# EVALUATION OF A THREE PHASE RECTIFICATION OF HIGH DYNAMIC PERFORMANCE

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#### Abstract

The paper is concerned with a three phase configuration devised for rectification. The power is transferred from the network to the output by high frequency sinusoidal current pulses and it is controlled by the pulse frequency and/or by the clamping of the voltage across the switched capacitor. The pulse frequency is lower for thyristors and higher for transistors. It offers unit power factor in the whole control range, fast dynamic response and makes filtering easier because of the high pulse frequency.

The operation of the circuit has been described in a previous paper (NAGY, 1989). In this paper the train of the clamped voltages of the switched capacitor for symmetrical and asymmetrical energy supply is determined and an approximate investigation of the harmonic components in the input filter is described.

Keywords: power electronics, high frequency rectification, three phase rectifier.

## Introduction

A three phase rectifier operating with high frequency pulses has been suggested in (NAGY, 1989). This paper is the continuation of the previous one.

The three phase rectifier described in (NAGY, 1989) offers some advantages over the full bridge thyristor rectifier. They are as follows:

- the power factor can be kept close to unity,
- due to the high frequency operation, the size of the filter components is relatively small,
- the dynamic behaviour is improved.

The energy is delivered by high frequency current pulses from the input to the output with the help of a switched capacitor. The output power is controlled by the switching frequency and by the clamping of the voltage across the switched capacitor. The clamping is done by controlled switches.

The paper describes the control of the clamping switches for symmetrical (SES) and asymmetrical energy supply (AES). An approximate I. NAGY

investigation of the high frequency current pollution of the supply network is discussed as well. Computer results are presented.

Referring to the equations and figures of the previous paper (NAGY, 1989), letter p will be added after the number e. g. Eq. (3p) and Fig. 2p.

#### Configuration

The configuration discussed is shown in Fig. 1. Its operation in pulse mode has been described in detail in a previous paper so only a brief summary will be given here. Only one controlled switch out of the six plus two is turned on in an interval. In the time span  $30^{\circ} < \omega t < 90^{\circ}$  of phase voltage  $u_{ia} = U_{im} \sin \omega t$ ,  $T_{pa}$  and  $T_{nb}$  are turned on, while in the interval  $90^{\circ} < \omega t < 150^{\circ}$  of  $u_{ia}$ ,  $T_{pa}$  and  $T_{nc}$  are alternately turned on. Turning on  $T_{pa}$ , a sinusoidal current pulse of high frequency flows in circuit  $T_{pa} - L_0 - C_0 - C - C_i$  and it increases voltage  $u_c$  ( $C \ll C_i$  and  $C \ll C_0$ ).



Fig. 1. Three phase configuration

Assuming thyristors in place of the controlled switches,  $T_{pa}$  can be turned off any time after  $u_c > u_{ia}$  by turning on the clamping thyristor  $T_{p1}$ . The energy trapped in choke  $L_0$  is supplied to the output. The next negative current pulse flows in the negative half of the configuration.

Applying transistors rather than thyristors,  $u_c > u_{ia}$  is not a precondition for turning off  $T_{pa}$ . Thyristors as controlled switches will be assumed later unless it is otherwise specifically stated.

### **Calculation of the Clamped Condenser Voltages**

The expressions for the output energies carried by one current pulse (NAGY, 1989)

THREE PHASE RECTIFICATION OF HIGH DYNAMIC PERFORMANCE

$$W_{op} = C[u_{cp}(k) - u_{cn}(k-1)]\{u_{ip}(k) - [u_{cp}(k) + u_{cn}(k-1)]/2\}$$
(1)

and

$$W_{on} = C[u_{cp}(k) - u_{cn}(k)] \{ -u_{in}(k) + [u_{cp}(k) + u_{cn}(k)]/2 \}$$
(2)

[Eqs. (41p) and (42p)]. Knowing the input voltages the clamped capacitor voltages  $u_{cp}(k)$  and  $u_{cn}(k)$  can be determined from given  $W_{op}$ ,  $W_{on}$  and  $u_{cn}(k-1)$ .

#### Symmetrical Energy Supply (SES)

The clamping thyristors are supposed to be fired in a manner that each current pulse carries the same energy portion to the output Eq. (25p):  $W_{op} = W_{on} = W_o$ . By solving the equations of second order, voltages  $u_{cp}(k)$  and  $u_{cn}(k)$  are expressed from Eqs. (1) and (2) at given  $W_o = W_{op} = W_{on}$  and  $u_{cn}(k-1)$ , respectively:

$$u_{cp}(k) = u_{ip}(k) + \sqrt{u_{ip}^2(k) - b_p(k)}, \qquad (3)$$

$$u_{cn}(k) = u_{in}(k) - \sqrt{u_{in}^2(k) - b_n(k)}, \qquad (4)$$

where

$$b_p(k) = 2W'_o + u_{cn}(k-1)[2u_{ip}(k) - u_{cn}(k-1)], \qquad (5)$$

$$b_n(k) = 2W'_o + u_{cp}(k)[2u_{in}(k) - u_{cp}(k)], \qquad (6)$$

and  $W_o' = W_o/C$ .

