IDENTIFICATION OF LINEAR SYSTEMS IN THE PRESENCE OF NONLINEAR DISTORTIONS[1]

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Abstract

A method is presented to measure and identify a linear system in the presence of nonlinear distortions. The method is based on a two-step approach. In the first step the influence of nonlinear systems up to degree 4 is eliminated. In the second step the remaining linear system is identified using a weighted least squares method. The kernel of the proposed technique is the excitation of the system with a pure sinusoid. Special attention is paid to the elimination of higher harmonics in the excitation signal which are due to the nonlinear load of the generator.

Keywords: system identification, nonlinear systems, Volterra series.

Introduction

Identification is a powerful technique for building accurate models of complex systems from noisy data (EYKHOFF, 1974; NORTON, 1986; SORENSON, 1980). In this article we will focus on the identification of dynamic systems. Among all possible systems, linear systems form an important class due to their well-known properties and their mathematical suitability. A large number of identification methods have been developed to deal with linear systems in the time domain (LJUNG and SÖDERSTRÖM, 1983; LJUNG, 1986) and in the frequency domain (SCHOUKENS and PINTALON, 1991). There are also toolboxes available which allow the less experienced user to apply the identification methods to his specific problem (KOLLÁR et al., 1991; LJUNG,1986). However, in practice most systems are only approximately linear, which will introduce model errors due to the nonlinear contributions. A first possibility to deal with this problem is to apply small excitations around a working point and to fit the linear model which approximates best the nonlinear behaviour in some sense (BENDAT, 1990). A second ap-

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proach is to model the nonlinear system as a Volterra system (SCHETZEN, 1980) to eliminate the higher-order nonlinear contributions and to model the remaining underlying linear system. If the major interest is in the global behaviour of the system for a specific class of excitations (e.g. noise excitation) the first approach is preferred, but if the estimates are used to make a physical interpretation of the results based on a linear model, it is obvious that the second approach is required. In this article the second approach is studied. In Section 2 it is shown how the higher-order nonlinear contributions can be eliminated and the influence of the measurement noise is analyzed. In Section 3 a robust identification method is presented to model the underlying linear model. Finally, the proposed method is verified in Section 4.

2. Elimination of the Nonlinear Contributions

2.1. Measurement of the Underlying Linear System
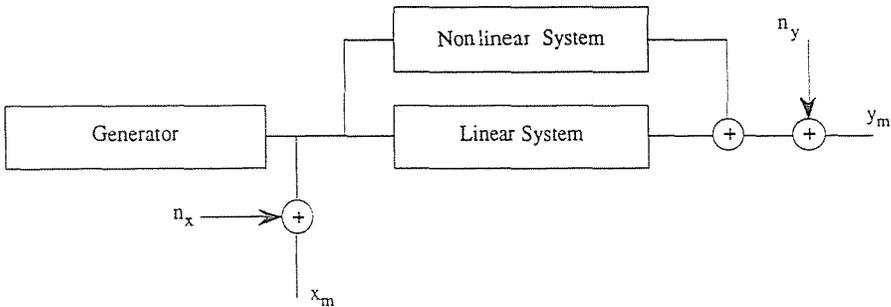
Fig. 1. Model setup with noise on the measured excitation and response

Consider the setup in Fig. 1. A linear system with a nonlinear distortion is excited with an excitation signal $x(t)$. The corresponding response is $y(t)$. The measurements of these signals $x_m(t)$ and $y_m(t)$ are disturbed by noise n_x and n_y. The data processing is performed in the frequency domain. We assume that the excitation $x(t)$ is periodic and, for simplicity, an entire number of periods is measured. Under these conditions the discrete Fourier transform allows to calculate the Fourier coefficients $X(f)$ and $Y(f)$ without introducing systematic errors. The noises $N_x(f)$ and $N_y(f)$ on the Fourier coefficients are assumed to be zero mean complex normal distributed with independent real and imaginary parts, independent between input and output, and to have variances σ_x^2 and σ_y^2 (the variance

equals two times the variance of the real and imaginary parts, under the previous assumptions). In BRILLINGER, (1975) it is shown that this noise model is valid if the noise is mixing and the length N of the data record is increasing. Moreover, it is shown that this noise model gives also a good description, even for quite short data records (SCHOUKENS and RENNEBOOG, 1986).

