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MODELING, IDENTIFICATION AND ADAPTIVE CONTROL OF A NONLINEAR SYSTEM

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Abstract

Mathematical or physical models offer a possibility to solve problems like to control unknown state variables. These models are often described by physical parameters. Some of these parameters can be determined beforehand, some of them can only be identified in the running process. In this paper an algorithm to identify physical parameters is presented in the state space. The mathematical model of a nonlinear thermal system is developed and together with the identification algorithm the adaptive feedback control of not measurable state variables is shown. A simple and robust feedback controller is presented based on the power balance. A brief discussion of the results of the control and — at the same time — the estimation of unknown parameters is closing the paper.

Keywords: Modeling, identification, adaptive control.

Introduction

A common problem in technology is that a variable has to be controlled but cannot be measured. Models of the systems offer a possibility to solve these problems.

These models are described by physical parameters, some of them can be determined beforehand, some of them can only be identified in the running process. This procedure will be demonstrated for a process with the following characteristics:

I) the system is nonlinear
II) there is an extensive delay between input and output (this means: the time constants are large)
III) not all of the variables which are needed for the controller are measurable
IV) parameters of the process are unknown and have to be identified
V) the system is closed, that means: only power exchange but no mass exchange take place with the environment.

A nonlinear system of this kind can be described by the following equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\, \boldsymbol{x} + \boldsymbol{B}\, \boldsymbol{u}\ , \tag{1.1}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}\ . \tag{1.2}$$

In this paper the identification algorithm for a nonlinear system will be shown in the state space, the modeling of the system will be done by separating the system in regions with discrete parameters. The nonlinearities will be described by physical laws.

The considered thermal system is running as a batch process. Fluid will be filled in and the system will be closed. A temperature inside the system is to be determined and to be controlled by the measurement of the temperature at the output. The mathematical model of the physical structure of the system will be developed. The system parameters (e.g. the capacity of the fluid, which depends on the filling level and is relevant changing) are unknown and have to be identified.

The identification of unknown parameters will be shown for this process and a simple adaptive, nonlinear controller will be developed based on the power balance. The control of the unknown temperature inside the system and — at the same time — the estimation of unknown parameters will be shown.

Furthermore a brief discussion of the results will be given.

Identification Algorithm

The advantages of the state space model are the following possibilities:

1. To choose the state variables as physical variables (e.g. the current and the angular velocity of an electrical plant, or temperatures of different points in a thermal system). If this is done, the calculating of a physical value inside the system is very easy.
2. Physical parameters of a system, which can be determined beforehand, reduce the number of the parameters to estimate (often there were only a few parameters of a system relevant changing).

Choosing the state space description the problem which remains to solve is, that the output $\boldsymbol{y}$ is nonlinear in these parameters. The performance criteria such as of the minimum of the least squares estimation error need a model linear in the parameters. This will be accomplished here by choosing a gradient method to estimate the parameters $\boldsymbol{\lambda}$. That means developing the output of the model in a Taylor series with respect to the unknown parameters $\boldsymbol{\lambda}$ at the point $\widehat{\boldsymbol{\lambda}}(i)$, where $\widehat{\boldsymbol{\lambda}}(i)$ describes the estimation of $\boldsymbol{\lambda}$

after the i-th iteration. This series will be broken off after the linear term:

$$\widehat{\boldsymbol{y}}(t)=\widehat{\boldsymbol{y}}(t)\mid_{\widehat{\boldsymbol{\lambda}}(i)}+\frac{\partial\widehat{\boldsymbol{y}}(t)}{\partial\boldsymbol{\lambda}}\mid_{\widehat{\boldsymbol{\lambda}}(i)}\Delta\widehat{\boldsymbol{\lambda}}(t)\ . \tag{2.1}$$

This equation will be considered at the time points $t=kT$ (T sampling time; $k=0\ldots N$), these values will be put together to vectors. $\widehat{\boldsymbol{y}}(t)\mid_{\widehat{\boldsymbol{\lambda}}(i)}$ and $\frac{\partial\widehat{\boldsymbol{y}}(t)}{\partial\boldsymbol{\lambda}}\mid_{\widehat{\boldsymbol{\lambda}}(i)}$ can be calculated by equations (1.1) and (1.2). Unknown parameters can be parts of the elements of the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ and parts of the elements of $\boldsymbol{B}$.

