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MODELLING ERRORS OF NONSINUSOIDAL CAPACITANCE MEASURING METHODS

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Abstract

In the last few years several new digital methods have been developed for capacitance measurements. Articles introducing them usually deal only with the measuring errors of the instruments and ignore the modelling errors caused by the non-ideal behaviour of the real objects. After reviewing the models of capacitors and the nonsinusoidal capacitance measuring methods, the paper gives the upper bounds of the modelling errors and determines the equivalent frequency of the methods. The calculated modelling errors give the theoretically attainable accuracy of the methods.

Keywords: capacitance measurement, nonsinusoidal test signal.

Introduction

Instruments for precision capacitance measurement are generally based upon impedance bridges, for example Schering or Glynne bridges. Their accuracy is determined basically by the reference components and effect of leads. The measuring time depends on the convergency of balancing algorithms. Though these devices have been developed much further, both their accuracy and speed are limited, and these limits can hardly be exceeded.

To avoid these problems, in the last few years several new digital methods have been developed for capacitance measurement (see references). Most of them measure frequency, time or voltage which are directly the function of the capacitance to be measured and use nonsinusoidal measuring signals. These methods could decrease the measuring time considerably and a few of them are suitable to increase the accuracy as well. However, they are not used frequently in precision capacitance measurement, because the exact meaning of the measured value is unknown, if the measured capacitor is non-ideal.

Since the new nonsinusoidal methods differ in principle from the bridges, the measured values also can differ, even if ideal instruments are assumed. The reason of the difference is the modelling error.

The total error of a measured value consists of two parts, that is of the modelling error and the measurement error. Modelling error derives from the difference between the behaviour of the supposed model and that of the real object. The source of measurement error is the measuring system which cannot realize the measuring method perfectly.

To estimate the modelling error of the nonsinusoidal methods, first we have to choose the proper capacitor model.

Models of Capacitors

The most commonly used model of capacitors in wide frequency range is shown in *Fig. 1.* The serial components of the model can also represent the leads used, besides the serial loss resistance and inductance of capacitor. The parallel conductance derives from the DC leakage current of capacitors.

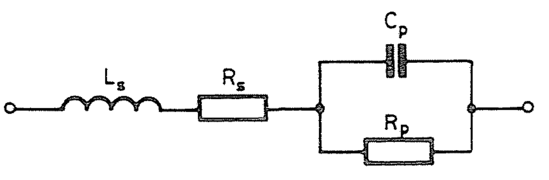

Fig. 1. The wide frequency range capacitor model

Though this approach is much better than the simple serial or parallel model, it does not characterize the behaviour of non-ideal dielectrics. If the permittivity has considerable frequency dependence, the C_p capacitor in the model is also a complex element. To describe this component the complex permittivity has been introduced :

$$\varepsilon^*(j\omega) = \varepsilon'(\omega) - j\varepsilon''(\omega). \tag{1}$$

The capacitance is determined by the real part of $\varepsilon^*(j\omega)$:

$$C(\omega) = \varepsilon'(\omega)\, C_g\,, \tag{2}$$

where C_g is the so called geometrical capacitance (the capacitance which can be measured without dielectrics).

The reason of the frequency dependence of the permittivity is the finite settling time of polarization of dielectrics. The Debye equation can be used for many types of polarization to describe this frequency dependence:

$$\varepsilon^*(j\omega) = \varepsilon(\infty)\left[1 + \frac{k_d}{1 + j\omega\tau_{\mathrm{D}}}\right], \tag{3}$$

where τ_{D} is the so called relaxation time which can change in very wide range from about 10^{-12} to 10^4s, and k_d is the dispersion factor that shows the degree of frequency dependence of the capacitor:

$$k_d = \frac{\varepsilon(0) - \varepsilon(\infty)}{\varepsilon(\infty)}. \tag{4}$$

$\varepsilon(\infty)$ and $\varepsilon(0)$ are the permittivities measured at very high and very low frequencies, with respect to $1/(2\pi\tau_{\mathrm{D}})$.

Frequency dependence of C_p can be described with an equivalent circuit as well (*Fig. 2*).

