

ADJUSTMENT OF STOCHASTIC STOCK MODELS WITH LEARNING

M. MOLNÁR

Département Informatique
INSA de Rennes, France

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Abstract

The algorithms handling the problems of inventory control have already been developed relatively early; the realization of the optimal inventory policy for big production systems and for the trading is helped by known mathematical models. Creation of a mathematical model produces the constraints determining the limits of its validity. If the constraints correspond to the system to be modelled, the parameters of the system can be set on the base of the model.

What happens if the conditions determined during the modelling process are not fulfilled or there is no a priori knowledge of the effects influencing the system? This question occurs when the store supply cannot be scheduled and/or the change of the demand is unknown, even the distribution cannot be determined in advance.

In the paper we are dealing with a possible solution for the problem of stock management in the case of unknown and/or unidentifiable input and output effects. The method proposed is the stochastic approximation. Its application possibility will be shown for a periodic stochastic stock control model provided the stationarity of the processes. The requirement of periodicity and stationarity is not strict. The principle of generating the model can be used for other cases, too, e. g. for the solution of nonperiodic problems. We will show how the method can be applied for following the slow changes in nonstationary cases.

Keywords: stock control, adaptive system, stochastic approximation.

Problem of Inventory Control, Periodic Models

For following the generalization of the method, we define the primary objective of inventory control. As an example a periodic store management will be shown, where the store revision is done with a given periodicity. (In other cases the repeat orders are not emitted periodically, but the events depend on a threshold; it can easily be seen that adaptive modelling is possible in that case, too.)

For simplicity there is only one product in the stock, and the storage is not limited either in space or in time. The detailed models can be found in the literature [HADLEY and WHITIN (1963)], [BENASSY (1991)].

$$K_{st} = IC \int_{T_+} r(t) dt,$$

where: $r(t)$ describes the stock (called 'stock net'),
 IC is the cost coefficient of storing,
 T_+ is the interval of positive stock net.

The shortages mean losses according to the quantity and often to the time, therefore these losses can be expressed by the negative area and/or by the maximum of the shortage:

$$K_{lo} = -\Pi \int_{T_-} r(t) dt - \Pi' \int_{T_-} \lambda(t) dt,$$

where: $\lambda(t)$ describes the demand,
 Π and Π' are the cost coefficients of the shortages,
 T_- is the interval of negative stock net.

The cost of control can be generally considered to be constant:

$$K_{co} = J.$$

The handling cost of the repeat order is independent of the quantity and it is present, when a supply was initialized:

$$K_{rep} = A \operatorname{sgn}(Q),$$

where: $\operatorname{sgn}(Q)$ notifies the indicator function of a repeat order Q .

The structure of costs and the estimation of cost coefficients are detailed in the literature [BENASSY (1991)].

The cost (averaged to the time unit) according to the above is as follows (the costs can be completed by other components if necessary):

$$K(x, u) = \frac{J}{T} + \frac{A}{T} \operatorname{sgn}(Q) + \frac{IC}{T} \int_{T_+} r(t) dt - \frac{\Pi}{T} \int_{T_-} r(t) dt - \frac{\Pi'}{T} \int_{T_-} \lambda(t) dt. \quad (1)$$

The 'stock net' of an $\langle R, T \rangle$ periodic model is shown in *Fig. 1*. In this model the repeat order determined at the time of revision completes the stock up to level R . The supply arrives with a certain delay.

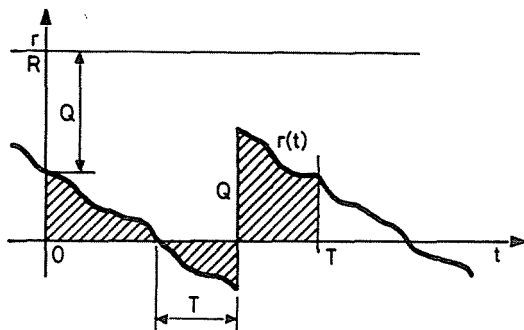


Fig. 1. Stock net of the $\langle R, T \rangle$ model

In the majority of the systems the aim is the minimization of the costs. In some cases it means an exact solution. In stochastic cases the expected values should be minimized.

$$J(\mathbf{u}) = E_{\mathbf{x}} \{K(\mathbf{x}, \mathbf{u})\} \rightarrow \min_{\mathbf{u}}, \quad (2)$$

\mathbf{x} : one of the state vectors,

\mathbf{u} : the vector of the control parameters.

