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IMPROVEMENT OF THE RATE OF CONVERGENCE OF THE CONJUGATE GRADIENT FAST FOURIER TRANSFORM METHOD FOR ANALYZING PLANAR FREQUENCY SELECTIVE SURFACES

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Abstract

In this paper two methods are shown to improve the rate of convergence of the Conjugate Gradient Fast Fourier Transform (CG-FFT) method for analyzing planar frequency selective surfaces (FSS) with finite conductivity and with arbitrary angles of incidence.

After formulating the physical model an operator equation is written for the induced surface current which is first solved by the CG-FFT method. Here the norm and inner product are defined in detail. Then the problem of the preconditioning is discussed and a new procedure called Biconjugate Gradient fast Fourier transform (BiCG-FFT) method is developed. It is demonstrated that this procedure requires a smaller number of iterations than the original or the preconditioned CG-FFT method.

At the end of the paper results of the analysis of an infinite rectangular grid obtained by the different methods are given and compared according to precision and rate of convergence.

Keywords: periodic structures, frequency selective surfaces, preconditioning, conjugate gradient FFT method, generalized biconjugate gradient FFT method.

Introduction

The-CG-FFT method has been applied by a number of authors (CWIK and MITTRA, 1985; SARKAR, ARVAS and RAO, 1986; CWIK and MITTRA, 1987; CHRISTODOULOU and KAUFFMAN, 1986) to the analysis of scattering from FSS consisting of arbitrarily shaped patches with finite conductivity. The CG-FFT method is an iterative technique that employs the CG method to improve upon each iterate, utilizing the FFT. This method requires no matrix inversions. A detailed summary of the CG method may be found in (SARKAR and ARVAS, 1985). In spite of its advantages several authors have observed that in some cases the CG algorithm may converge too slowly to be useful (CHRISTODOULOU and KAUFFMAN, 1986; KAS and YIP,

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1987; SARKAR, 1987; PETRE and ZOMBORY, 1978). This occurs when the condition number of the operator is too large. To overcome this difficulty two procedures are presented in this paper.

One possible way to reduce the condition number is by preconditioning (KAS and YIP, 1987). Application of the preconditioned CG algorithm to an operator equation requires additional reprocessing of the original one. In our experience the rate of convergence of the preconditioned CG method is better than that of the conventional one only in cases where the condition number of the original operator is large.

The other possible way to improve the convergence of the conventional CG method is by the application of the generalized BiCG-FFT method. This procedure was first introduced to solve electromagnetic scattering problems by SARKAR (1987) and was first applied to FSS by PETRE and ZOMBORY (1988). The generalized BiCG-FFT method does not minimize the residual or the error in the solution at each iteration, but reduces some power norm. Since our operator is a nonsymmetrical one, this method requires additional $2N$ storage locations, where N is the number of unknowns.

At the end of the paper numerical results are presented to compare the necessary number of iterations of the conventional CG-FFT and the generalized BiCG-FFT method.

Formulation

Let us consider the problem of a uniform plane wave scattered from free standing periodically located conducting patches shown in *Fig. 1.*