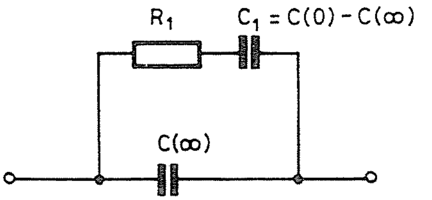

Fig. 2. The Debye model ($\tau_D = R_1C_1$)

The Debye model outlined above can be used only if the dielectric has a single relaxation time. Several dielectrics, however, can be described only with a distribution of relaxation times (Böttcher and Bordewijk (1978))

$$\varepsilon^*(j\omega) = \varepsilon(\infty)\left[1 + k_d \int_0^\infty \frac{G(\tau)}{1 + j\omega\tau}\, d\tau\right], \tag{5}$$

where $G(\tau)$ is the density function of relaxation times, and

$$\int_0^\infty G(\tau)\, d\tau = 1. \tag{6}$$

For the Debye model:

$$G(\tau) = \delta(\tau - \tau_{\mathrm{D}}), \tag{7}$$

where $\delta(\tau)$ is the delta function.

Eq. (4) can be described also in discrete form:

$$\varepsilon^*(j\omega) = \varepsilon(\infty)\left[1 + k_d \sum_{i=1}^{N} \frac{g_i}{1 + j\omega\tau_i}\right], \tag{8}$$

where g_i is the weight, τ_i is the relaxation time of an elementary polarisation and N is their number.

$$\sum_{i=1}^{N} g_i = 1. \tag{9}$$

For the Debye model:

$$\tau_i = \tau_{\mathrm{D}} \qquad \text{for any } i. \tag{10}$$

Eq. (8) can be described also with an equivalent circuit, similarly to the Debye model (*Fig. 3*).

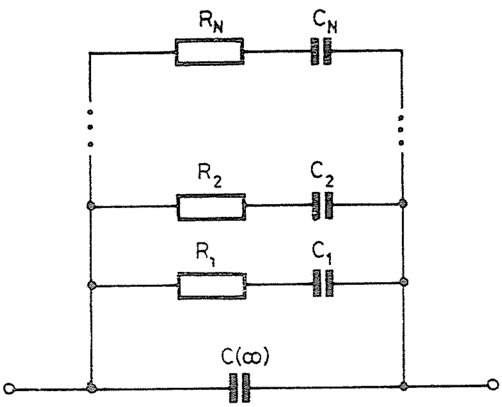

Fig. 3. Capacitor model with a distribution of relaxation times ($\tau_i = R_iC_i$; $k_d = \sum_{i=1}^{N} C_i/C(\infty)$)

The Pessimistic Dielectric Model

To evaluate the E modelling error from the C_m measured and C correct values of capacitance, we have to know the $G(\tau)$ distribution function of relaxation times. Since $G(\tau)$ depend on the dielectrics, it is difficult to determine the modelling error with universal validity. We can give its upper bound if we can find the model and its $G(\tau)$ which cause the highest modelling error.

It is proved below that this pessimistic model is the Debye model, and

$$\max_{\omega}\{|E[\omega,k_d,G(\tau)]|\} \leq \max_{\omega}\{|E_{\mathrm{D}}[\omega,k_d,\tau_{\mathrm{D}}]|\} , \tag{11}$$

if we can express the $E[\omega,k_d,G(\tau)]$ modelling error as follows:

$$E[\omega,k_d,G(\tau)] = k_d \sum_{i=1}^{N} g_i\, E_0(\omega\tau_i) , \tag{12}$$

where $E_{\mathrm{D}}[\omega,k_d,\tau_{\mathrm{D}}]$ is the modelling error calculated from the Debye model and $k_d g_i E_0(\omega\tau_i)$ is the modelling error caused by a single τ_i relaxation time polarisation having weight g_i.

From Eq. (12) we can write:

$$|E[\omega,k_d,G(\tau)]| \leq k_d \sum_{i=1}^{N} g_i\, |E_0(\omega\tau_i)| . \tag{13}$$

Since E_0 depends on the product of ω and τ_i, it is true for any τ_{D} and τ_i, that:

$$\max_{\omega}\{|E_0(\omega,\tau_{\mathrm{D}})|\} \geq |E_0(\omega\tau_i)| . \tag{14}$$

From Eq. (9), Eq. (13) and Eq. (14):

$$|E[\omega,k_d,G(\tau)]| \leq k_d \max_{\omega}\{|E_0(\omega,\tau_{\mathrm{D}})|\} . \tag{15}$$

If we use Eq. (12) for the Debye model, we can write:

$$E_{\mathrm{D}}[\omega,k_d,\tau_{\mathrm{D}}] = k_d\, E_0(\omega\tau_{\mathrm{D}}) . \tag{16}$$

Combination of Eq. (15) and Eq. (16) results Eq. (11).

Having been proved Eq. (11), we can estimate the modelling error of a capacitance measuring method using the Debye model, if Eq. (12) is true.