In most cases the solution in the literature presumes the knowledge of the distribution functions of the random effects for the determination of this expected value.

Optimization without a priori Knowledges

One of the methods in identification of stochastic systems, in recognition of signals and forms, in filtering and in optimization is the stochastic approximation; its detailed analysis can be found in the literature [TSYPKIN (1973)], [BENVENISTE et al, (1987)].

This adaptive algorithm approaches the vector of parameters to be optimized (or to be identified, etc.), in general:

$$\mathbf{u}[n] = \mathbf{u}[n-1] + \gamma[n]H(\mathbf{u}[n-1], \mathbf{x}[n]) + \gamma^2[n]\varepsilon(\mathbf{u}[n-1], \mathbf{x}[n]), \quad (3)$$

where $\gamma[n]$: the gain of the process,

- $H(\mathbf{u}, \mathbf{x})$: the function determining how the control parameters should be updated in the bases of the observations,
 $\varepsilon(\mathbf{u}, \mathbf{x})$: a residual low noise.

This iteration results in a series of elements converging to the desired optimal u^* value (if certain conditions not detailed here are met).

The updating function $H(\mathbf{u}, \mathbf{x})$ depends on the aim of the optimization (or the criterion of identification, etc.). Evidently the gradient of the cost function (or a function related to the gradient) can be chosen as updating function. Thus the learning algorithm (not considering the noise):

$$\mathbf{u}[n] = \mathbf{u}[n - 1] - \gamma[n] \nabla_{\mathbf{u}} K(\mathbf{u}[n - 1], \mathbf{x}[n]). \quad (4)$$

For optimizing in stationary cases a diminishing gain $\gamma[n]$ can be used and tracking the slow variations is possible with a constant gain.

An example of Adaptive Stock Control: Adaptive Model $\langle R, T \rangle$

For demonstration the already mentioned model $\langle R, T \rangle$ is chosen. To describe the cost function the geometrical approximations are used, which are known in the literature [HADLEY and WHITIN (1963)] (i.e. the stock net function is approximated by straight lines, therefore the integrals can be replaced by triangles and trapezoids). We can distinguish three cases (cf. *Fig. 2*):

- a) some positive stock is left over,
- b) a shortage is produced in the period,
- c) the delivery supplies only the deficiency.

If the mean loss of shortage is proportional to the time of shortage, then the cost function of such a model is as follows (averaged to time unit):

$$K(R, T, \lambda, \mu)$$

$$= \begin{cases} \frac{L}{T} + IC \left(R - \mu \frac{\lambda T}{2} \right) & \text{a) if } R \geq \lambda T + \mu \\ \frac{L}{T} + \frac{IC}{2\lambda T} (R - \mu)^2 + \frac{\Pi}{2\lambda T} (R - \mu - \lambda T)^2 & \text{b) if } \mu \leq R < \lambda T + \mu, \\ \frac{L}{T} - \Pi \left(R - \mu \frac{\lambda T}{2} \right) & \text{c) if } R < \mu \end{cases} \quad (5)$$

where: $L = J + T$, constant cost of control and handling,
 λ is the mean of demands,
 μ notifies the sum of demands during the delay of supply.

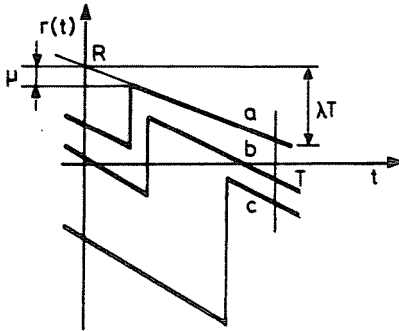


Fig. 2. Behaviour of variables according to the expression (5)

In stochastic cases the delay of supply and the demands are random variables. For us the interesting circumstances are, when the distribution of these variables is unknown or their handling is difficult (e. g. it is multimodal). In that case μ and λT are unknown random variables.

The stock management applying $\langle R, T \rangle$ policy can set the system by the parameters R and T , the vector of the control parameters is:

$$\mathbf{u}^T = (R, T). \quad (6)$$

Knowledges of the state of the system can be obtained by observing the random variables μ and λT :

$$\mathbf{x}^T = (\mu, \lambda T). \quad (7)$$

In the practice the period T is often fixed, in the paper only the algorithm determining the level R will be shown in more details (optimization for T can be performed in the same way).

