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NON-UNIQUELY SOLVABLE NETWORK MODELS

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Abstract

Many methods are known from the literature to perform the analysis of a linear electrical circuit by its nullator norator pairs model. A procedure based on such a method is very elegant from the aspect of the network theory.

If a linear electrical circuit is uniquely solvable, then its nullator norator pairs model is also solvable but not necessarily uniquely. The procedure mentioned above can be applied only if the model has a unique solution. For example, if the electrical circuit contains a two-port network part without a common vertex, then its model cannot be calculated in the previous manner, in general.

The present paper releases the limit of this procedure. First the author deals with non-uniquely solvable nullator norator pairs networks of different sort, and selects those networks which occur most often during the modelling. After defining the notion of the quasiregular network its basic properties are introduced. Further properties are summarized in two theorems, which enable the calculation of quasiregular networks, such that this calculation is traced back to uniquely solvable nullator norator pairs networks. Finally, an example is given for the application of the author's procedure.

Keywords: linear circuits, nullator, norator, solvability.

Introduction

For computer calculation of a linear electrical circuit the use of nullators and norators mentioned in DAVIES (1966) offers many advantages. Considering these elements the network consisting of controlled sources, ideal transformers, impedance converters, gyrators, operational amplifiers, etc., can be transformed to models containing only sources, RLC elements and nullator norator pairs (BRUTON, 1980). At the same time, the graph of the model is always connected, therefore a very simple computer program can be constructed for the numerical analysis. Often it can be decided from the model whether the original circuit is solvable or not (PÁVÓ, 1988). Finally, the nullator norator pairs model gives a possibility for the realization (synthesis).

To produce a nullator norator pairs model one has to exchange some two-port network parts of the original circuit for their nullator norator pairs models as it is shown in VÁGÓ (1985) and in HOLLÓS (1981a). It is known from the literature that there exist many models of a two-port network. In a concrete case the choice of the model depends on practical points of view. If the model described above has a unique solution, then its analysis can be performed by one of the methods presented in VÁGÓ (1985) and in HOLLÓS (1981b, c).

A network or a network model is called *regular* if the voltages and the currents of all its elements are uniquely determined. In case of a two-port network this determination refers to both the input and the output variables. In practice the regularity of an electrical circuit is an indispensable requirement. When a circuit is solvable, so is its network model and vice versa. But a model of a regular network is not necessarily a regular one.

To illustrate this situation let us consider networks of *Fig. 1* together with their nullator norator pairs model. Observe that the first network is unsolvable and so is its model. The second and the third networks are regular, the model of the second network is also regular, but that of the third model is not regular since it has more solutions.

Let us remark that the third network in *Fig. 1* is a two-port one without common vertex. In general, although a two-port network without common vertex is regular, its nullator norator pairs models found in HOLLÓS (1981a) are always non-uniquely solvable. Networks which contain a model of the previous property can be calculated with none of the methods described in the literature up to now.

In this article a method is developed which removes the limit of the modelling of nullator norator pairs networks mentioned above.

Types of Non-uniquely Solvable Models

In *Fig. 2* we see some nullator norator pairs networks which have more solutions. The elements of undetermined voltage or current are marked by $*$ in the networks.

The models in *Fig. 2* are examples for non-uniquely solvable nullator norator pairs networks. In general, the undetermined voltage or current may occur at every element of the model, except the nullators. In what follows we are interested in models where the undetermined voltage or current occur only at norators, such as in the network in *Fig. 2b*.

Definition. The nullator norator pairs network is called *quasiregular*, if it is not regular, but after arbitrarily fixing the voltage or the current of