The nonlinear system is described by its Volterra representation, and we assume that this series converges. This assumption is not always met in practice, but we can always restrict the amplitude of the excitation to a finite band and consider a series which describes the system in some least squares sense. In this article a method is proposed which eliminates the contributions up to degree 4, but it is possible to generalize the technique in order to eliminate higher-order contributions.

If the input is restricted to be a pure sinusoid with frequency f and complex amplitude $X(f)$ the output can be written as a sum of sinusoids with complex amplitudes $Y(f_k), k = 0, 1, 2, 3$. Due to the restriction of the nonlinear system to a maximum degree of 4, the maximum output frequency is $4f$. The contributions at DC and $2f$, which are generated by the nonlinear contributions of second and fourth degree, do not disturb the measurement at frequency f. Consequently, only the contribution of degree 3 at frequency f will disturb the measurements. The complex amplitude $Y(f)$ of the system response can be written as:

$$Y(f) = H(f)X(f) + a(f)X^2(f)X^*(f), \tag{1}$$

where $*$ denotes the complex conjugate. Consider two experiments with excitation amplitudes X_1 and X_2 and the corresponding output amplitudes Y_1 and Y_2. Define the scaling values S_i:

$$S_i = X_i^2 X_i^* \tag{2}$$

and the scaled Fourier coefficients

$$U = \frac{X_1 S_2 - X_2 S_1}{\sqrt{|S_1|^2 + |S_2|^2}} \qquad V = \frac{Y_1 S_2 - Y_2 S_1}{\sqrt{|S_1|^2 + |S_2|^2}} \tag{3}$$

Eq. (1) shows that the following relation holds:

$$V(f) = H(f)U(f). \tag{4}$$

From *Eq.* (4) it is seen that it is possible to eliminate the nonlinear contributions of degree 3 by making a combination of two experiments. The first

disturbing contribution will be at least of degree 4. Two problems remain to be studied: the first one is the study of the influence of the measurement noise on the previous result, and the second one is the generation of a pure sinusoidal excitation on a nonlinear device.

The scaling factor in (3) is introduced to simplify the formulation of the estimator in Section 3.

2.2. Study of the Noise Influence on the Measured Transfer Function

Due to the noise, the measured value of the transfer function will not be equal to the true value. Defining

$$\mathbf{Z}_m = (X_{1m} \quad X_{2m} \quad Y_{1m} \quad Y_{2m} \quad S_{1m} \quad S_{2m}), \tag{5}$$

where the index m denotes the measured values, Eq. (4) can be written as

$$H_m = \frac{V_m}{U_m} = H_m(X_{1m}, X_{2m}, Y_{1m}, Y_{2m}) = H_m(\mathbf{Z}_m) = H(\mathbf{Z}) + h, \tag{6}$$

where h is the measurement error due to the noise. For simplicity, we have omitted the dependence on the frequency f. An approximated value of h is found using the Taylor series of $H_m(\mathbf{Z}_m)$ around $H(\mathbf{Z}_e)$ with $\mathbf{Z}_e$ the exact value of $\mathbf{Z}$

$$h = \frac{\partial H}{\partial \mathbf{Z}}\Big|_{Z_e} (\mathbf{Z}_m - \mathbf{Z}) + \frac{1}{2}(\mathbf{Z}_m - \mathbf{Z})^t \frac{\partial^2 H}{\partial \mathbf{Z}}\Big|_{Z_e} (\mathbf{Z}_m - \mathbf{Z}) \tag{7}$$

A detailed analysis shows that the bias $E[h]$ of H_m is not zero, and contains contributions in the complex variances σ_x^2 and σ_y^2. For sufficiently large S/N ratios, this bias will completely be masked by the uncertainty on the measurements. In the next section we will deal in more detail with the systematic errors of the identified models. The complex variance of h, defined as

$$\sigma_H^2 = E[(h - E[h])^*(h - E[h])] \approx E[h^*h] \tag{8}$$

is given by

$$\sigma_H^2 = (\sigma_y^2 + |H|^2\sigma_x^2)\frac{|X_1|^6 + |X_2|^6}{|X_1|^2|X_2|^2(|X_1|^2 - |X_2|^2)^2}. \tag{9}$$

