renewal process. Hence the throughput per user is simply

$1/T$, where $T=\mathcal{E}[\tau]$, and since we have assumed that different user's cycles are independent, then the average throughput is given by

$$S=\frac{\lambda k}{T}.$$

It is known from the theory of Markov chains that $T=1/\pi(k)$, and the throughput can be expressed as

$$S=\lambda k\pi(k). \tag{11}$$

We can find T, without solving for the set Π as follows, define

$$\tau_i=\min\{t;\mathcal{X}(t)=k|\mathcal{X}(0)=i\},$$

then, using standard techniques, we can determine $T_i=\mathcal{E}[\tau_i]$ by using the following recursion

$$T_i=1+\sum_{j=1}^{k-1}p_{ij}T_j. \tag{12}$$

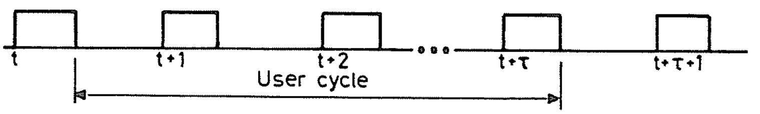

Fig. 2. User cycle, packets depart at t and $t+\tau$ $(X(t)=X(t+\tau)=k)$

3.1. Independent Erasure Model

The same method described in the previous section can be applied for both slotted as well as for unslotted systems, with independent erasure model. However, for the independent model, we can determine the distribution of τ directly. For the type-II hybrid ARQ we have

$Pr\{\tau>i\}$ {probability that the number of successful symbols out of iL is less than k},

$$Pr\{\tau>i\}=\sum_{j=0}^{k-1}\binom{iL}{j}(1-\varepsilon)^j\,\varepsilon^{iL-j}, \tag{13}$$

and for Memory type-I ARQ we have

$Pr\{\tau > i\}$ {probability that the number of successful symbols after the i'th transmission of the same data packet is less than k}.

$$Pr\{\tau > i\} = \sum_{j=0}^{k-1} \binom{L}{j} \left(1 - \varepsilon^{i}\right)^{j} \varepsilon^{i(L-j)}, \tag{14}$$

and the average number of transmissions is then

$$\mathcal{E}[\tau] = \sum_{i=1}^{\infty} Pr\{\tau \geq i\} = 1 + \sum_{i=1}^{\infty} Pr\{\tau > i\}. \tag{15}$$

For scheme A, we can avoid the infinite summation involved in (15) as follows,

$$\mathcal{E}[\tau] = 1 + \sum_{i=1}^{\infty}\sum_{j=0}^{k-1} \binom{iL}{j} (1-\varepsilon)^{i} \varepsilon^{iL-j},$$

$$\mathcal{E}[\tau] = 1 + \sum_{j=0}^{k-1} \frac{(1-\varepsilon)^{j}}{j!} \sum_{i=1}^{\infty} (iL)(iL-1)\dots(iL-j+1)\,\varepsilon^{iL-j},$$

$$\mathcal{E}[\tau] = 1 + \sum_{j=0}^{k-1} \frac{(1-\varepsilon)^{j}}{j!} \frac{\mathrm{d}^{j}}{\mathrm{d}\varepsilon^{j}} \left(\frac{\varepsilon^{L}}{1-\varepsilon^{L}}\right). \tag{16}$$

On the base of the above introduced model, the throughput of the type-II hybrid ARQ can be upper and lower bounded using the following theorem.

Proposition 1

For large q, the throughput of the type-II hybrid ARQ FH-PRNs is bounded by

$$\frac{\lambda L}{\exp(2\lambda L) + \dfrac{L}{k}} < S \leq \lambda L \exp(-2\lambda L). \tag{17}$$

Proof

Since for large q erasures are independent, then the average number of symbols being transmitted from an arbitrary data packet in the channel until success is given by $k/(1-\varepsilon) = k\exp(2\lambda L)$. Then it is easy to show that

$$(\mathcal{E}[\tau] - 1)\,L < k\exp(2\lambda L)\ , \qquad \text{or} \qquad T < \frac{k}{L}\exp(2\lambda L) + 1\,,$$

on the other hand

$$\mathcal{E}[\tau]L \geq k\exp(2\lambda L)\ , \qquad \text{or} \qquad T \geq \frac{k}{L}\exp(2\lambda L)\ ,$$

and (17) follows directly.

Similarly it is easy to prove that each packet will be delivered through this channel with probability 1 in certain circumstances as it is given by the following theorem.

Proposition 2

For the proposed schemes; provided $\varepsilon < 1$ the delivery of every new packet is certain with a probability one.

