# NEW MARKER CENTRE ESTIMATION ALGORITHM OF HIGH ACCURACY IN MOTION ANALYSIS 

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#### Abstract

The non-contact measurement method of motion means almost no discomfort to the moving person or animal. Furthermore, the use of CCD cameras and stroboscopic lighting makes daylight applications possible. As a result a popular and widely used solution in motion analysis is to place markers to landmark points and to characterise motion on the basis of tracking the markers. The actual position of a marker is determined by estimating its centre. The estimation is based on the covered pixels, if several pixels are covered then sub-pixel resolution and accuracy can be achieved. The paper surveys the most commonly used geometric centroid calculation and shows that this method gives a distorted estimation if the set of covered pixels is asymmetrical. In the case of ideal marker images circle fitting does not give a better result, it can be useful if the marker image is partly occluded. A new marker centre estimation algorithm is introduced. It is optimal in the sense that the expected value of the deviation between the actual and the estimated value of a circular marker image is minimal.


Keywords: marker centre estimation, motion analysis, accuracy.

## Introduction

Visual observation has always been considered an important part of motion analysis. Since 1888, when E-J. Marey presented the chrono-photographic box, objective observation of motion is possible. (FURNÉE, 1989) both gives a survey on the history of motion analysis and describes the different existing analyzers. The motion analysis systems, being used presently generally apply CCD cameras thus eliminating the tedious work of film transcription. The landmark points of a moving object are designated by markers. If the markers are passive retroreflectors then neither wiring nor control circuitry must be carried by the moving object. However, in this case the identification of markers is an extra burden: clustering of the pixels, belonging to

[^0]the same marker image is required as well as the identification of markers on consecutive frames. Clustering methods are summarised in (JOBBÁGY et ail., 1991) while an automatic method for marker classification is given in (Ferrigno et al., 1988).

The shape of a marker is either disk or sphere or hemisphere, all resulting in a circular image on the CCD surface in the ideal case. While processing the digitised video output signal of the camera the centres of the circular images are estimated. It has two advantages. On the one hand it means a substantial information reduction, because only one point has to be further processed instead of the several elementary points (covered elementary pixels of the CCD chip) constituting the marker image. On the other hand sub-pixel accuracy can be achieved.

This paper concentrates on the estimation of the centres of circular marker images. Simulation programs were used to analyze the effectiveness of different estimation methods, provided the marker image is an ideal circle.

## Marker Centre Estimation

The task is to estimate where the centre of a marker image is. If the image of a marker is projected onto only one pixel of the CCD chip (see Fig. 1a) then the resolution and accuracy of measuring the position of the marker are determined by the number of columns and lines. If the image of the marker covers several elementary pixels (see Fig. 1b) then sub-pixel resolution and accuracy can be achieved by estimating the midpoint of the marker on the basis of all the covered pixels.


Fig. 1. Marker image of different size projected onto the surface of a CCD chip. Only one pixels is touched (a). Several pixel are touched (b).

During the investigation the marker image on the surface of the CCD chip was considered to be an ideal circle with even luminosity. The implied simplifications do not substantially diminish the generality of the results as it is shown in (Jobbágy and Furnée, 1993).

In present day motion analyzers the video output signal of a CCD camera is processed by a 1 bit A/D converter. It means that a pixel is evaluated as covered if the corresponding video output signal is above a given threshold level. As a result of this quantisation marker images with different radii and centres can be placed on the CCD surface so that the set of elementary pixels, evaluated as covered is the same. Fig. 2 shows two marker images with different radii and centres. In those pixels, which are partly covered by the images the percentage of coverage is written: the upper number is for the continuous circle while the lower one is for the dashed circle. If the threshold level used for quantising the video signal corresponds to $50 \%$ coverage of a pixel then after digitising the video output signal the same set of covered pixels belongs to both marker images. It means that when the set of covered pixels is known it determines neither the centre nor the radius of the image. In other words it is the result of the finite resolution and it is independent of the method used for estimating the centre on the basis of the covered pixels.