The value of the capacitor voltage  $u_{cn}(k-1) = u_{cn}(0)$  at the angle  $(30^\circ + \epsilon + \varphi)$  must be chosen in a way that the periodicity condition

$$u_{cp}(N+1) = -u_{cn}(0) \tag{7}$$

be met at a given  $W'_o$  where  $u_{cp}(N+1)$  is the capacitor voltage at the angle  $(\omega t_2 + \varepsilon + \varphi)$  (Fig. 6p), that is, at the very beginning of the next 60° cycle starting with a negative current pulse in phase c. After an iteration procedure, relation (7) can be met.

I. NAGY

#### Asymmetrical and General Energy Supply (AES, GES)

Now the sum of the output energy in each  $\gamma$  cycle is kept constant

$$w'_{op}(k) + w'_{on}(k) = 2w'_{o1} = 2W'_o$$
(8)

and the ratio

$$w'_{op}(k)/w'_{on}(k) = \beta(k) \tag{9}$$

is given.

 $w'_{op}(k)$  and  $w'_{on}(k)$  can be calculated from the two relations. Eqs. (3) ... (6) hold for the determination of  $u_{cp}(k)$  and  $u_{cn}(k)$ , but  $w'_{op}(k)$  and  $w'_{on}(k)$  has to be substituted into Eqs. (5) and (6), respectively, in the place of  $W'_{o}$ .

Selecting  $\beta(k)$  according to the relation:

$$\beta(k) = u_{ip}(k)/[-u_{in}(k)] \tag{10}$$

the state AES is realized. In the state of SES  $\beta = 1$ . In cases of any other  $\beta$  the state of GES prevails.

#### **Calculation Results**

In this Section  $u_{cp}^*$  and  $u_{cn}^*$  denotations are applied for the clamped peak values.

The development of the 300 Hz component in the resultant output voltage  $(u_{op}(k)+u_{on}(k))$  can be avoided by delivering constant energy in each cycle. The configuration works as an active filter by the clamping thyristor.

Symmetrical Energy Supply 
$$(W_{op} = W_{on})$$

The calculations have been carried out in the way described above.

The output energy  $W_o$  as a function of the initial value of the capacitor voltage  $u_{cn}(0)$  is shown in Fig. 2. N is the parameter.

The time functions of capacitor voltage amplitudes  $u_{cp}^*(k)$  and  $u_{cn}^*(k)$  for various initial conditions and for different pulse numbers were calculated and are shown for N = 7 in Fig. 3. The conditions  $u_{cp}^*(k) > u_{ia}$  and  $u_{cn}(k) < u_{ib}^*$  must always be satisfied.

On the other hand, the voltages  $u_{cp}(k)$  and  $u_{cn}(k)$  obtained without firing the clamping thyristors in the k'th cycle must be higher than the ones



Fig. 2. Output energy versus initial condition  $|u_{cn}^*(0)|$ 



Fig. 3. Capacitor voltages clamped, unclamped.  $(W_{op} = W_{on}, N = 12)$ 

shown in Fig. 3. The values  $u_{cp}(k)$  and  $u_{cn}(k)$  depend on the input and the output voltages as well as on the clamped peak values of the capacitor voltages. The time functions of  $u_{cp}(k)$  and  $u_{cn}(k)$  at  $u_{cn}(0) = -1.5$  for

 $\mathbf{34}$ 

output voltages  $U_o = 0.75$  and 0.9 are shown as well. They were calculated from the relations [see Eq. (1p)]:

$$u_{cp}(k) = -u_{cn}^{*}(k-1) + 2[u_{ip}(k) - U_{o}]$$
(11)

and

$$u_{cn}(k) = -u_{cp}^{*}(k) - 2[u_{in}(k) - U_{o}].$$
<sup>(12)</sup>

The output voltage  $U_o = 0.9$  cannot be realized since the clamped values are higher in certain intervals. The negative sequence component of the capacitor voltage amplitudes  $u_{c2}^*$  as a function of time is drawn as well. The maximum value of the still realizable output voltage  $U_{omax}$  as a function of the initial condition  $u_{cn}^*(0)$  is shown in *Table 1* for different N values. The maximum value of the output voltage is 0.75.