Proof

Consider the chain associated with an arbitrary data packet. It is the same as $\mathcal{X}$ but with $p_{ki} = 0$, i. e. 'k' becomes an absorbing state. It is easy to show, using (4), that if $\varepsilon < 1$ then all states $i = k$ are transient since,

$$\lim_{n\to\infty} p_{ii}^{(n)} = \lim_{n\to\infty} p_{ii}^{n} = \lim_{n\to\infty}\left[\mathcal{E}_m\left[(\varepsilon(m))^L\right]\right]^n = \begin{cases} 0, & \varepsilon < 1 \\ 1, & \varepsilon = 1 \end{cases}.$$

4. Adaptive Packet Length

In this section we shall investigate the performance of type-II hybrid ARQ system when the packet lengths are made adaptive, and the subsequent retransmissions of a given packet have a reduced packet length. This will reduce the average packet length on the channel and hence the multiple-access interference is reduced. Two possible distinct approaches for controlling the length of the subsequent retransmissions can be considered. These are

1. Feedback adaptation,
2. Local adaptation.

1. Feedback adaptation

The decision on the length of the packet to be transmitted at t'th access instant depends on the number of accumulated successful symbols yet received by the receiver, i. e. $L(t) = f(\mathcal{X}(t))$. An example is to set the packet length proportional to the number of symbols which is still required by the receiver to reconstruct the packet, i. e. $L(t) = \alpha(k - \mathcal{X}(t))$, where α is a control parameter that may depend on the traffic, i. e. $\alpha = \alpha(\lambda)$.

The information about $\mathcal{X}(t)$ can be made available at the transmitter by a direct feedback from the receiver.

The analysis presented in the last section can be applied here as well because, the number of accumulated successful symbols is again a Markov chain with transition probabilities given by

$$p_{ij} = \begin{cases} \beta(L(i), j-i), & j = i, \ldots, k-1 \\ \sum_{c=k}^{i+L(i)} \beta(L(i), c-i), & j = k \\ 0, & \text{otherwise.} \end{cases}, \qquad i < k, \tag{18}$$

However, to calculate the transition probabilities, we have to calculate the average packet length on the channel, because the average erasure rate depends on this average, $\epsilon = 1 - \exp\left(-2\lambda\overline{L}\right)$.

The average packet length on the channel is defined by

$$\overline{L} = \lim_{t\to\infty} \frac{\sum_{r=1}^{t} L(r)}{t},$$

where $L(r)$ is the transmitted packet length at the t'th access instant, and since $L(r) = f(\mathcal{X}(r))$, then we can write

$$\overline{L} = \sum_{i=0}^{k} L(i)\pi(i). \tag{19}$$

Note that $\pi(i)$ depends on $\overline{L}$, and we have to solve a set of nonlinear equations which can be solved iteratively. An alternative method to calculate $\overline{L}$ is to utilize the regenerative property of the user's cycle and set

$$\overline{L} = \frac{\mathcal{E}\left[\sum_{r=1}^{\tau} L(r)\right]}{\mathcal{E}[\tau]} = \frac{Z}{T}. \tag{20}$$

To find Z, let

$$\zeta_i = \min\left\{\sum_{r=1}^{t} L(r); \mathcal{X}(t) = k|\mathcal{X}(0) = i\right\},$$

then using the same standard techniques used to derive (11), we can find $Z_i = \mathcal{E}[\zeta_i]$ recursively by

$$Z_i = L(i) + \sum_{j=i}^{k-1} p_{ij} Z_j . \tag{21}$$

Note that the throughput can be put in the form

$$S = \frac{\lambda k}{T} = \frac{\lambda k \overline{L}}{Z} = \frac{k\rho}{Z}, \tag{22}$$

where $\rho = \lambda\overline{L}$.

For the special case $L(i) = k - i$, $Z = k\exp\left(2\lambda\overline{L}\right)$, and the throughput can be put in the form

$$S = \lambda\overline{L}\exp\left(-2\lambda\overline{L}\right) = \rho\exp\left(-2\rho\right), \tag{23}$$

which is the maximum possible achievable throughput in FH-SSMA predicted in (HEDGE, 1990; SOUSA, 1989) for very long packet lengths. However, as we shall see in the next section, the price of this choice is a larger packet delay.

2. Local adaptation

In this case, the decision on the length of the packet transmitted at transmission instant t, depends on the number of previous unsuccessful transmissions of the current data packet. i. e, $L(t) = f(r)$, where r is the index of transmission instants within the user cycle. As an example, we set $L(r) = k\alpha(r)$, where $\alpha(r)$ is a parameter that controls the successive packet lengths. As in the feedback adaptation, α can be made traffic dependent, i. e. $\alpha(r) = \alpha(\lambda, r)$.