In this expression, it is assumed for simplicity that the noise variance does not depend upon the amplitude of the excitation. The expression can also be written as

$$\sigma_H^2 = \sigma_H^2(X_2)\frac{1 + k^3}{(1 - k)^2}, \tag{10}$$

with $\sigma_H^2(X_2)$ the variance on the non-compensated transfer function measurement with amplitude X_2 and $k = |X_2|/|X_1|$. In *Fig. 2* it is shown how the standard deviation depends on the ratio of $|X_2|/|X_1|$, with a fixed amplitude $|X_2|$. It is clear that the optimum ratio, resulting in a minimum variance, is about 2. In addition, it can be remarked that the variance of the compensated measurement will be considerably increased with respect to the non-compensated measurement. This increase is due to the introduction of the additional complex parameter a in the measurement, and it is the price which has to be paid for the elimination of the systematic errors due to the nonlinear contribution.

scaled standard deviation
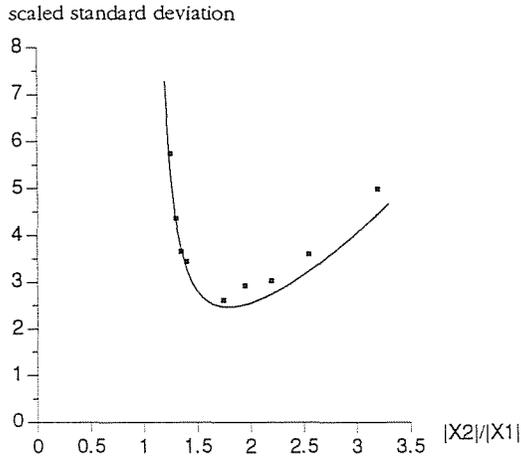
Fig. 2. Evolution of the standard deviation as a function of the ratio of the input amplitudes. Full line: theoretical value; dots: experimental value

2.3. Generation of a Pure Sine Wave on a Nonlinear System

To generate the excitation signals, an arbitrary waveform generator is used, which allows to generate a computed waveform $\mathbf{X}_g$. Due to the output impedance of the generator and the nonlinear input impedance of the device under test, the actually generated signal $\mathbf{X}_{d,l}$ will not be the desired signal $\mathbf{X}_d$. If a pure sine wave is to be generated, also the harmonic components will be present in $\mathbf{X}_{d,l}$ due to the nonlinear pull of the system on the generator. To deal with this problem, we will use an iteration algorithm which will correct the generator signal in order to get the desired excitation. In a first step, the transfer function $\mathbf{H}(f_k)$ between $\mathbf{X}_g(f_k)$ and $\mathbf{X}_d(f_k)$ is measured, assuming that the linear part is dominant. This can

be done using the classical transfer function measuring techniques. Next the following iteration procedure is followed:

$$\mathbf{X}_{g,l+1}(f_k) = \mathbf{X}_{g,l}(f_k) + \mathbf{H}^{-1}(f_k)(\mathbf{X}_d(f_k) - \mathbf{X}_{d,l}(f_k))) \qquad (11)$$

where $\mathbf{X}_{g,l}(f_k)$ denotes the generator signal in the lth iteration. To start the process, we can choose $\mathbf{X}_{g,0}(f_k)=\mathbf{X}_d(f_k)$. The iteration is continued until the moment has improves, or a maximum number of iterations is reached. This method was checked on experiments and turned out to converge fast, even for strongly nonlinear loads.

Mathematical interpretation

The previous problem can be formulated as an optimization problem where the following cost function should be minimized with respect to the generator coefficients $\mathbf{X}_g$:

$$(\mathbf{X}_d - \mathbf{X}_{d,l}(\mathbf{X}_{g,l}))^{*T}(\mathbf{X}_d - \mathbf{X}_{d,l}(\mathbf{X}_{g,l})).$$

Using the Newton optimization algorithm the following iterative procedure is found:

$$\mathbf{X}_{g,l+1} = \mathbf{X}_{g,l} - \left(\frac{\partial^2 K}{\partial X_{g,l}^2}\right)^{-1}\frac{\partial K}{\partial X_{g,l}}. \qquad (12)$$

If the linear term is dominant, the following approximations are valid:

$$\left(\frac{\partial^2 K}{\partial X_{g,l}^2}\right) = 2 \ \mathrm{diag}(H^2(f_k)), \qquad (13)$$

$$\frac{\partial K}{\partial X_{g,l}(f_k)} = 2H(f_k)(\mathbf{X}_d(f_k) - \mathbf{X}_{d,l}(f_k)). \qquad (14.)$$