Table 1 Maximum value of output voltage  $U_o$ 

	$U_{o\max}$					
$u_{cn}^*(0)$	N = 15	N = 12	N = 7	N=2		
2	0.752	0.753	0.755	0.762		
1.5	0.756	0.758	0.763	0.771		
1.3	0.759	0.761	0.768	0.778		
1.1				0.787		

#### Voltage Control

The configuration is a device delivering an approximately constant output power. It keeps its average output power constant at a given current pulse rate with changing load at constant input voltages and at constant capacitor voltage change [see Eq. (9p)].

The following voltage control strategy is recommended for smooth, continuous voltage change. Starting from rated output voltage and from the maximum number of current pulses as well as supposing constant current load, first the clamped capacitor voltage amplitude has to be reduced by gating the clamping thyristors at the right time. Each particular initial condition  $u_{cn}^*(0)$  together with the pulse number N determines the output energy and the train of condenser voltage amplitudes within 60° in steadystate. After reducing the output power and voltage to a certain level, two input variables have to be changed at the same time in order not to change the output power and voltage stepwise:

- the number of current pulse pairs N has to be reduced by one,
- the capacitor voltage amplitudes corresponding to a new  $u_{cn}^*(0)$  initial value have to be increased by the right firing of the clamping thyristors in order to keep the average output power constant in spite of the sudden reduction in N.

The new  $\gamma$  cycles with an increased length have to be evenly distributed along the 120 degree time span as has been previously shown in (*Fig. 6p*). Altogether four pulses have to be omitted along a 120° interval, namely two positive and two negative pulses. The pulse pattern remains the same and the power factor for the fundamental components is still unity.

Further reduction in the output voltage can be carried out in the same way.

A ratio 10:1 in the reduction of output power can be achieved corresponding to the same ratio in the reduction of output voltage at constant current load.

#### Input Filter

The input filter has chokes  $L_i$  and capacitors  $C_i$  (Fig. 4). It prevents the high frequency current from flowing into the line.

An approximate investigation of the high frequency current pollution of the network is discussed. It is assumed that three current generators (Fig. 4a) impress current pulse trains consisting of half wave current pulses with equal amplitudes shown for phase 'a' in Fig. 4b. The other two current pulse trains of the other two phases are the same as  $i_a$ , only one of them is shifted forward, the other one is shifted backward by  $120^\circ$  in respect to  $i_a$ .

First the Fourier spectrum of the pulse train  $i_a$  was determined for N = 15. The half period of the current pulse was supposed to be  $100 \,\mu s$  corresponding to  $5 \,\mathrm{kHz} \,(\omega_n/\omega = 10^{-2})$ . The results are shown in Table 2.  $\omega_o/\omega_n = 9$  and 10 have been chosen. Here  $\omega_o$  is the resonant frequency of the input filter,  $I_1$  is the amplitude of the fundamental component of the current  $i_a$ ,  $I_n$  is the amplitude of the n'th harmonic component of the current  $i_a$ .

The following remarks throw some light on the results:

- no 3(1+2q) harmonics exist (q=0, 1, 2, ...),

<sup>-</sup> no even harmonics and



Fig. 4. Input filter

Table 2Fourier spectrum. N = 15

N		15	$I_1$	=	0.326	Irms	= 0.394
			$I_{Ln}/I_1$		$I_{cn}/I_1$		
n	$ I_{n}/I_{1} $						$100 \sum_{1}^{n} \frac{I_{n}^{2}}{2I_{rms}^{2}}$
			$\omega_o/\omega_n$		$\omega_o/\omega_n$		
			9	10	9	10	
1	1.0		1.0125	1.01	0.0125	0.01	27.7
5	0.2		0.29	0.27	0.09	0.07	35.8
7	0.144		0.365	0.28	0.22	0.138	36.5
11	0.09		0.19	0.44	0.28	0.53	36.8
13	0.08		0.07	0.11	0.15	0.194	37.0
93	0.982		0.01	0.011	5 0.99	0.994	79.3
185	0.4		<0.001	0.001	2 0.4	0.403	93.6
187	0.39		<0.001	0.001	2 0.39	0.393	98.8

— only (6m+1) harmonics occur (m = 1, 2, 3, ...), beside the fundamental one,

- local maximuma are around frequencies 3(2N+1)p, (p = 1, 2, 3, ...), since (2N+1) evenly distributed pulses occur in the time span  $\omega_n(t_3-t_1)=120^\circ$ .
- the first several line current harmonics  $I_n/I_1$ , (n = 5, 7, 11, 13, 17, 19) are practically the same as in normal three phase operation with constant dc current for N = 7, 12, 15. In the case of N = 2 the local maxima follow each other very frequently and the pattern for  $I_n/I_1$  is quite different from the others. The fundamental components being small, the higher percentage value of harmonics produces still small harmonic current.

Around the resonant frequency of the input filter the current in the input choke  $I_{Ln}$  and in the input capacitor  $I_{cn}$  is considerably increased above  $I_n$ . It can be  $2 \sim 3$  times higher than  $I_n$ .