Fig. 2. Marker images with different radii and centres may result in the same set of covered pixels as a result of quantisation

## The Theoretical Limits of Accuracy

It is possible to derive the theoretical limits of accuracy, if accuracy is regarded as the expected value of the distance between the actual and the estimated centre of a circular marker image, which is randomly projected onto the CCD surface. The theoretical limits of accuracy (as a function
of marker size) were computed by a simulation program. The program scanned a surface equivalent to that of an elementary pixel. The shape of the equivalent surface must be selected so that 'whole areas' be included in it. The definition of an 'area': while the centre of a circular marker image is within an area, the same (and only those) pixels are reported to be covered (cf Fig. 3). It means that independently of the estimation method, the finite resolution results in these areas. The expected value of the deviation of the estimated centre from the actual is:


Fig. 3. The selection of the surface to be scanned shows that it must be made up of whole areas.

$$
\begin{equation*}
E(d)=\int_{A} p(A) \mathrm{d}(A) \mathrm{d} A \tag{1}
\end{equation*}
$$

where $p(A)$ gives the probability distribution of the centre of a randomly projected marker image along the scanned equivalent surface and $d(A)$ is the deviation distribution. As the calculation was done by selecting a finite number of discrete points within the scanned surface, (1) must be modified:

$$
\begin{equation*}
E(d)=\sum_{i=1}^{S} p_{i} \cdot d_{i}, \tag{2}
\end{equation*}
$$

where $S$ is the number of discrete points, $p_{i}$ is the probability that the centre of a randomly projected circular marker image is in the $i$ th discrete point and $d_{i}$ is the deviation of the estimated centre from the actual one if this
latter is in the $i$ th discrete point. If the marker image is projected randomly then its centre can be in any discrete point with the same probability:

$$
\begin{equation*}
p_{i}=\frac{1}{S} \tag{3}
\end{equation*}
$$

The estimated centre was computed in the following way. Given the size (the radius) of a circular marker image we grouped the $S$ points into classes: a class contained those points which were within the same 'area' (cf Fig. 3). We computed the centre of gravity of each area and regarded this as the estimation for all the points within this area. It means that the theoretical limits of accuracy imply the knowledge of the marker image radius. In the practical applications it is unknown, the available information is the set of covered pixels only.

## Conventional Marker Centre Estimation Methods

On the basis of the information, which pixels are covered the most frequently used estimation is to calculate the geometric centroid of these pixels:

$$
\begin{equation*}
X_{c}=\frac{1}{N} \sum_{i=1}^{N} x_{i}, \quad Y_{c}=\frac{1}{N} \sum_{i=1}^{N} y_{i} \tag{4}
\end{equation*}
$$

where the number of covered pixels is $N$, the coordinates of the $i$ th covered pixel are $x_{i}, y_{i}$ and the estimated marker centre coordinates are $X_{c}, Y_{c}$.

Using only the contour pixels it is possible to fit a circle on them. In order to eliminate the iterations from the conventional least squares fit the modified error function to be minimised is:

$$
\begin{equation*}
E F=\sum_{i}\left[\left(X_{c}-x_{i}\right)^{2}+\left(Y_{c}-y_{i}\right)^{2}-R_{s}^{2}\right] \tag{5}
\end{equation*}
$$

where $x_{i}, y_{i}$ are the coordinates of the contour points, $X_{c}, Y_{c}$ are the coordinates of the centre of the fitted circle and $R_{s}$ is its radius. This method is detailed in (CHERNOV and Ososkov, 1984) and (MOURA and Kitney, 1991) and is reported to be applied in (Morris, 1990). If the marker image is ideal there is no substantial difference between the results gained using the geometric centroid calculation and circle fitting. The latter requires much more computation and can only be advantageous if the marker image is distorted by occlusion. Even in this case unsolved is the problem of eliminating the contour points which are not on the circumference.

## New Marker Centre Estimation Method

Examining different sets of pixels, covered by circular images we concluded that
a) a given set of pixels can be covered by circular images with different radii,
b) applying the method, described at the theoretical limits of accuracy, different estimated centres may be assigned to the same set of covered pixels assuming different radii.

The latter is the case if the set of pixels is asymmetrical. Fig. 4 shows a part of a CCD chip surface and the circular image of a marker projected onto it. Those elementary pixels are hatched that are qualified as covered after digitisation. The ratio of the two sides of an elementary pixel corresponds to that of the NXA 1011 frame transfer sensor of Philips ( $10 \mu \mathrm{~m}$ horizontally and $15.6 \mu \mathrm{~m}$ vertically). This chip is applied in a measurement quality camera used in the PRIMAS PRecIsion Motion Analysis System, developed at the Motion Studies Lab., TU Delft (cf Furnée, 1989). Fig. 5 shows estimations that can be derived with the method, described at the theoretical limits, supposing different marker radii. The base of a pyramid shows the $x, y$ coordinates of the centre of gravity of the area, within which the centre of a circular marker image may move with the given radius so that the pixels covered are those, hatched on Fig. 4. The height of a pyramid is proportional to the size of the corresponding area. It can be seen that the geometric centroid calculation gives a distorted estimation. We consider an estimation optimal if the expected value of the distance between the actual and the estimated centre of a circular marker image is minimal. Of course, in a practical application the radius of the image can not be considered as known. (Jobbágy and Furnée, 1993) introduced the BEWRI (Best Estimation Without Radius Information) method. This method requires the calculation of the weighted average of the centres of gravity of the areas, which correspond to the same set of covered pixels and to different supposed radii. The realisation of the method in its original form needs a great number of computation and as a result can only be applied for off-line estimations.