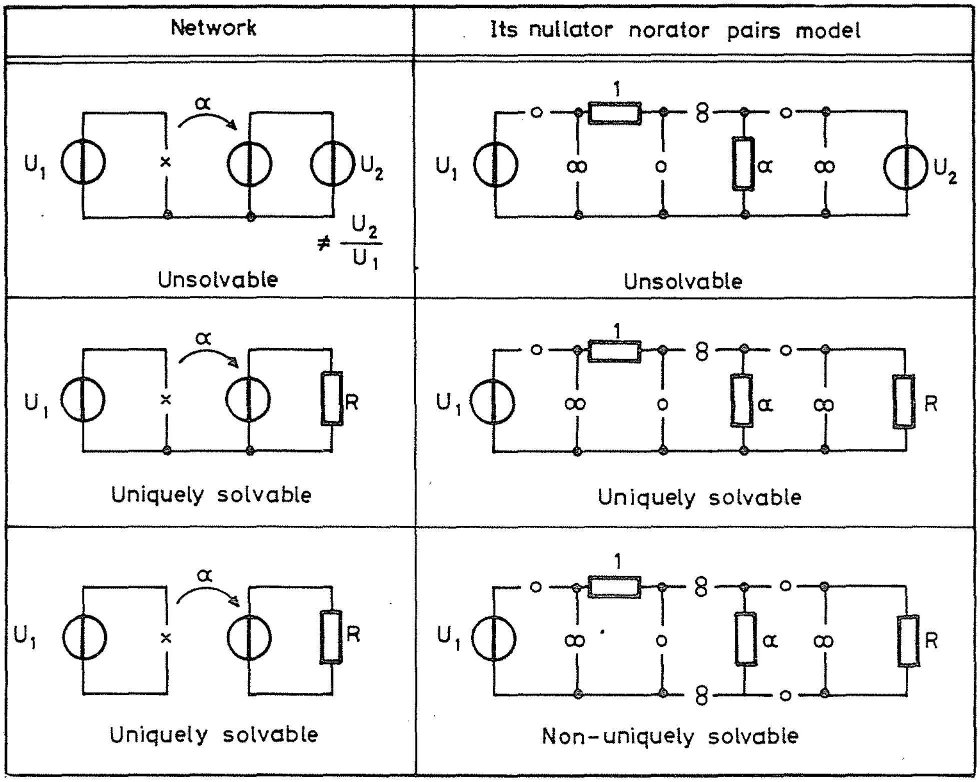

Fig. 1. Electric circuits and their models

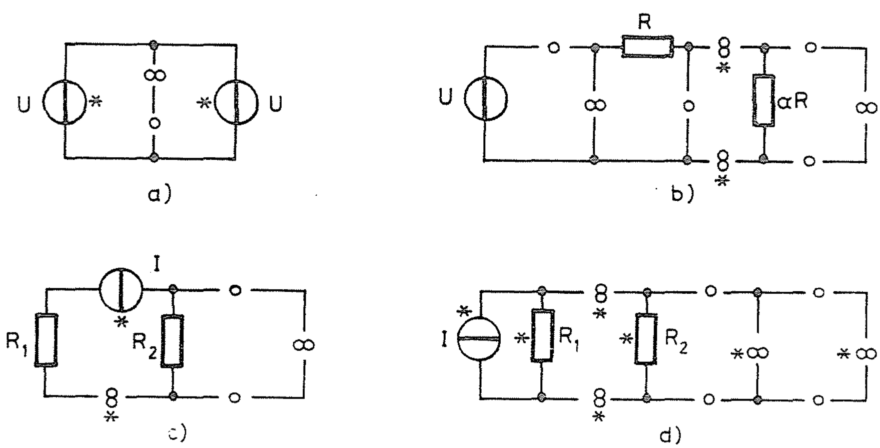

Fig. 2. Some non-uniquely solvable network models

solely norators the network becomes a regular one. The total number of the fixed voltages and currents is called the *order* of the network.

The network in *Fig. 2b* is an example for a quasiregular network of order 1. Namely if we arbitrarily fix the voltage of one of the two norators marked by ∗ both the voltage and the current of each element of the obtained network are determined. Another example is given for a quasiregular network of order 2 in *Fig. 3*.

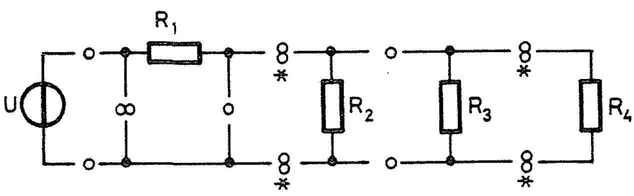

Fig. 3. Quasiregular network of order 2

Some Properties of Quasiregular Networks

In a quasiregular network
Property (*a*): there exists a cutset or a circuit formed purely by norators. The total number of the maximal independent cutsets and of the maximal independent circuits is exactly the order of the network;
Property (*b*); there exists a cutset or a circuit formed purely by nullators. The total number of the maximal independent cutsets and of the maximal independent circuits is exactly the order of the network.