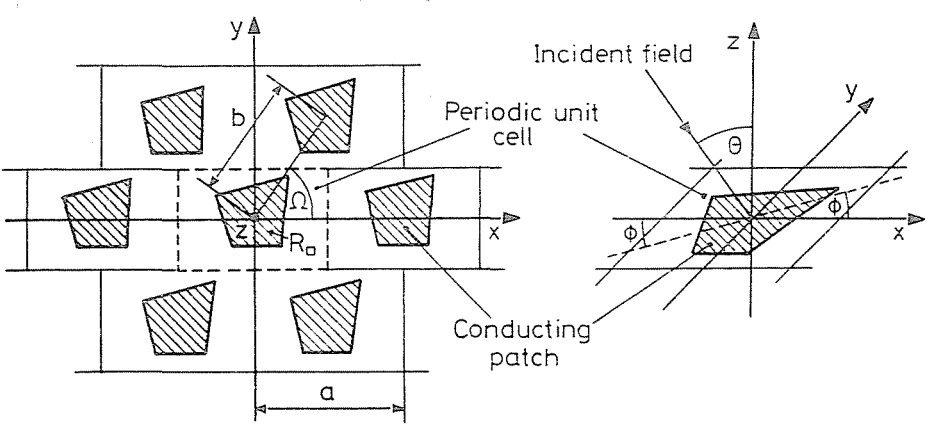

Fig. 1. Free standing periodically located patches illuminated by a plane wave

The coordinate system is chosen as shown in the figure, the FSS is located in the plane $z = 0$, and a plane wave is incident upon the FSS from the $z > 0$ half-space. The geometry is determined by a, b and Ω, and the loss of the patches is characterized by the surface resistance ($R_\square$).

It is important to note that in our physical model the conducting patches are sufficiently thin so that the tangential electrical field is the same on both sides of the FSS.

For transverse electrical (TE) polarization the incident field components are:

$$E_x^i = E_0^i \sin(-\Phi) , \qquad E_y^i = E_0^i \cos\Phi , \tag{1}$$

where E_0^i is the amplitude of the incident electrical field.

For transverse magnetic (TM) polarization the incident field is:

$$E_x^i = E_0^i \cos\Theta \cos\Phi , \qquad E_y^i = E_0^i \cos\Theta \sin\Phi . \tag{2}$$

Let $\mathbf{J}$ be the current induced on the FSS surface due to a given incident field and let $\mathbf{A}$ be the magnetic vector potential due to this current $\mathbf{J}$. At the $z = 0$ plane $\mathbf{A}$ can be written as (with the time convention $\exp(j\omega t)$ suppressed):

$$\mathbf{A}(x, y) = \mathbf{J}(x, y) * G(x, y) , \tag{3}$$

where $*$ is the sign of the convolution and

$$G(x, y) = \frac{1}{4\pi} \frac{e^{-jk_0 r}}{r} , \qquad r = (x^2 + y^2)^{1/2} \tag{4}$$

is the free-space Green's function. Here k_0 is the free-space wave number, and $\mathbf{J}$ and $\mathbf{A}$ are column vectors with only the x and y components included.

The scattered electrical field $\mathbf{E}^s$ at $z = 0$ can be derived from $\mathbf{A}$:

$$\mathbf{E}^s = \frac{1}{j\omega\varepsilon_0} [\operatorname{grad}\operatorname{div}\mathbf{A} + k_0^2 \mathbf{A}] . \tag{5}$$

Fourier transforming (5), we obtain the following equation in the spectral domain

$$\tilde{\mathbf{E}}^s(\alpha, \beta) = \frac{1}{j\omega\varepsilon_0} \tilde{\mathbf{G}}(\alpha, \beta) \tilde{\mathbf{J}}(\alpha, \beta) , \tag{6}$$

where

$$\tilde{\mathbf{G}}(\alpha, \beta) = \frac{-j}{2(k_0^2 - \alpha^2 - \beta^2)^{1/2}} \begin{bmatrix} k_0^2 - \alpha^2 & -\alpha\beta \\ -\alpha\beta & k_0^2 - \beta^2 \end{bmatrix} \tag{7}$$

is the spectral Green's matrix, where α and β are the transform variables of x and y, respectively.