The last equation can be interpreted as a least squares problem (KRONMÜLLER, 1988/1989) and leads to (m parameters are to be identified):

$$\Delta\widehat{\boldsymbol{\lambda}}(i)=\left(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{\mathrm{T}}\left(\boldsymbol{y}-\widehat{\boldsymbol{y}}\mid_{\widehat{\lambda}(i)}\right)\ , \tag{2.2}$$

$$\widehat{\boldsymbol{\lambda}}(i)=\widehat{\boldsymbol{\lambda}}(i-1)+\Delta\widehat{\boldsymbol{\lambda}}(i)\ , \tag{2.3}$$

with

$$\boldsymbol{X}=\begin{pmatrix}\widehat{\boldsymbol{y}}_{\lambda 1}(0) & \widehat{\boldsymbol{y}}_{\lambda 2}(0) & \ldots & \widehat{\boldsymbol{y}}_{\lambda m}(0)\\ \ldots & \ldots & \ldots & \ldots\\ \widehat{\boldsymbol{y}}_{\lambda 1}(N) & \widehat{\boldsymbol{y}}_{\lambda 2}(N) & \ldots & \widehat{\boldsymbol{y}}_{\lambda m}(N)\end{pmatrix}\ ,$$

where

$$\widehat{\boldsymbol{y}}_{\lambda j}(k)=\frac{\partial\widehat{\boldsymbol{y}}(k)}{\partial\lambda_j}\mid_{\widehat{\lambda}(i)}\ .$$

The proposed algorithm is known as Gauss-Newton algorithm (HOFFMANN, 1971). The convergence of this algorithm is not guaranteed, but if the initial values of the iteration $\widehat{\boldsymbol{\lambda}}(0)$ are not too wide away from the correct ones $\boldsymbol{\lambda}$, the convergence is in most applications nearly quadratic.

The control is now done in two steps:

1. At the beginning, there is no information from the output available, the control will be started based on initial values.
2. The first measurements are done, the described algorithm will be started and if the iteration is successfully ended, the new parameter set $\widehat{\boldsymbol{\lambda}}$ will be given to the model. Caused by the identification of parameters of $\boldsymbol{A}$, the state $\widehat{\boldsymbol{x}}$ has also to be calculated.

For real-time conditions it is possible to move a window over the data. The values of the state vector can be updated at the beginning t_0 of each window by:

$$\widehat{\boldsymbol{x}}_{\text{new}}(t_0)=\widehat{\boldsymbol{x}}_{\text{old}}(t_0)\mid_{\widehat{\lambda}}+\frac{\partial\widehat{\boldsymbol{x}}_{\text{old}}(t_0)}{\partial\lambda}\mid_{\widehat{\lambda}}\Delta\widehat{\boldsymbol{\lambda}}\ , \tag{2.4}$$

where $\boldsymbol{x}_{\text{old}}$ is the state vector based on the iteration in the last window. The block diagram of the system and the model (observer) is shown in *Fig. 1*.