From property (a) immediately follows that the order of the network is unique. Taking into account properties (a) and (b) there is a possibility to pair the independent cutsets and circuits formed by different degenerated elements as follows: let each independent norator cutset or circuit belong to an independent nullator cutset or circuit and vice versa. The number of the cutset-circuit pairs is the order of the network. Properties (a) and (b) are called shortly cutset-circuit pairs property. Namely the quasiregular network is always of cutset-circuit pairs property.

Theorem 1. In a quasiregular network the voltage and the current of each element are determined and they do not depend on the arbitrarily fixed norator voltages and currents, except the voltage of a norator belonging to a norator cutset and the current of a norator belonging to a norator circuit.

Theorem 1 guarantees that the undetermined voltages and currents occur only at norators. Moreover, a two-port network can be correctly modelled by a quasiregular network unless the input or output voltages or currents refer to norators belonging to a norator cutset or circuit.

The proofs of the properties and of the theorem can be found in the Appendix.

Solving the Quasiregular Network

To solve a quasiregular network means to determine the unique voltage and current of the network elements. According to Theorem 1 it is sufficient to solve a regular network obtained from the quasiregular network after arbitrarily fixing voltages and currents of purely norators.

If the norators to be fixed are known, the regular network is immediately obtained. But, in practice, these norators are not given in advance. During the analysis, the following problems arise: how to select norators for fixing voltages or currents (which is possible in several manners, in general) and how to decide at all whether the considered model is quasiregular or not. For solving these problems we introduce some notions and a procedure as follows.

A nullator norator pair of a network is called *distinguished* if the nullator of the pair belongs to a cutset or a circuit formed from purely nullators of the network and, at the same time, the norator of the pair belongs to a cutset or a circuit formed from purely norators.

To eliminate a distinguished nullator norator pair means the substitution of the pair elements. Namely the cutset element has to be replaced by a short-circuit, while the circuit element by an open-circuit.

Let us consider a quasiregular network. Because of its cutset-circuit pairs property, it trivially has a distinguished nullator norator pair. To eliminate this pair means to fix either the voltage or the current of a norator element as 0. In case of a quasiregular network the elimination decreases the order of the network by exactly one.

To reduce a nullator norator pairs network means the elimination of its distinguished nullator norator pairs, respectively, as follows. First an arbitrary distinguished nullator norator pair is eliminated, then the elimination is repeated for the obtained network, etc., until the last obtained network does not contain any distinguished nullator norator pair. Following this procedure to a quasiregular network of order n the elimination will be finished in n steps. The network obtained finally contains neither nullator cutset or circuit nor norator cutset or circuit.

The *reduced network* of a nullator norator pairs network is the network obtained at the end of the procedure described above. A nullator norator pairs network has more reduced networks, in general.

Comparing the notion of the reduced network with the solving of the quasiregular network it follows:
Property (*c*). Any reduced network of a quasiregular network is always regular.

Moreover it is sufficient to solve one of the reduced networks instead of the quasiregular network.

A regular electrical circuit has either regular or quasiregular nullator norator pairs network model. In *Fig. 4* we summarize the possibilities from the aspect of the solvability of the models. Considering this diagram we often have to decide whether the model for the calculation is quasiregular or not. For this decision it is useful to consider the following theorem.

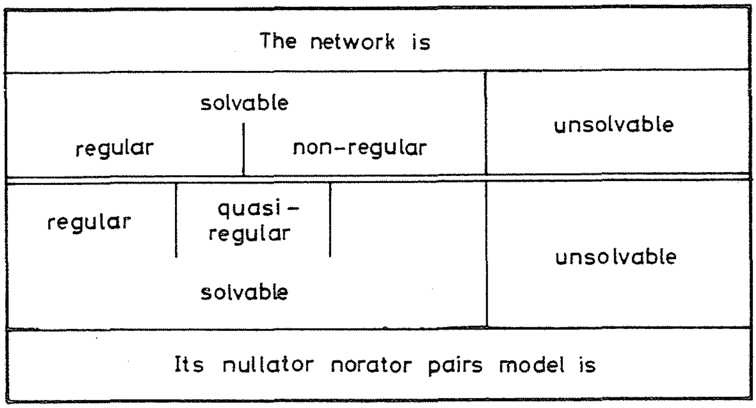

Fig. 4. Classification of networks and their models from the aspect of the solvability

Theorem 2. A nullator norator pairs network is quasiregular if and only if it has a distinguished nullator norator pair and its reduced network is regular.