Since the structure is periodic, $\mathbf{J}(x,y)$ has a discrete spectrum in the transformed domain corresponding to the Floquet modes. The explicit expressions for $\alpha_{m,n}$ and $\beta_{m,n}$ are

$$\alpha_{m,n} = 2\pi m/a - k_0 \sin\Theta \cos\Phi ,$$

$$\beta_{m,n} = 2\pi n/(b \cdot \sin\Omega) - 2\pi m \cot\Omega/a - k_0 \sin\Theta \sin\Phi . \tag{8}$$

Taking the inverse transform of (6) and enforcing the boundary condition $\mathbf{E}^s(x,y) + \mathbf{E}^i(x,y) = R_{\square}(x,y)\mathbf{J}(x,y)$ on the conducting surfaces of the FSS, we arrive at the final equation:

$$f_c\left\{\frac{1}{j\omega\varepsilon_0}\sum_{m,n=-\infty}^{\infty}\sum \widetilde{\mathbf{G}}(\alpha_{m,n},\beta_{m,n})\widetilde{\mathbf{J}}(\alpha_{m,n},\beta_{m,n})e^{j(\alpha_{m,n}x+\beta_{m,n}y)} - R_{\square}\mathbf{J}\right\}$$

$$= -f_c\mathbf{E}^i , \tag{9}$$

where f_c is a truncation operator restricted to the conducting surfaces.

Equation (9) may be written in a more compact form:

$$f_c\left\{\frac{1}{j\omega\varepsilon_0}\mathcal{F}^{-1}\left[\widetilde{\mathbf{G}}\mathcal{F}(f_c\mathbf{J})\right] - R_{\square}\mathbf{J}\right\} = -f_c\mathbf{E}^i , \tag{10}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent the discrete Fourier and the inverse discrete Fourier transform, respectively.

Equation (10) is the operator equation for the induced current and will be solved using the CG-FFT method discussed in the next section.

Solution of the Operator Equation Using the CG-FFT Method

In this section the basic principle of the CG method is described and applied to the operator equation (10). This procedure is an iterative one unlike the conventional matrix method and possesses the following properties:

The CG method requires much less storage ($\sim 5N$) than the conventional matrix methods ($\sim N^2$), where N is the number of unknowns.

With an arbitrary initial guess the CG method always converges to the solution in a finite number (N) of iterations.

After each iteration the quality of the solution is known and the magnitude of the residuals decreases monotonously.

The operator equation (10) can be written in a very simple form:

$$L(\mathbf{J}) = \mathbf{Y} , \tag{11}$$

where L denotes the operator

$$L(\mathbf{J}) = f_c \left\{ \frac{1}{j\omega\varepsilon_0} \mathcal{F}^{-1} \left[\widetilde{\mathbf{G}} \mathcal{F}(f_c \mathbf{J}) \right] - R_{\square} \mathbf{J} \right\} \tag{12}$$

and $\mathbf{Y}$ denotes the known excitation

$$\mathbf{Y} = -f_c \mathbf{E}^i \,. \tag{13}$$

It is important to note that the CG method has some versions (special cases) that yield simple algorithms (SARKAR and ARVAS, 1985). From these versions the following has given the fastest convergence for our problem.

The functional to be minimized is:

$$F(\mathbf{J}_i) = \|L(\mathbf{J}_i) - \mathbf{Y}\|^2 \,, \tag{14}$$

where $\mathbf{J}_i$ is the value of the unknown quantity $\mathbf{J}$, after the i-th iteration and $\|\cdots\|$ means the norm. For the CG method one starts with an initial guess $\mathbf{J}_1$ and lets

$$\mathbf{R}_1 = \mathbf{Y} - L(\mathbf{J}_1) \,; \qquad \mathbf{P}_1 = L^*(\mathbf{R}_1) \,. \tag{15}$$