## Realisations of the Estimation Method

The BEWRI method in its original form requires the following steps when the covered set of pixels is given.


Fig. 4. A circular marker image on the surface of a CCD chip. The hatched elementary pixels can be qualified as covered, depending on the threshold level.


Fig. 5. Estimations of the centre of a marker image, that can cover the hatched pixels of Fig. 4.
a) Compute the minimal $\left(r_{\min }\right)$ and the maximal ( $r_{\max }$ ) radius of a circular image, that can cover the given set of pixels (those and only those).
b) Regarded $r_{\text {min }}$ as known determine the corresponding area (within which the centre of the marker image with the radius $r_{\text {min }}$ may move so that the same set of pixels is covered): both its size ( $A_{\text {min }}$ ) and its centre of gravity ( $C G_{\text {min }}$ ).
c) Repeat b) supposing a radius $r_{i}=r_{\text {min }}+i \cdot \Delta r$, until $r_{i}=r_{\text {max. }}$. The size and the centre of gravity for $r_{i}$ is $A_{i}$ and $\mathrm{CG}_{i}$, respectively.
d) Compute the weighted average of centres of gravity using the sizes of areas as weighting factors and regard it as the BEWRI result:

$$
B E W R I=\frac{\sum_{i} A_{i} \cdot C G_{i}}{\sum_{i} A_{i}} .
$$

To reduce the necessary computation the following two algorithms have been developed. The MAXARE (MAXimal ARea Estimation) algorithm searches for the maximal area and considers the centre of gravity of this area as the estimation for the centre of the covering marker image:

$$
M A X A R E=C G_{m}, \quad m=i:\left\{A_{i}=\max .\right\}
$$

The AVRAE (AVerage RAdius Estimation) algorithm computes $r_{\text {min }}$ and $r_{\text {max }}$ the same way as it is described in the a) point of BEWRI method. Then the average radius is determined: $r_{a v}=\left(r_{\text {min }}+r_{\text {max }}\right) / 2$. Finally, the centre of gravity of the area, that corresponds to $r_{a v}$ is considered as the estimation for the centre of the covering marker image:

$$
A V R A E=C G_{(\min +\max ) / 2}
$$

The three estimations for the covered set of Fig. 4 have been computed. Fig. 6 shows the $X, Y$ plane of Fig. 5 together with the three above detailed estimations. In order to evaluate the different estimations in general a simulation program has been written. This program scans a part of the surface of the CCD chip the same way as it is described at the theoretical limits of accuracy. We supposed a pixel geometry of $10 \mu \mathrm{~m}$ (hor.) $\times 15.6 \mu \mathrm{~m}$ (vert.) and scanned a surface equivalent to the surface of an elementary pixel in 3900 steps: 50 horizontally and 78 vertically. The investigated marker sizes conform to the most frequently used ones: marker images between 4 and 12 lines were analyzed, the step in marker size was 0.032 lines. Fig. 7 shows the results.

## Conclusions

The geometric centroid calculation is the most frequently used marker centre estimation method in CCD camera based motion analysis. We showed that this method results in a distorted estimation if the marker image is asymmetrical. Assuming the radius of the marker image as known we derived the theoretical limits of accuracy. This accuracy cannot be reached


Fig. 6. Different estimations for the centre of a marker image that covers the pixels hatched on Fig. 4.
$\mathrm{E}(\mathrm{d}), \%$ of line (vertical pixel side)


Fig. 7. Comparing the accuracy of the geometric centroid calculation and of the BEWRI method to the theoretical limits.
as the marker image radius is unknown in the practical applications. We derived a method called BEWRI for estimating the centre of a circular marker image without knowing its radius. To reduce the great number of calculations required by the method we introduced two slightly simplified realisations: the MAXARE and the AVRAE methods which provide almost the same accuracy as BEWRI. Fig. 7 shows that the results gained by using

## (MAXARE - BEWRD / MAXARE



Fig. 8. Comparing the accuracy of the MAXARE and BEWRI methods as a function of marker size.
the BEWRI method are much closer to the theoretical limits than to the accuracy of the geometric centroid calculation. In this figure we included neither the MAXARE nor the AVRAE results because they would have been too close to BEWRI data. Instead the differences between MAXARE and BEWRI are separately shown on Fig. 8. This demonstrates that the two estimations yield almost the same accuracy. It means that in practical applications MAXARE should be preferred because it requires less computation.

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