Note that in this case $\mathcal{X}(t)$ is inhomogeneous Markov chain with transition probability given by

$$p_{ij}(r) = Pr\left\{\mathcal{X}(r+1) = j|\mathcal{X}(r) = i\right\},$$

$$p_{ij}(r) = \begin{cases} \beta\left(L(r), j-i\right), & j = i, \ldots, k-1 \\ \sum_{c=k}^{i+L(r)} \beta\left(L(r), c-i\right), & j = k \\ 0, & \text{otherwise.} \end{cases} \quad , \qquad i < k\,. \tag{24}$$

However, we can find the distribution of τ by utilizing the assumption that erasures are independent,

$$Pr\left\{\tau > t\right\} = \sum_{j=0}^{k-1} \binom{\sum_{i=1}^{t} L(i)}{j} (1-\varepsilon)^j \, \varepsilon^{\left(\sum L(i) - j\right)}\,. \tag{25}$$

The average cycle length, $T = \mathcal{E}[\tau]$, can be found from (15). Similarly Z can be put in the form

$$Z = \mathcal{E}\left[\sum_{r=1}^{\tau} L(r)\right] = L(1) + \sum_{i=1}^{\infty} L(i+1) Pr\left\{\tau > i\right\}\,. \tag{26}$$

The average throughput is found then by (22), and the average packet length $\overline{L}$ can be found using the regenerative property of the user cycle as in (20).

5. Numerical Results and Conclusion

In *Fig. 3* we compare the average channel throughput of the two proposed schemes with the conventional type-I hybrid ARQ scheme, for different system and jamming parameters, in the slotted case. The corresponding delay performances, as measured in the average numbers of access attempts, are shown in *Fig. 4*. The improvement by employing the new schemes is recognizable in all cases.

We note from these figures that although $L = k$, for scheme A, maximize the throughput, the average number of access attempts is increased. This is because the chance of success from the first attempt is very small, and it is better to add some redundancy to increase the chance of success from the first attempt.

In *Figs. 5*, and *6* we study the performance of the feedback adaptive system for different α and we conclude that the choice $\alpha = 1$ maximizes the throughput (as mentioned in the text, we reach the capacity of the system), but the price of that is the excess packet delay.

In *Figs. 7* and *8* we study the performance of the local adaptive system and we conclude that the choice $L(r) \geq k$ is better than $L(r) < k$.

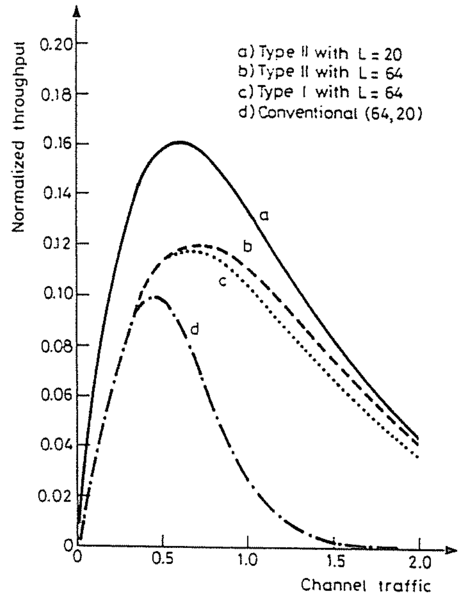

Fig. 3. Normalized throughput versus normalized offered traffic for fixed packet length case ($k=20$, $q=10$, $\varrho_t=0$, $\varrho_f=0$)

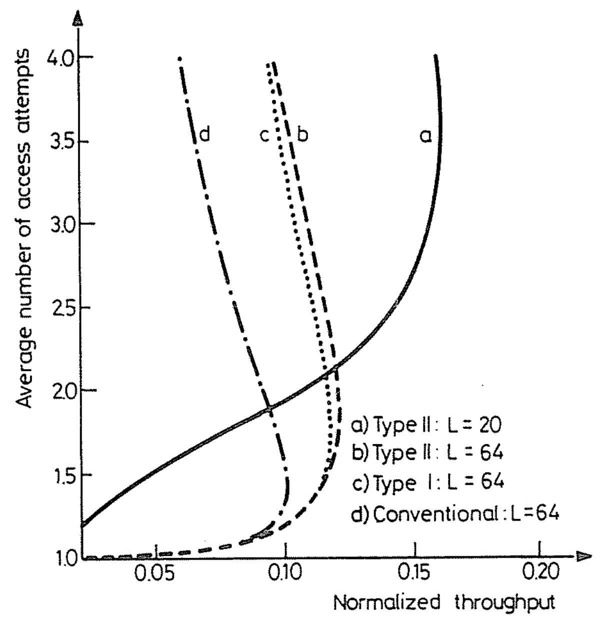

Fig. 4. Average number of access attempts versus throughput for fixed packet length case ($k=20$, $q=10$, $\varrho_t=0$, $\varrho_f=0$)