At the i-th iteration the CG method develops the following:

$$a_i = \frac{\|L^*(\mathbf{R}_i)\|^2}{\|L(\mathbf{P}_i)\|^2} \,, \tag{16}$$

$$\mathbf{J}_{i+1} = \mathbf{J}_i + a_i \mathbf{P}_i \,, \tag{17}$$

$$\mathbf{R}_{i+1} = \mathbf{R}_i - a_i L(\mathbf{P}_i) \,, \tag{18}$$

$$b_i = \frac{\|L^*(\mathbf{R}_{i+1})\|^2}{\|L^*(\mathbf{R}_i)\|^2} \,, \tag{19}$$

$$\mathbf{P}_{i+1} = L^*(\mathbf{R}_{i+1}) + b_i \mathbf{P}_i \,. \tag{20}$$

Here L^* denotes the adjoint operator for L and is defined by the following inner product:

$$\langle L(\mathbf{f}), \mathbf{g} \rangle = \langle \mathbf{f}, L^*(\mathbf{g}) \rangle \,, \tag{21}$$

where the inner product is

$$\langle \mathbf{f}, \mathbf{g} \rangle = \int_{\Omega} \mathbf{f} \overline{\mathbf{g}} \, \mathrm{d}\Omega \,. \tag{22}$$

Here $\mathbf{f}$ and $\mathbf{g}$ are vector functions, Ω is the domain of the operator L and the overbar denotes the complex conjugate. The norm in (16) or (19) is defined as

$$\|\mathbf{f}\|^2 = \langle \mathbf{f}, \mathbf{f} \rangle = \int_{\Omega} |\mathbf{f}|^2 \, \mathrm{d}\Omega \,. \tag{23}$$

The explicit expression for the adjoint operator L is the following:

$$L^*(\mathbf{J}) = f_c \left\{ \frac{1}{j\omega\varepsilon_0} \mathcal{F}^{-1} \left(\overline{\widetilde{\mathbf{G}}} \mathcal{F}(f_c \mathbf{J}) \right) - R_{\square} \mathbf{J} \right\} \,. \tag{24}$$

Equations (16)-(20) are applied repeatedly until the desired error criterion is satisfied. In our case the error criterion is defined as $\|\mathbf{R}\|^2 / \|\mathbf{Y}\|^2 \leq 10^{-4}$.

Improving the Rate of Convergence

In this section two procedures will be shown to improve the rate of convergence of the CG method.

a) Preconditioning

The principle of preconditioning is well known from the matrix theory (KAS and YIP, 1987). To formulate it let us consider the following matrix equation:

$$\mathbf{M}\mathbf{x} = \mathbf{b} \,. \tag{25}$$

The condition number $k(\mathbf{M})$ of the matrix $\mathbf{M}$ is defined as

$$K(\mathbf{M}) = \sigma_{\max} / \sigma_{\min} \,, \tag{26}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the largest and the smallest singular values of $\mathbf{M}$, respectively. Preconditioning of a matrix equation consists of finding a pair of matrices $\mathbf{P}$ and $\mathbf{Q}$ and transforming the equation (25) into the equivalent pair of equations

$$\mathbf{PMQy} = \mathbf{Pb}, \quad \mathbf{x} = \mathbf{Qy} \,. \tag{27}$$

The CG algorithm will generally converge faster on the preconditioned system than on the original one if the condition number of $\mathbf{PMQ}$ is smaller than that of $\mathbf{M}$. It is to be noted that the eigenvalues of the operator L are the same as the eigenvalues of the spectral Green's matrix $\widetilde{\mathbf{G}}$. Our purpose

is to transform the original operator L into a new form in such a way as to have a new transformed spectral Green's matrix which has a smaller condition number than that of $\widetilde{\mathbf{G}}$.

According to KAS and YIP (1987), one can rewrite equation (10) into the following preconditioned form:

$$(f_c - f_f)\left\{\frac{1}{j\omega\varepsilon_0}\mathcal{F}^{-1}\left[(\widetilde{\mathbf{G}} - \mathbf{E})(\widetilde{\mathbf{G}} + \mathbf{E})^{-1}\mathcal{F}(\mathbf{K})\right]\right\} + \mathbf{K} = -2f_c\mathbf{E}^i\,, \quad (28)$$

where

$$\mathbf{J} = \mathcal{F}^{-1}\left[(\widetilde{\mathbf{G}} + \mathbf{E})^{-1}\mathcal{F}(\mathbf{K})\right] \quad (29)$$

and f_f is a truncation operator restricted to the nonconducting surfaces.