Substitution of these expressions into (11) finally results in (10). This interpretation shows that for strong non-linearities it is possible to improve the optimization process (10) by using a better approximation for the derivative or by using the full form (12). It can be remarked that the harmonics can be completely suppressed because the number of constraints is equal to the number of free frequencies.
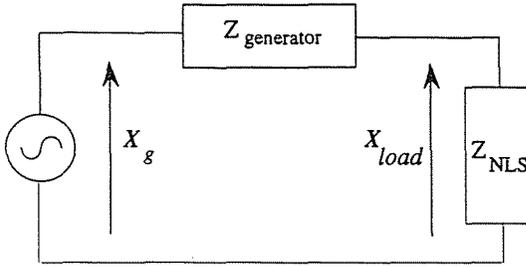
Fig. 3. Generation of an arbitrary excitation signal
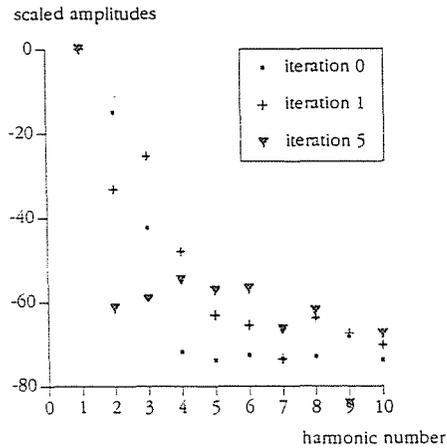
Fig. 4. Optimization of the input spectrum

2.4. Experimental Verification

Some experiments were made to verify the previous results. In the first experiment, the evolution of the input spectrum is shown during the optimization process. We intended to generate a pure sinusoid on a linear circuit which was strongly loaded by a nonlinear diode circuit. The spectrum is shown in *Fig. 4*, before optimization and after 1 and after 5 iterations.

In the second example, a linear circuit disturbed by a nonlinear circuit is measured. Again it was necessary to optimize the input signal in order to get a pure sinusoidal excitation. The measurements were made for 3 amplitudes of the excitation signal (1.3 V, 1.9 V and 3.8 V). The results are shown in *Fig. 5* where the compensated and the noncompensated measurements are compared to the measurement of the undisturbed linear system.
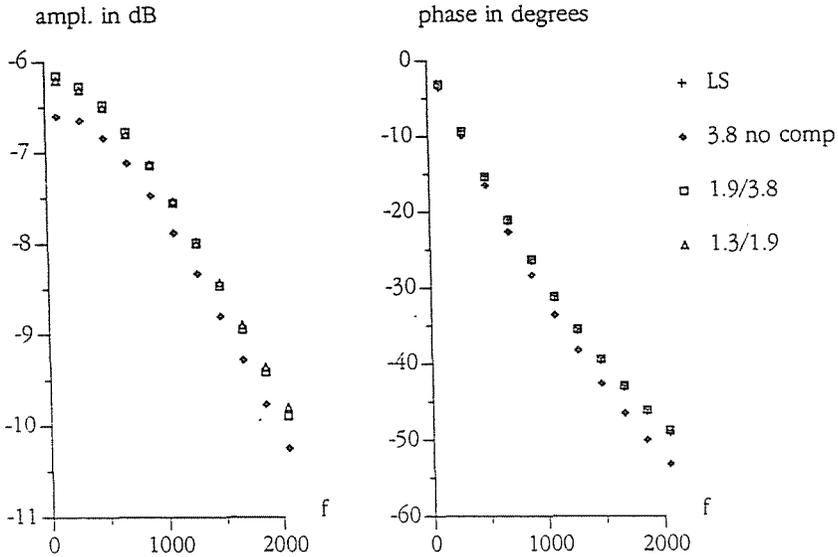
Fig. 5. Measurement of the underlying linear system in the presence of nonlinear distortions

3. Identification of the Underlying Linear System

The identification of the transfer function of the underlying linear system is based on a nonlinear weighted least squares method. It is an errors-invariables method which starts from the measured input and output spectrum. The difference between the measured and the modelled Fourier coefficients is minimized. It can be shown that this finally results in the minimisation of the following cost function:

$$K = \sum_{k=1}^{F} \frac{|V_m(f_k)D(f_k, \mathbf{P}) - U_m(f_k)N(f_k, \mathbf{P})|^2}{|D(f_k, \mathbf{P})|^2 \sigma_y^2(f_k) + |N(f_k, \mathbf{P})\sigma_y^2(f_k)}. \tag{15}$$