The proof of Theorem 2 can be found in the Appendix, too.

Application

Let us consider the two-port network in *Fig. 5* with feedback by an ideal transformer where Z is an arbitrary two-terminal RLC impedance, the transmission of the transformer is positive and differs from 1. We are interested in the voltage u of the secondary side of the transformer.

The analysis of the network is performed in two special cases as follows.

First let the network be an ideal operational amplifier shown in *Fig. 6a*. *Fig. 6b* shows the nullator norator pairs model of this network.

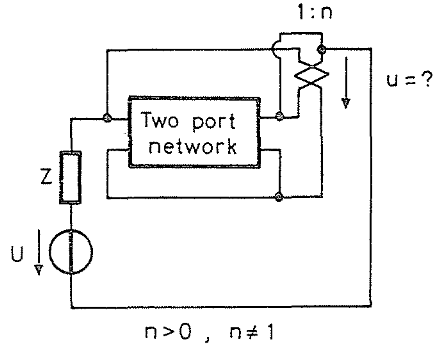

Fig. 5. Ideal transformer feedback by two-port network

It can immediately be seen that the model contains a norator cutset, a norator circuit and, at the same time, a nullator cutset as well. Therefore, it has not the cutset-circuit pairs property, it is not a quasiregular one, moreover, by a simple analysis we can prove that the model is unsolvable. Then the voltage of the secondary side is undetermined.

Secondly let the two-port network be a current generator controlled by voltage with transmission $r \neq 0$ according to *Fig. 7a.* The model of the network in question is shown in *Fig. 7b.* For the sake of the following investigation, the elements with the same parameters 1 and n are distinguished by single and double commas. To demonstrate the quasiregularity it is sufficient to show that the model has a regular reduced network. For example let us short-circuit the norators in *Fig. 7b* which are underlined. It is clear that all kernels of the reduced network (see in PÁVÓ (1988)) have exactly 3 elements. No kernel contains the element with parameter $1/r$, that is, the transmission of the controlled source does not influence the solvability. As pairs of elements $(1',n')$ and $(1'',n'')$ do not occur in a kernel, the element of parameter Z is the element of any kernel, moreover element $1'$ is not. So all kernels are $\{Z,n',n''\}$ and $\{Z,n',1''\}$, and taking into account PÁVÓ (1988) a sufficient condition of the solvability is

$$\frac{1}{Zn} \pm \frac{1}{Z^2n^2} \neq 0\,,$$

that is

$$1 \pm n \neq 0\,,$$

which obviously holds. As the searched voltage u is the voltage of $1'$, taking into consideration Theorem 1, u is uniquely determined.