The eigenvalues of $(\widetilde{\mathbf{G}} - \mathbf{E})(\widetilde{\mathbf{G}} + \mathbf{E})^{-1}$ are of the form $(e-1)/(e+1)$, where e is an eigenvalue of $\widetilde{\mathbf{G}}$. In that way the large eigenvalues of $\widetilde{\mathbf{G}}$ are transformed close to one and it may be expected to improve the condition number to its square root.

*b) Biconjugate Gradient Method (*SARKAR, *1987)*

The generalised BiCG method is usually applied to solve non-Hermitian and poorly conditioned operator equations. The application of this method results in faster convergence. The generalized BiCG method does not minimize the residual or the error in the solution at each iteration, but reduces some power norms. This method, however, requires additional $2N$ storage locations.

The functional which is minimized:

$$F(\mathbf{I}, \mathbf{J}) = 2\mathrm{Re}\langle L(\mathbf{I}), \mathbf{J}\rangle = \langle L(\mathbf{I}), \mathbf{J}\rangle + \overline{\langle L(\mathbf{I}), \mathbf{J}\rangle}\,, \quad (30)$$

where 'Re' denotes the real part of the inner product.

The generalized BiCG algorithm related to the operator equation (11) is the following:

$$\mathbf{J}_1 \equiv 0\,; \qquad \mathbf{R}_1 = \mathbf{P}_1 = \mathbf{Y}\,; \qquad \mathbf{W}_1 = \mathbf{Q}_1 = \overline{\mathbf{Y}}\,, \quad (31)$$

and for $i = 1, 2\ldots$

$$a_i = \frac{\langle \mathbf{R}_i,\ \mathbf{Q}_i\rangle}{\langle L(\mathbf{P}_i),\ \mathbf{W}_i\rangle}\,, \quad (32)$$

$$\mathbf{J}_{i+1} = \mathbf{J}_i + a_i\mathbf{P}_i\,, \quad (33)$$

$$\mathbf{R}_{i+1} = \mathbf{R}_i - a_i L(\mathbf{P}_i)\,, \quad (34)$$

$$\mathbf{Q}_{i+1} = \mathbf{Q}_i - \overline{a}_i L^*(\mathbf{W}_i)\,, \tag{35}$$

$$c_i = \frac{\langle \mathbf{R}_{i+1}, \mathbf{Q}_{i+1} \rangle}{\langle \mathbf{R}_i, \mathbf{Q}_i \rangle}\,, \tag{36}$$

$$\mathbf{P}_{i+1} = \mathbf{R}_{i+1} + c_i \mathbf{P}_i\,, \tag{37}$$

$$\mathbf{W}_{i+1} = \mathbf{Q}_{i+1} + \overline{c}_i \mathbf{W}_i\,. \tag{38}$$

It is important to note that the operator L defined by equation (12) is a nonsymmetrical one, therefore the algorithm (31)-(38) has to be used (PETRE and ZOMBORY, 1988).

Scattering Parameters

The scattering parameters, i. e. the voltage and the power reflection and transmission coefficients are the quantities of greatest importance in characterizing a FSS. In order to define these coefficients for both polarizations, i. e. transverse electrical (TE) and transverse magnetic (TM), it is necessary first to define the scattered fields. We restrict our attention to the dominant mode where $(m,n) = (0,0)$, i. e. only the constant current components $\mathbf{J}(0,0)$ contribute to the scattered field (CHRISTODOULOU and KAUFFMAN, 1986).