The estimates $\mathbf{P}_{EV}$ are the values of $\mathbf{P}$ which minimize the cost function:

$$\mathbf{P}_{EV} = \arg\min_{\mathbf{P}} K(\mathbf{P}). \tag{16}$$

This is exactly the same expression as the cost function of ELiS, an estimator developed to estimate the transfer function of linear systems in the frequency domain (SCHOUKENS and PINTELON, 1991). Hence it is possible to solve this optimization problem using the previously developed routines, but replacing the measured Fourier coefficients by the scaled coefficients.
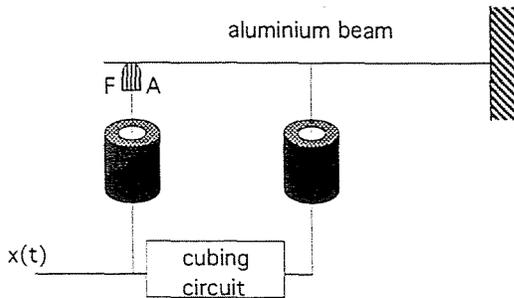
Fig. 6. Experimental setup

4. Experimental Verification

The previously developed technique is checked on a nonlinear system as shown in *Fig. 6*. Because we have only mini shakers in our lab, we were not able to excite a structure sufficiently to get a significantly nonlinear behaviour. For this reason, we created a nonlinear system by using a double excitation. The excitation signal $x(t)$ is fed to a first mini shaker, and after passing through a cubing device, to a second mini shaker. The transfer function between the acceleration A and the force F of the first mini shaker is modelled considering the acceleration as the driving force.

$$H(f) = \frac{F(f)}{A(f)}. \tag{17}$$

The excitation signals were designed to force a sinusoidal acceleration as explained in Subsection 2.3. Measurements were made at 10 different levels with a range of 1 to 50. The nonparametric frequency response function obtained before and after compensation is given in *Fig. 7*.

A detailed picture is also given of the evolution of the measurements at some frequencies. This figure shows clearly that at some frequencies a considerable improvement is found, while on other frequencies (for example at 175 Hz) the method fails. This behaviour will be discussed in some more detail later. Starting from this compensated measurement, a transfer function is identified. A model order of 6/8 is chosen. Two pole pairs are used to describe the two resonance peaks in the considered frequency band, while the two other pole pairs are used to model the influence of two neighbouring resonances. In *Fig. 8* the estimated poles and zeros corresponding to the resonances in the frequency band of interest are given for the non-compensated and the compensated measurements. This figure shows that the zeros (which are the poles of the transfer function acceleration/force)
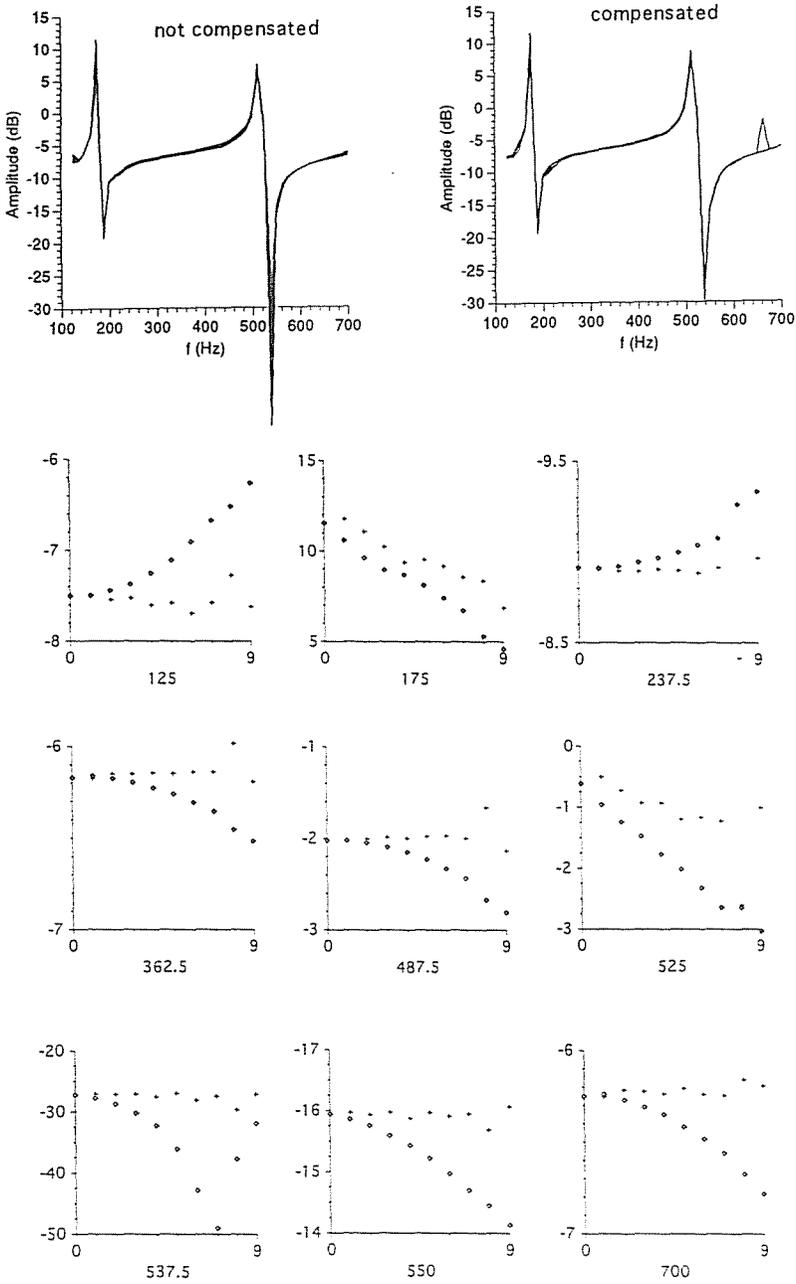
Fig. 7. Measured transfer function. (a) Amplitude as a function of frequency (b) Evolution of the measured (◇) and compensated (+) transfer function as a function of the excitation level (horizontal axis: experiment number, vertical axis: amplitude in dB)