The per unit value for the amplitudes of the choke currents:

$$I_{Ln}/I_n = 1/D \tag{13}$$

and that for the input capacitor currents:

$$I_{cn}/I_n = -[(\omega_n/\omega_o)^2 n^2]/D$$
(14)

and the amplitudes of the capacitor harmonic voltages:

$$U_n/(\omega_o L_i I_n) = (n/D)(\omega_n/\omega_o)$$
(15)

where

$$D = 1 - \left(\omega_n / \omega_o\right)^2 n^2 \,. \tag{16}$$

Considerable line currents can be found only among the first several harmonics. Most of the harmonic currents of the rectifier are short circuited by the filter capacitor.

 $\omega_o/\omega_n = 9$  is a better choice than  $\omega_o/\omega_n = 10$ , since it is in midway between the 7th and 11th harmonics.

The average value of  $i_a$  is  $(\omega_n/\omega = 10^{-2})$ 

$$I_{\text{ave}} = (2N+1)I_m \frac{2}{\omega T} \cdot \frac{\omega_n}{\omega} \int_0^{\pi} \sin \omega t \, d\omega t \,,$$
$$I_{\text{ave}} = (2/\pi)[(2+1)I_m](\omega_n/\omega) \,. \tag{17}$$

In normal rectifier operation with smooth dc current  $I_{dc}$ , the average of the absolute value of the line current is:

$$I_{\text{ave, }n} = (2/3)I_{dc}$$
 (18)

Assuming  $I_{\text{ave}} = I_{\text{ave}, n}$ 

$$I_{dc}/I_m = (3/\pi)(\omega_n/\omega)(2N+1).$$
 (19)

In the case of N=12 and  $\omega_n/\omega=10^{-2}$  the current pulse maximum has to be almost four times as high as  $I_{dc}$  to draw the same average current.

Without firing the thyristors, the peak value of the phase current  $I_s$  flowing in filter  $L_i - C_i$  is

$$I_s/I_1 = (U_s/U_1) \frac{1}{(\omega_o/\omega_n)^2 D^2},$$
(20)

where  $U_s$  is the supply phase voltage and  $U_1$  is the voltage developed across the capacitor  $C_i$  by the fundamental component  $I_1$ .  $U_s$  and  $U_1$  are peak values. Assuming  $U_s/U_1 = 10$ , the ratio  $I_s/I_1 = 1/8.1 = 0.12$ . The half wave of current  $i_c = I_m \sin \omega t$  produces a voltage change  $\Delta U_c$  across the capacitor  $C_i$ 

$$\Delta U_c/U_s = 2C/C_i \,, \tag{21}$$

where  $I_m = U_s/\sqrt{L/C}$ . Here it was assumed that  $i_c$  flows only in one capacitor  $C_i$ . At ratio  $C/C_i = 1/20$  the voltage ratio  $\Delta U_c/U_s = 0.1$ .

In order to reduce the output voltage some pulses have to be dropped out. There are basically two ways for it. Either the pulse pattern is maintained (*Fig. 6p*) or the rest of the pulses still flowing keep their location.

#### Conclusions

The three phase thyristor rectifier described above has some favourable features and numerous drawbacks. It has to be weighted in each particular case whether its application pays off or not.

Its advantageous features are as follows:

- its power factor for the fundamental components is always unity,
- the output filter can be less expensive, smaller and faster,
- its dynamic response is faster because of its high frequency operation.

The main disadvantages are:

- a number of additional components (chokes, capacitors, thyristors) have to be built in,
- the thyristor voltage rating is higher,

- some of the line harmonics are higher,
- its control is rather complex.

A further drawback is that the potential of the centre tap of the output capacitor is oscillating with voltage  $u_c$  around the potential of the zero point of the supply network.

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#### References

- ČUK, S. MIDDLEBROOK, R. D. (1977): A General Unified Approach to Modelling Switching DC to DC Converters in Discontinuous Conduction Mode. Proceedings of the IEEE Power Electronics Specialists Conference, Palo Alto, CA, June 14-16, 1977, pp. 36-57.
- CHIBANI, A. NAKAOKA, M. MARUHASHI, T. (1987): New High-Frequency Resonant PWM Inverter-Linked DC-DC Converter Using Insulated Gate Transistors. Proc. Second European Conference on Power Electronics and Applications. Grenoble, France, 22-24 Sept. 1987, Vol. 1. pp. 213-218.
- MIDDLEBROOK, R. D. (1981): Electronics: Topologies, Modelling and Measurement. Proceedings of the IEEE International Symposium on Circuits and Systems. Chicago, Il. April 27-29, 1981. pp.
- NAGY, I. (1989): Three Phase Rectification with High Frequency Pulses. Periodica Polytechnica Ser. Electrical Engineering, Vol. 34. No. 2. pp. 137–152.

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