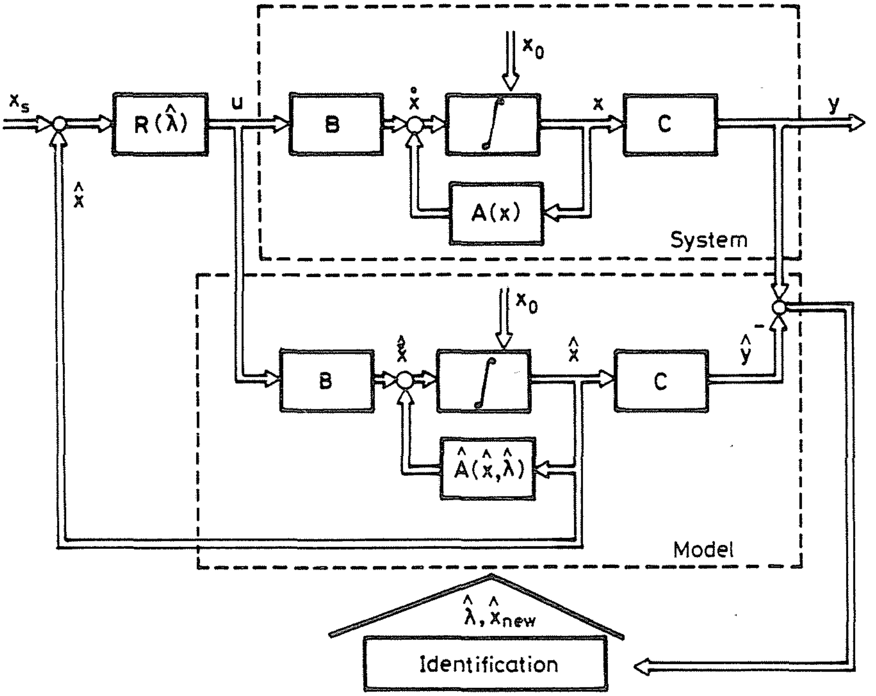

Fig. 1. Block diagram

A Simple Controller

The power balance of a system is given by

$$\begin{array}{ccccc} P_{\mathrm{in}} & = & P_{\mathrm{sa}} & + & P_{\mathrm{lo}} \\ \text{input power} & = & \text{saved power} & + & \text{losses} \end{array} . \tag{3.1}$$

By integration of equation (3.1) we obtain the energy balance:

$$\int_{t_1}^{t_2} P_{\mathrm{in}} dt = \int_{t_1}^{t_2} P_{\mathrm{sa}} dt + \int_{t_1}^{t_2} P_{\mathrm{lo}} dt . \tag{3.2}$$

If P_{in} is constant in $[t_1, t_2]$ the above equation becomes:

$$P_{\mathrm{in}} (t_2 - t_1) = \int_{t_1}^{t_2} P_{\mathrm{sa}} dt + \int_{t_1}^{t_2} P_{\mathrm{lo}} dt . \tag{3.3}$$

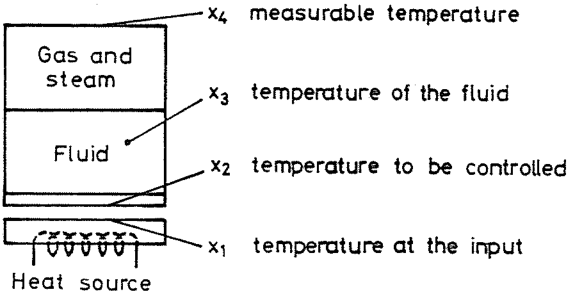

Fig. 2. A simplified structure

If t_2 is the time where the set-point is reached, the right-hand side is a function of the set-point $\boldsymbol{x}_\mathrm{s}$:

$$P_\mathrm{in}\,(t_2 - t_1) = f\left(\boldsymbol{x}_\mathrm{s}, \widehat{\boldsymbol{\lambda}}\right) \; . \tag{3.4}$$

Now two possibilities exist to solve (3.4):

1. It is possible to determine the necessary period $t_2 - t_1$ to reach the set-point $\boldsymbol{x}_\mathrm{s}$ with a given power P_in:

$$(t_2 - t_1) = \frac{f\left(\boldsymbol{x}_\mathrm{s}, \widehat{\boldsymbol{\lambda}}\right)}{P_\mathrm{in}} \; . \tag{3.5}$$

2. We can determine the power P_in which can be applied to the system in a given interval $(t_2 - t_1)$, e.g. one sampling interval T

$$P_\mathrm{in} = \frac{f\left(\boldsymbol{x}_\mathrm{s}, \widehat{\boldsymbol{\lambda}}\right)}{(t_2 - t_1)} \; . \tag{3.6}$$

This yields to the following nonlinear algorithm of the controller (power P in %):

$$P = \begin{cases} 100 & \text{for} \quad 100 \le \dfrac{P_\mathrm{in}}{P_\mathrm{max}} \, , \\ \dfrac{P_\mathrm{in}}{P_\mathrm{max}} & \text{for} \quad 0 \le \dfrac{P_\mathrm{in}}{P_\mathrm{max}} \le 100 \, , \\ 0 & \text{for} \quad \dfrac{P_\mathrm{in}}{P_\mathrm{max}} \le 0 \quad . \end{cases} \tag{3.7}$$

The behaviour of such a controller will be shown for example in the last section.

Modeling

A simplified structure of a thermal system is shown in *Fig. 2*. This system has all of the characteristics given in the introduction, but has the advantage that all temperatures are measurable:

I) The heat transition resistance R_{34} between x_3 and x_4 is strong nonlinear.

II) The delay between power inputs and the changes of the state variable x_4 is large.