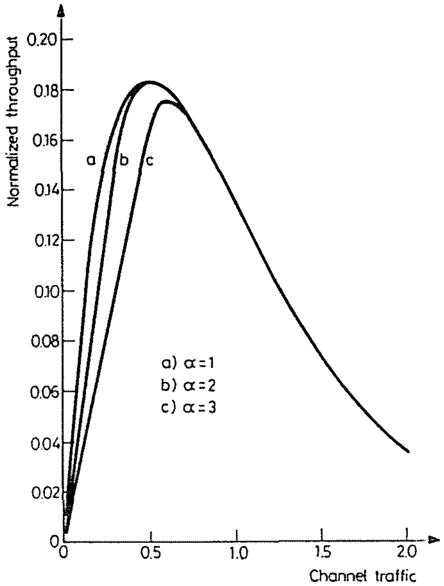

Fig. 5. Normalized throughput versus normalized offered traffic for the feedback adaptive scheme ($k=48$, $q=\infty$, $\varrho_t=0$, $\varrho_f=0$)

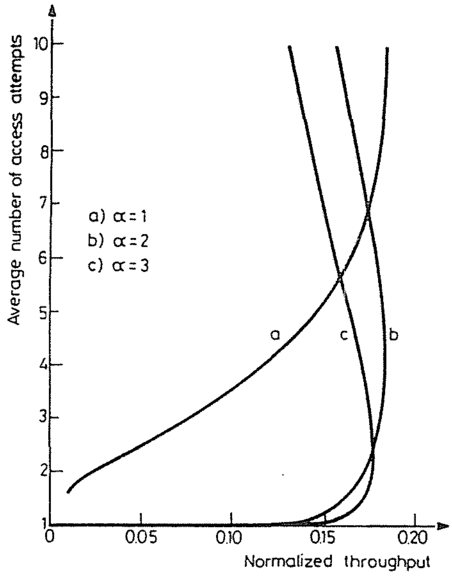

Fig. 6. Average number of access attempts versus throughput for the feedback adaptive scheme ($k=48$, $q=\infty$, $varrho_t=0$, $\varrho_f=0$)

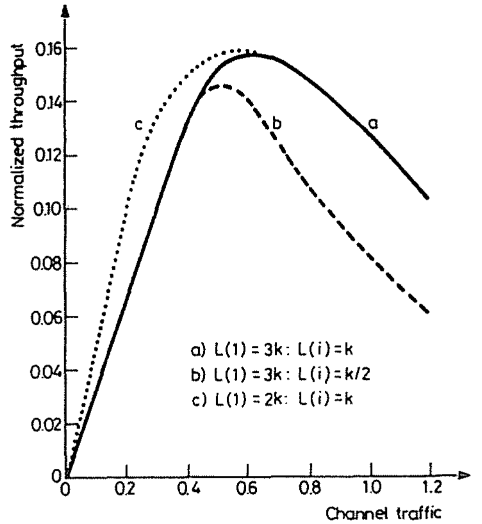

Fig. 7. Normalized throughput versus normalized offered traffic for the local adaptive scheme ($k=12$, $q=\infty$, $\varrho_t=0$, $\varrho_f=0$)

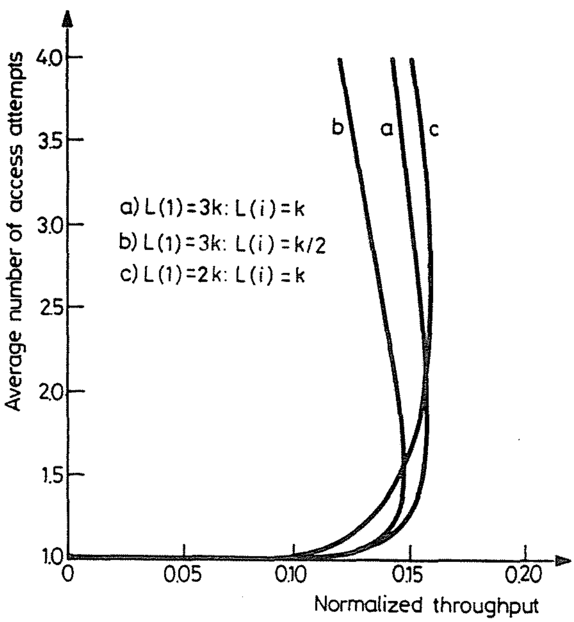

Fig. 8. Average number of access attempts versus throughput for the local adaptive scheme ($k=12$, $q=\infty$, $\varrho_t=0$, $\varrho_f=0$)

We conclude from this study that by employing memory at the receiver, the performance of FH-PRNs is enhanced, and an acceptable performance is achievable even at high jamming environment.

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