For large z/λ the scattered and the transmitted fields are the following:

$$\mathbf{E}^s = \frac{1}{j\omega\varepsilon_0} \widetilde{\mathbf{G}}(0,0)\, \widetilde{\mathbf{J}}(0,0)\, \mathrm{e}^{jk_0 z}\,, \qquad z > 0\,, \tag{39}$$

$$\mathbf{E}^t = \mathbf{E}^i + \frac{1}{j\omega\varepsilon_0} \widetilde{\mathbf{G}}(0,0)\, \widetilde{\mathbf{J}}(0,0)\, \mathrm{e}^{-jk_0 z}\,, \qquad z < 0\,. \tag{40}$$

For TE polarization the voltage reflection and transmission coefficient becomes:

$$\Gamma_{TE}^{TE} = \frac{E_{TE}^{TE}}{E_0^i}\,, \quad \Gamma_{TE}^{TM} = \frac{E_{TE}^{TM}}{E_0^i}\,, \quad T_{TE}^{TE} = 1 + \Gamma_{TE}^{TE} \quad T_{TE}^{TM} = \Gamma_{TE}^{TM}\,, \tag{41}$$

where

$$E_{TE}^{TE} = -E_x^s \sin\Phi + E_y^s \cos\Phi\,, \quad E_{TE}^{TM} = E_x^s \cos\Phi + E_y^s \sin\Phi\,.$$

The subscript indicates the incident field, and the superscript the scattered field.

For TM polarization the above coefficients are:

$$\Gamma_{TM}^{TM} = \frac{E_{TM}^{TM}}{E_0^i \cos\Theta}, \qquad \Gamma_{TM}^{TE} = \frac{E_{TM}^{TE}}{E_0^i \cos\Theta},$$

$$T_{TM}^{TM} = 1 + \Gamma_{TM}^{TM}, \qquad T_{TM}^{TE} = \Gamma_{TM}^{TE}, \tag{42}$$

where now $E_{TM}^{TM} = E_x^s \cos\Phi + E_y^s \sin\Phi$ and $E_{TM}^{TE} = E_x^s \sin\Phi - E_y^s \cos\Phi$.

If only the dominant mode is the propagating mode, then there are no grating lobes and the following equations will hold

$$P_d/P_i + |\Gamma_{TE}^{TE}|^2 + |\Gamma_{TE}^{TM}|^2 + |T_{TE}^{TE}|^2 + |T_{TE}^{TM}|^2 = 1,$$

$$P_d/P_i + |\Gamma_{TM}^{TM}|^2 + |\Gamma_{TM}^{TE}|^2 + |T_{TM}^{TM}|^2 + |T_{TM}^{TE}|^2 = 1, \tag{43}$$

where P_d is the power dissipated in the metal and P_i is the incident power.

Numerical Results

In this section results for the dominant mode voltage reflection and transmission coefficients of the analysis of an infinite rectangular grid obtained by the different methods are presented and compared according to precision and the rate of convergence.

Equation (11) has been solved using CG, preconditioned CG and BiCG-FFT methods as well. Application of these methods to the scattering problem investigated means that a unit cell of the screen is divided into $2^m \times 2^n$ segments and the $\mathcal{F}$ and $\mathcal{F}^{-1}$ transform are implemented by the FFT and the inverse FFT algorithms. In our example the CG algorithm used is given by equations (16)-(21), the preconditioned form is given by equation (28) and the general BiCG algorithm is given by equations (31)-(38). The above algorithms are applied to 16×16 or 32×32 grid divisions of the unit cell of the structure. The convergence criterion is the reduction of the relative residual to $\varepsilon^2 = 10^{-4}$.

Let us consider the geometry of an infinite rectangular grid (*Fig. 2*).

The sizes of the grid are determined by a, b and W, and the surface resistance is given by $R_\square$.