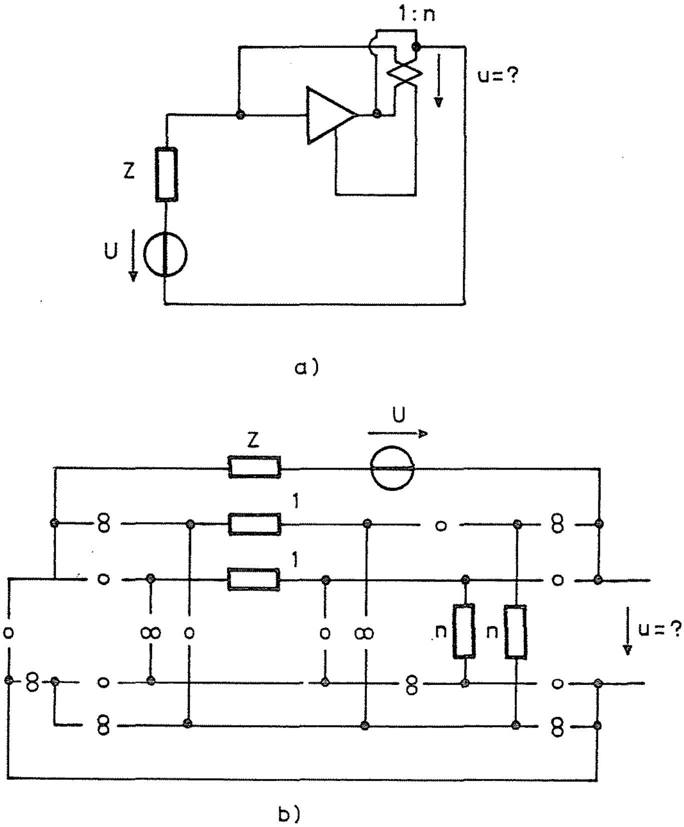

Fig. 6. The two-port network is an ideal amplifier

Remarks

To decide the quasiregularity of a network a topological procedure is needed. Nevertheless, this procedure is simpler than a calculation by topological formulas. Namely it need not generate all the subgraphs of the network graph (all the trees, circuits, cutsets, etc.), but only one step of the elimination has to be done, the network obtained in this way is already simpler. Naturally, a computer program can be constructed similarly for the procedure as it is mentioned in PÁVÓ (1988).

We suggest that the examination of the quasiregularity has to precede the network analysis procedure, the latter may be based on a traditional method known from the literature. Such an examination is especially useful when the parameter of some network elements has to be changed because of the uncertainty of the solvability in a synthesis procedure (see PÁVÓ

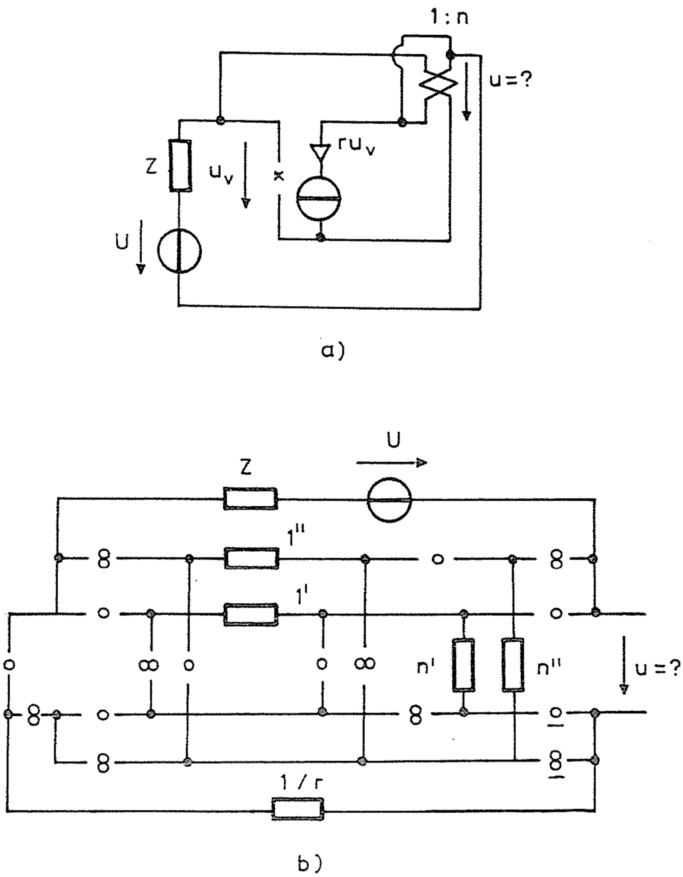

Fig. 7. The two-port network is a controlled generator

(1988)). Finally, by the above examination we can easily produce non-trivially unsolvable networks which we may use in education as well.