Because of the cost function (5) can be derived in pieces and for defining the optimal level R^* the projection of gradient by R is sufficient:

$$\frac{\partial K(R, T, \lambda T, \mu)}{\partial R} = \begin{cases} IC & \text{a) if } R \geq \lambda T + \mu \\ \frac{IC}{\lambda T}(R - \mu) + \frac{\Pi}{\lambda}(R - \mu - \lambda T) & \text{b) if } \mu \leq R < \lambda T + \mu \\ -\Pi & \text{c) if } R < \mu \end{cases} \quad (8)$$

Examples for Searching the Optimum by Learning

For demonstrating the optimization of stochastic stock control models by the stochastic approximation method and without a priori knowledges, a simulator program has been developed. The simulator can generate random signals for the input and output of the stock model, the variables of the state vector can be observed in the course of process simulation, and optimal setting of control parameters can be done using adaptive algorithms.

The first example was taken from the book containing the basic models [HADLEY and WHITIN (1963)]. In this case a company stores bicycle wheels and uses the $\langle R, T \rangle$ strategy. In our case a period is 3 months long and the supply arrives within 6 months after the repeat order. The yearly distribution of the demands can be characterized as normal, its expected value is 600 units, its variance is 900. The price of a wheel is 15\$, the cost rate of store is 20\$ and the shortage of a wheel means the cost of 25\$. The costs of the revision and the repeat order are 30\$. Applying the method of the authors (on the basis of normal distribution) the optimal level of R :

$$R^* \sim 499 \text{ units.}$$

Although according to the theorem of central distribution limit, the common distribution is approximate normal, nevertheless the case was optimized by the above method, too, generating different daily distribution at the output of the simulator.

Creating the daily development of the demands on the basis of normal distribution the iteration starting on different initial values and using a hyperbolic gain can be seen in *Fig. 3*. The common limit of the curves results in the same value of

$$R^* \sim 499 \text{ units.}$$

It can be seen that in the domain of gravity the series (4) reaches its final value rather fast (in 10 – 15 steps or even, sometimes 5 – 6 steps) by the above algorithm (in the paper the choice of the initial value and gain of the process are not detailed).

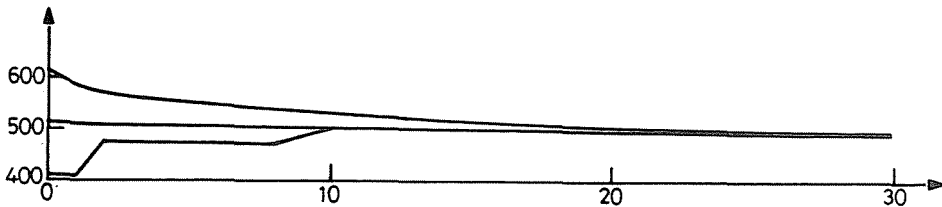


Fig. 3. Case of daily normal distribution

The level of the stock is diminishing, if the daily demands are generated on the base of uniform distribution. The method of stochastic gradient approaches the approximate value of

$$R^* \sim 469 \text{ units.}$$

The simulator makes it possible to compare the real costs at different values of R . Taking into account the cost of 100 periods simulated randomly, the following comparison table for the two methods:

Model	R^*	Costs	Profit
HADLEY-WHITIN	479	222.2	
adaptive	469	215	3.2%

In the third case a bimodal was applied at the output of the stock which keeps the daily expected value unchanged. It could be realized by mixing two normal distributions with the expected values 1 and 3, and the variances 0.5 and 1.5, respectively. Here the stochastic approximation gives an optimal value equal to 470.

The results of program runs of 100 periods and of identical input and output can be seen in the following table:

Model	R^*	Costs	Profit
HADLEY-WHITIN	493	231.4	
adaptive	470	215	6.9%

Adaptation in the Case of Random Supply and Random Demands

In the next example the joint case of random supply and random demands is investigated. The supply delays are determined by a random number generator (in our case with normal distribution), in the same way the demands are changed randomly (with uniform distribution). *Fig. 4* shows the convergence of such a system starting with different initial values.