of the compensated system are much less amplitude dependent than those corresponding to the unprocessed data. The zeros can even move to the right half plane if no compensation is applied. The compensation did not allow to stabilize the poles corresponding to the resonance at 175 Hz. A careful analysis shows that the relation between the amplitudes at this frequency is of the form $y = x^z$ with z not an unteger number. For such a relationship, the Taylor series and hence the Volterra series do not exist at $x = 0$. As a consequence, we cannot define an underlying linear system which dominates for small amplitudes, and the method will fail.
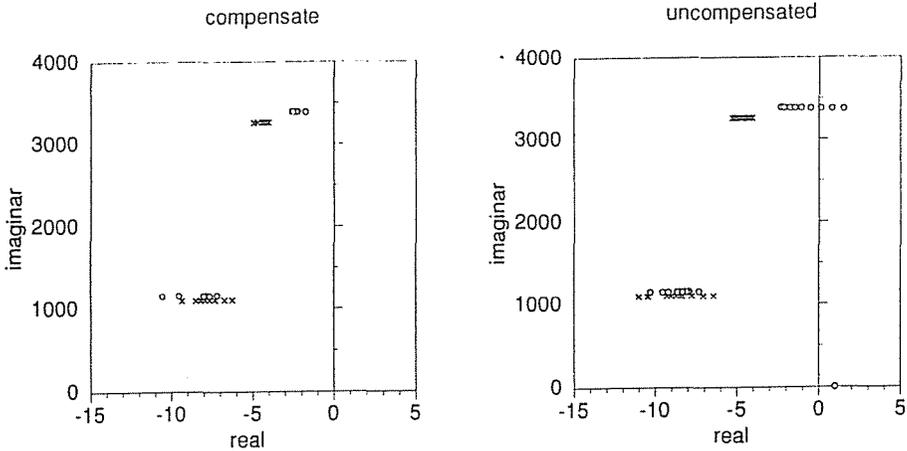
Fig. 8. Evolution of the poles/zeros in the frequency band of interest as a function of the excitation level.

5. Conclusion

In this article a method is presented to measure and model the underlying linear system of a nonlinear device. Such an underlying linear system, which will become dominant for low excitation levels, can only exist if the Volterra series exists. To simplify the measurement problem, a pure sinusoidal signal is selected as excitation. To avoid the creation of higher-order harmonics due to the nonlinear load of the generator, an iterative method is proposed, allowing to get signals with a high harmonic purity. A noise analysis of the measured transfer function is made, and a simple rule to get a minimum noise sensitivity is proposed. In the second part,

a weighted least squares method is developed to model the linear part. It turned out that the estimation could be done with ELiS, using scaled Fourier coefficients. Because the scaled Fourier coefficients do not satisfy the basic noise assumptions of ELiS a bias will be introduced, proportional to the variance of the noise.

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