Appendix

The Proof of the Properties

Let us denote by U, I, Y, A and B the sets of the voltage and current sources, operator admittances of the passive elements, nullators and norators of the network, respectively. Starting from PÁVÓ (1988) we can write the network equations system in the following form:

$$\begin{array}{cccccccccc} U^u & I^u & B^u & A^u & Y^u & Y^i & A^i & B^i & U^i & I^i \end{array}$$

$$\left[\begin{array}{cccccccccc} \multicolumn{5}{c}{\boxed{\qquad \mathbf{B} \qquad}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \multicolumn{5}{c}{\boxed{\qquad \mathbf{Q} \qquad}} \\ 0 & 0 & 0 & 0 & \boxed{\mathbf{Y}} & \boxed{-\mathbf{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boxed{\mathbf{1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boxed{\mathbf{1}} & 0 & 0 & 0 \end{array}\right] \cdot \begin{bmatrix} \mathbf{u}_u \\ \mathbf{u}_i \\ \mathbf{u}_b \\ \mathbf{u}_a \\ \mathbf{u}_y \\ \mathbf{i}_y \\ \mathbf{i}_a \\ \mathbf{i}_b \\ \mathbf{i}_u \\ \mathbf{i}_i \end{bmatrix} = 0 \qquad (*)$$

In $(*)$ each $\mathbf{u}$ is a vector, components of which are Laplace transformed voltages, the subscript of $\mathbf{u}$ refers to the sort of the elements, while each $\mathbf{i}$ is a vector formed similarly from current components. $\mathbf{B}$ and $\mathbf{Q}$ are circuit and cutset matrices based on the same fundamental tree of the network graph, $\mathbf{Y}$ is a diagonal matrix the non-zero entries of which are the operator admittances of the passive elements, $\mathbf{1}$ denotes the unit-, $\mathbf{0}$ the null matrix. Symbols U^u, I^u, B^u, A^u, Y^u, Y^i, A^i, B^i, U^i and I^i denote the set of the columns of the matrix in $(*)$, respectively.

Let us suppose that $(*)$ refers to a quasiregular network of order n. So the rank of the matrix in $(*)$ is less by n than the number of its rows. Therefore, there are n homogeneous linear equations among the column vectors of the matrix. This equations system contains independent vector equations and n is maximal. Vectors occurring in any equation belong to either the set B^u or B^i, that is, they refer either to norator voltages or to norator currents. Because of the construction of the matrix in $(*)$ there exist sets of linear dependent column vectors in $\mathbf{B}$ and in $\mathbf{Q}$, which differ at least in one element, and their total number is exactly n, from which Property (a) immediately follows.

As the rank of the matrix is invariant with respect to transforming, there exist n homogeneous linear equations among the row vectors of the matrix in (∗) and they form an independent system of equations with maximal n. Again because of the construction of the matrix, vectors belonging to any equation of the system arise either from the rows of **B** and of nullator voltage rows or from the rows of **Q** and of nullator current rows. Repeating the former argument the validity of Property (b) follows as well.

The Proof of the Theorems

We prove Theorem 1 by induction on the order of the quasiregular network.

Let $n = 1$. Fix the voltage of a norator according to the definition. We can suppose that the corresponding column vector of B^u in (∗) has the only non-zero element in its first component. Move this column to set U^u. Leave a row vector from (∗) which refers to either nullator voltage or nullator current depending on whether the network has nullator circuit or cutset. The system of equations derived from (∗) in this way relates to a regular network. Consider any element of this network which does not belong to the norator cutset. The voltage or the current of this element is the quotient of two determinants and only the numerator can depend on the fixed norator voltage. After some elementary transformations of this determinant we can always get a column the unique non-zero element of which is in its first component. Computing the value of this determinant according to this column we get the proof. This procedure can be performed similarly in the case of fixed norator current.

Let $n > 1$, and suppose that the theorem is valid for any quasiregular network of order $n-1$. Let us fix a norator voltage or current. Because of the induction hypothesis the voltage or current to be calculated can depend only on the fixed voltage or current. As we can repeat the above procedure with another norator voltage or current the proof of the theorem is completed.

To prove Theorem 2 we note that the necessity of the condition is obvious. After this let us suppose that (∗) relates to a network which contains a distinguished nullator norator pair and its reduced network is regular. Let n be the number of the steps necessary to get the reduced network. Observe that if we fix in (∗) the voltages or currents of norators substituted by short-circuit or open-circuit during the reducing procedure as 0, then we similarly get the reduced network. Therefore, the rank of the matrix in (∗) is less by n than the number of its rows. From this it already follows that the network in question is quasiregular of order n.

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