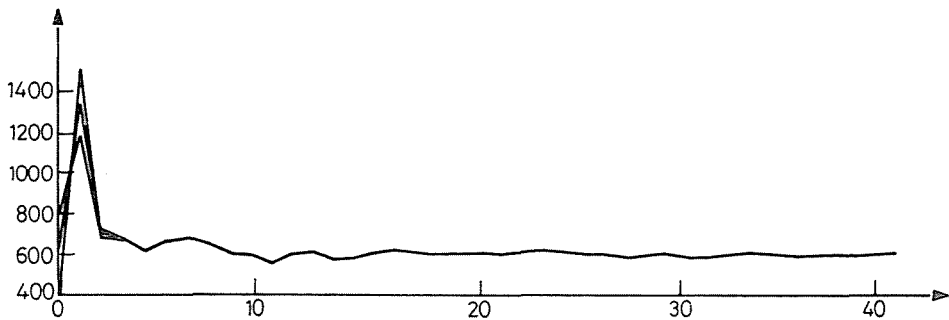


Fig. 4. Case of random supply and random demands

Fig. 5 illustrates the ability of the model for learning at changing conditions. Here the variance of the random number generator of the delays is changed, and the following conclusion can be stated: the bigger the variance (the supply is more uncertain), the bigger the value of R^* .

The above examples prove that the learning algorithms set the system parameters according to the 'conclusions' gained from the given samples.

Tracking Slow Variations

The method mentioned earlier can be applied in non-stationary cases, too [BENVENISTE et al. (1987)]. We have investigated the case, when the demands change as a function of time. *Fig. 6* illustrates the situation

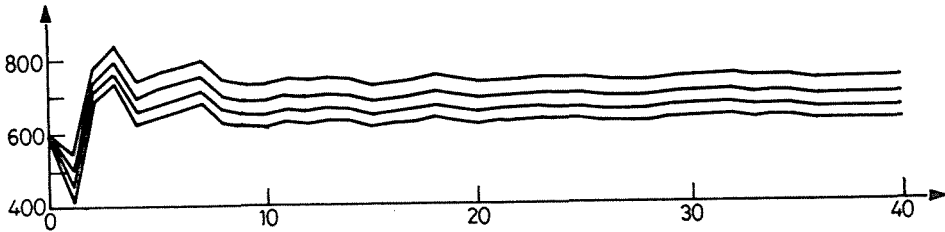


Fig. 5. Changes of variance of supply delay

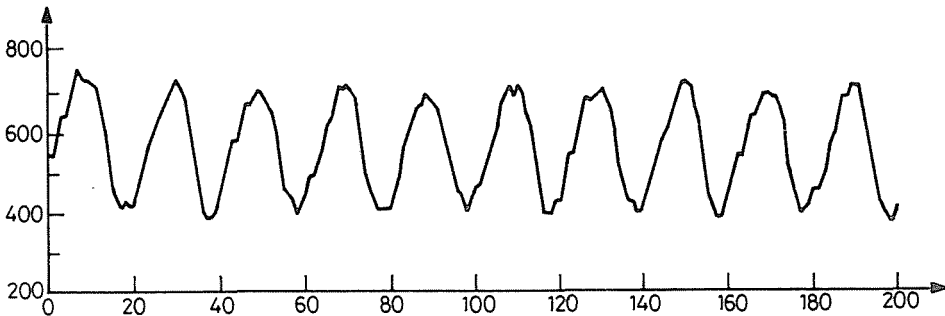


Fig. 6. Mean of demands change according to a sine-wave

when the mean of demands changes according to a sine-wave. It can be seen that the algorithm is capable to follow the variations of the mean.

Conclusions

The problems of inventory control can be handled in those cases, when unknown effects occur at the input and output of the stock. As learning algorithm we applied the stochastic approximation, more precisely the method of stochastic gradient for the above task.

The method can be effectively applied for optimization of inventory control having carefully set up the cost function.

The algorithms converge fast by choosing the convergency coefficient properly. Considering that there are few measuring data for determination

of inventory control strategy, it is important to reach the optimum relatively fast by the method. Our example shows that in most cases it can be achieved within 5 – 10 periods.

Significant cost savings can be expected in case of a control parameter defined by adaptive algorithm, relative to the well known solution of the stochastic $\langle R, T \rangle$ model.

Using constant gain in the iterations, the algorithms are capable of following the changes of the system in time; although in this solution the elimination of statistical noises can only be achieved by processing many measurement data.

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Address:

Miklós MOLNÁR
Département Informatique
INSA de Rennes
20; avenue des Buttes de Coësmes
35043 Reunes Cedex, France