III) The temperature to be controlled is x_2 inside the system. This is the place of the highest temperature of the fluid.

IV) The parameters of the system are unknown and have to be determined.

V) The system is running as a batch process. Only power exchange with the environment is possible.

The modeling is done by separating the system in four regions with discrete parameters: heat source, bottom, fluid and eventually the metal lid. The room between fluid and lid has only a small capacity. This capacity will be neglected. A critical point of modeling is the resistance R_{34} between x_3 and x_4.

Description of the Resistance R_{34}

Is the filling level lower than the maximal possible filling level, there is a room filled with gas and steam, where the heat conduction is interrupted, and where the heat transport is done by convection.

Under the assumption that the system is balanced, that is, the partial pressure p_{w} of fluid in the gas is the maximal possible pressure, the saturation pressure $p_{\mathrm{s}}(T)$:

$$p_{\mathrm{w}} = p_{\mathrm{s}}(T) \; , \tag{4.1}$$

we can find an analytical description (Antoine equation) for p_{w} from (BAEHR, 1988):

$$\ln\left(\frac{p_{\mathrm{s}}(T)}{\mathrm{mbar}}\right) = 19.016 - \frac{4064.95}{\frac{T}{^{\circ}\mathrm{C}} + 236.25} \; , \tag{4.2}$$

with the law of ideal gas the density ρ_{ws} can be described as a function of p_{w}:

$$\rho_{\mathrm{ws}}(T) = \frac{p_{\mathrm{w}}}{R_{\mathrm{w}}T} = \frac{p_{\mathrm{s}}(T)}{R_{\mathrm{w}}T} \; , \tag{4.3}$$

with R_{w}, the special gas constant of water.

Using the assumption that R_{34} is reciprocal to ρ_{ws}, we obtain:

$$R_{34}(T) \sim \left(\frac{R_{\mathrm{w}}T}{p_{\mathrm{s}}(T)}\right) \; , \tag{4.4}$$

where the proportional factor in equation (4.4) is independent of the filling level and therefore can be determined beforehand. A comparison of the computed solution by measurement of x_3 and x_4 with the analytical one is shown in *Fig. 3*.

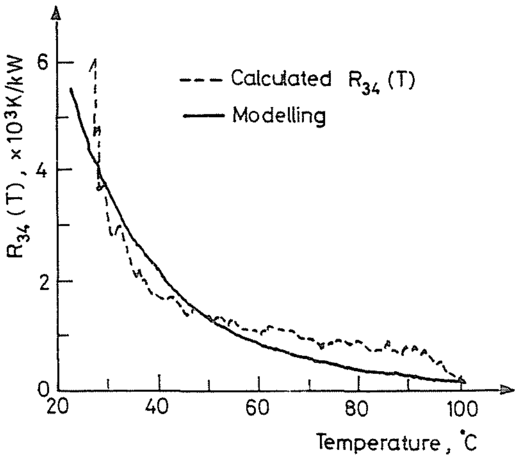

Fig. 3. Comparison of the analytical and computed solution of R_{34}

Results

The results of this model combined with the identification are illustrated in *Fig. 5*. At the top there is the temperature x_1, followed by the temperature x_2 and eventually temperature x_4, all of these are compared with measured temperatures. The identification algorithm knows only the first five minutes x_4 and the whole input power sequence (to estimate was the capacity of the fluid C_3).The dashed curves show the predicted temperatures.

In contrast to *Fig. 4*, in *Fig. 5* the results of control and identification in the closed loop are plotted. The control depends on the chosen initial values. The system and the model are excited, the identification begins when the first measurements are available.

Fig. 6 shows the input power sequence P. If the system becomes stationary, a limiting oscillation in observable, which is caused by the nonlinear controller. The push-pull behaviour of the control element is acceptable, because the applied element is realized by an electronic switch.