Fig. 3 shows the number of iterations required for convergence for a simple rectangular grid over a range of a/λ from 0 to 1.0 in steps of 0.1. Here $a = b$; $\Omega = 90°$, the polarization is TE, $\Theta = \Phi = 0°$ and $R_\square = 0$ Ohm. This figure indicates that the improvement for the preconditioned CG method is realized only when the condition number of the original operator is large i.e. the structure is near resonance. On the other hand

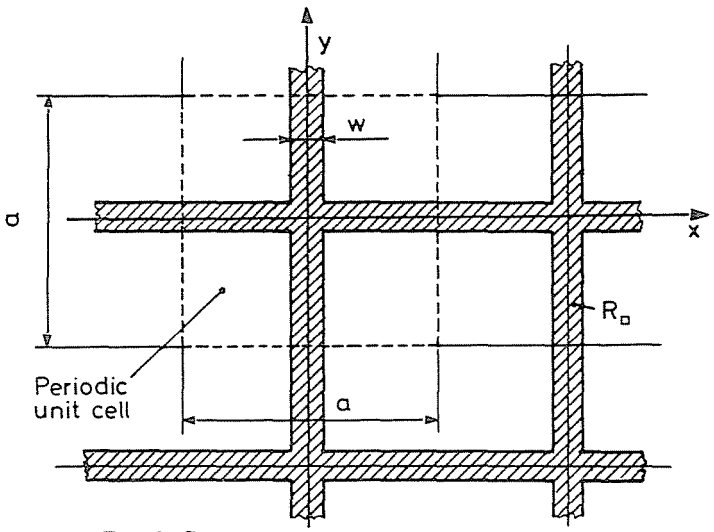

Fig. 2. Geometry of an infinite rectangular grid

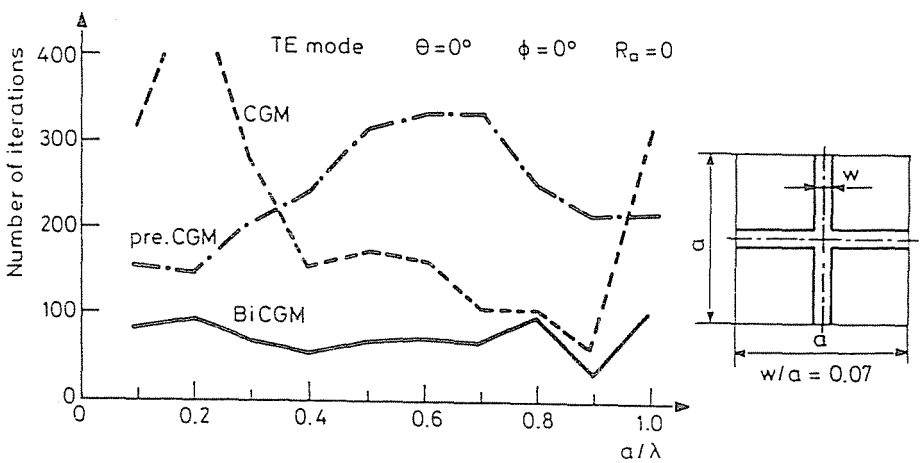

Fig. 3. Number of iterations required for convergence as a function of a/λ

the generalized BiCG method gives the fewest number of iterations over the entire domain regardless of the condition number of the operator.

Fig. 4 shows the relative errors of the different methods as a function of the number of iterations. Here $a = b$, $a/\lambda = 0.25$, the polarization is TE, $\Theta = \Phi = 0°$ and $R_\square = 0$ Ohm.