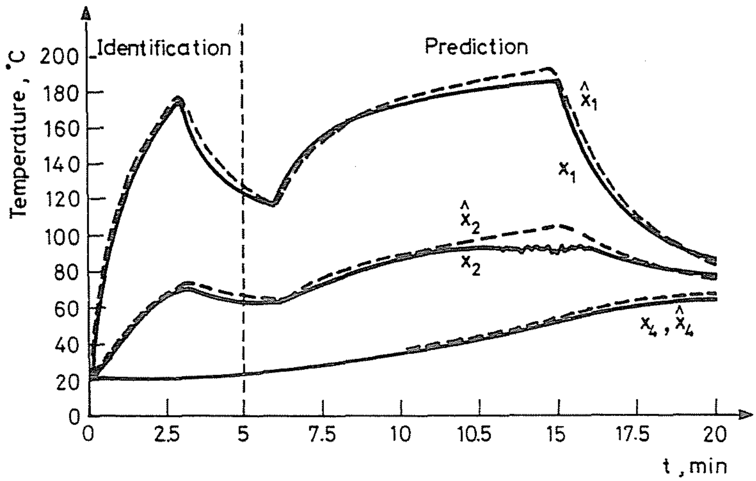

Fig. 4. Comparison: measured and predicted temperatures
solid line: measurement
dashed line: prediction

Conclusion

In common problems to obtain the state of a system it is often sufficient to determine the state by an observer (Luenberger observer, Kalman filtering, e.g.: FÖLLINGER (1985), NAHI (1969), LEE (1960),...). In this application it is not enough to estimate the state but it is also necessary to identify unknown parameters of the system. To solve this, a nonlinear model of a thermal system was developed. With this model control of bottom temperature and — at the same time — estimation of unknown parameters like the capacity of the fluid (depending on the filling level) is possible. The quality of the identification algorithm depends relevant on the chosen model. In this application the remaining errors are small enough, so that the proposed Gauss-Newton algorithm converges very fast. Other gradient or sensitivity methods proposed in the literature (Newton, Newton-Raphson, steepest descent, ... e.g.: BARD (1970), BERGER (1973)) lead usually to more extensive calculations. To save computation time and because of the good convergence and robustness of the algorithm described above it will be used in this application.

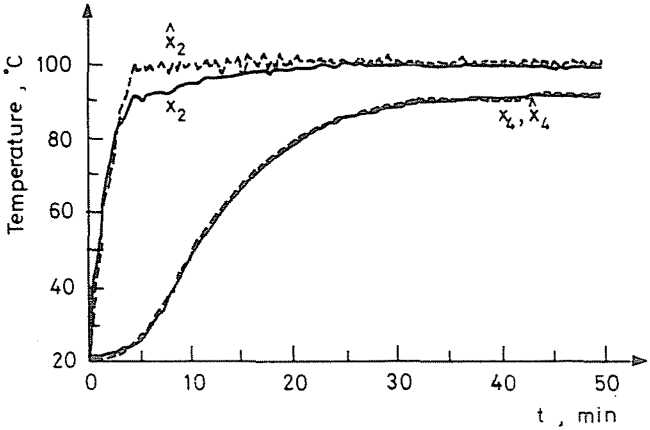

Fig. 5. The estimation and measurement for x_4 are scarcely distinguishable from each other
solid line: measurement
dashed line: estimation

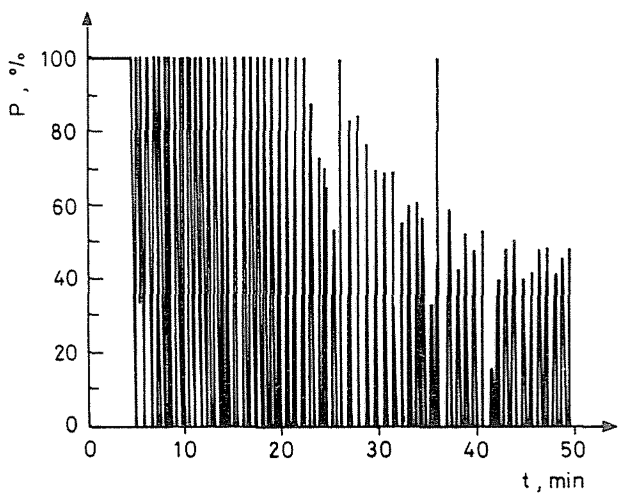

Fig. 6. The input sequence by applying the controller based on power balance

First, the variation in the output signal is poor, only a limited amount of information is available, because of the large delay between input and output. The control depends first on the chosen initial values, the identification begins when the first measurements are available. The applied controller is a simple nonlinear, adaptive controller, which is based on the power balance. This controller is very robust, that means even if the deviation of the estimated values $\widehat{\boldsymbol{\lambda}}$ becomes ca. $\pm 10\%$ with respect to the correct parameters, it is possible to reach the set-point.

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