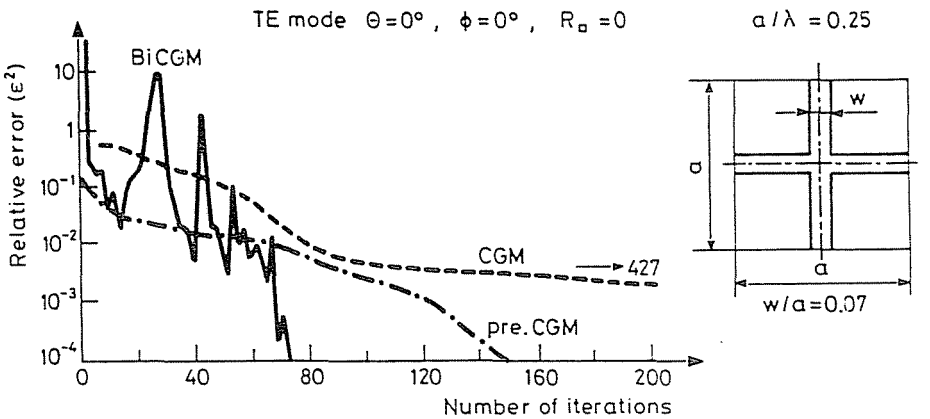

Fig. 4. Relative errors as a function of the number of iterations

It can be seen from the figure that the magnitude of the residuals decreases monotonously for the CG and the preconditioned CG method unlike the generalized BiCG method, where the fall of the residuals is not monotonous.

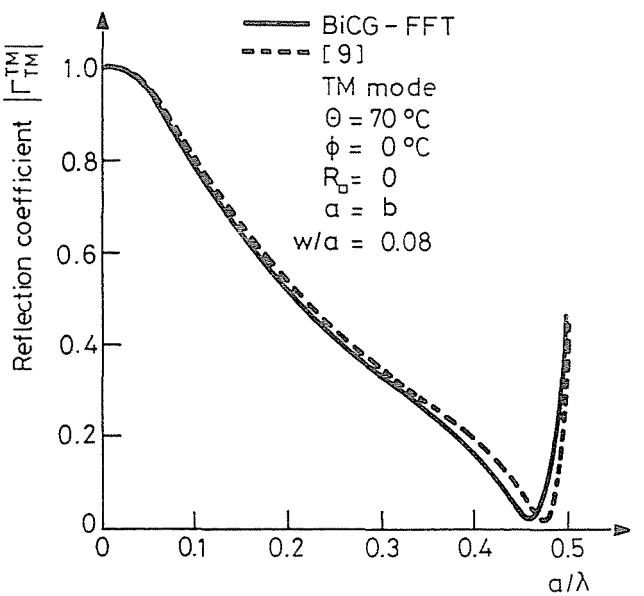

Fig. 5. Reflection coefficient for TM polarization as a function of a/λ

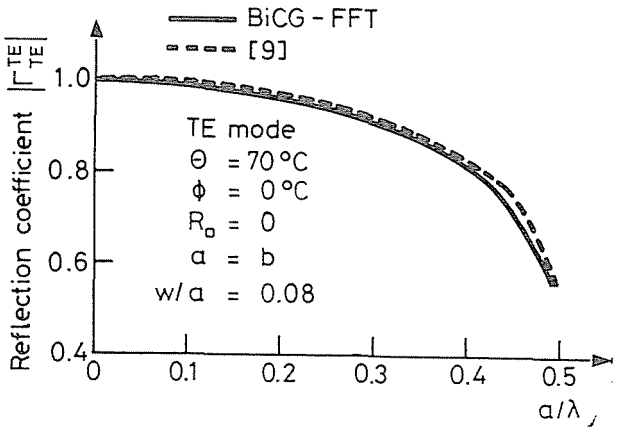

Fig. 6. Reflection coefficient for TE polarization as a function of a/λ

Figs 5-6 show the voltage reflection coefficients for both transverse electrical and transverse magnetic polarizations as a function of a/λ. Here $a = b$, $\Omega = 90°$, $\Theta = 70°$, $\Phi = 0°$ and $R_□ = 0$ Ohm.

Calculations using the generalized BiCG-FFT method are compared with another method (USLENGHI, 1978) which is based on a Fourier series expansion method solution. Our results are in good agreement with the results derived from (USLENGHI, 1978).

Summarizing the results of this paper we may conclude that the generalized BiCG-FFT method can be used in a very efficient manner for analyzing planar FSS. It has been demonstrated that the generalized BiCG method requires a smaller number of iterations than the CG or the preconditioned CG method and the precision of these methods is suitable for numerical calculations.

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