The Nonsinusoidal Methods

The nonsinusoidal capacitance measuring methods can be divided into three groups. These are the Switched-Capacitor Method (MATSUMOTO et al (1987)) the RC Discharge Method (HAGIWARA and SAEGUSA (1983); RUSEK and MAHMUD

(1986)) and the Linear Charge-Discharge Method (TSAO (1974), ZOLTÁN and SZENN (1985)).

The principle of the Switched-Capacitor Method is the following. First the unknown C_x capacitor is charged to V_r voltage. In the second phase the Q_x charge stored in C_x is transferred onto C_r reference capacitor. Measuring the charge on C_r for example, by measuring its V_x voltage we can calculate the unknown capacitance:

$$C_m = \frac{Q_x}{V_r} = C_r \frac{V_x}{V_r}, \tag{17}$$

where C_m is the measured value of C_x. During the measurement these processes are repeated with $T/2$ charging and discharging time.

The RC Discharge Method first charges the unknown C_x capacitor to voltage V_r. In the second period C_x is discharged through a known R resistor. The voltage of C_x will be changing during discharging, as follows:

$$V_c = V_r e^{-\frac{t}{RC_x}}. \tag{18}$$

Measuring the T_x time while V_c decays to a αV_r voltage ($0 < \alpha < 1$), T_x will be proportional to the capacitance:

$$C_m = -\frac{T_x}{R} \frac{1}{\ln \alpha}. \tag{19}$$

The basis of the Linear Charge-Discharge Method is an integrator consisting of the capacitor under test, a precision resistor and an operational amplifier. If we switch a symmetrical square wave signal of V_r amplitude to the input of the integrator and measure the time while the triangular wave on the output passes between two voltage levels $\pm\alpha V_r$, the measured time T_x will be proportional to the capacitance to be measured. From T_x the measured value of C_x is:

$$C_m = \frac{T_x}{2R} \frac{1}{\alpha}, \tag{20}$$

The Modelling Errors

Eq. (17), Eq. (19) and Eq. (20) give the correct value of C_x if the instruments and the capacitor under measurement are ideal. If the capacitor can be described only with a complex model, the results may include modelling error.

These modelling errors will be calculated for the wide frequency range model (*Fig. 1*) and the Debye model (*Fig. 2*), assuming that the errors are added if neither of them are too high.

The Switched-Capacitor Method

The main source of the modelling error, caused by a capacitor model according to circuit in *Fig. 1* is the following phenomenon: while the Q_x charge stored in the capacitor under test is transferred to C_r reference capacitor through R_s and L_s, a small amount of charge gets lost flowing through R_p. If C_p was charged to voltage V_r, the V_{cp} voltage of C_p will change as follows—supposing that $R_s \ll R_p$ and $L_s/R_p \ll R_sC_p$:

$$V_{cp} = V_r e^{-\alpha t}\left[\frac{\alpha}{\beta}\sin\beta t + \cos\beta t\right],$$
$$\alpha = \frac{R_s}{2L_s} \quad ; \quad \beta = \sqrt{\frac{1}{L_sC_p} - \frac{R_s^2}{4L_s^2}}. \tag{21}$$

If the time of the charge transfer is much longer than the time-constants in Eq. (21), the value of the lost charge is given by:

$$\Delta Q_x = \int_0^\infty \frac{V_{cp}(t)}{R_p}\,dt = V_rC_p\frac{R_s}{R_p}. \tag{22}$$

From Eq. (22) the modelling error is:

$$E = \frac{\Delta Q_x}{V_rC_p} = \frac{R_s}{R_p} = \tan\delta_s \tan\delta_p, \tag{23}$$

where $\tan\delta_s = \omega R_sC_p$ and $\tan\delta_p = 1/(\omega R_pC_p)$ are the loss factors caused by the serial resistance and the parallel DC leakage.

Eq. (23) shows that the method is insensible to the serial inductance , but the serial resistance deriving from the leads, from the residual resistance of switches, etc., can cause an error if the capacitor has conductance.

If the capacitor under measurement can be described with the model in *Fig. 3*, the charging and discharging currents consist of two parts:

$$i_c(t) = i_0(t) + \sum_{i=1}^{N} i_i(t), \tag{24}$$

where $i_0(t)$ is a current impulse charging $C(\infty)$ and $\sum_{i=1}^{N} i_i(t)$ is the so called absorption current which charges the C_i capacitances in *Fig. 3*. The voltage on the capacitor is a square wave signal with levels 0 and V_r. After the settling time $i_i(t)$ changes as follows,—assuming that discharging of the capacitor starts at $t = 0$:

$$i_i(t) = \frac{V_r}{R_i} \frac{e^{-\frac{t}{\tau_i}}}{1 - e^{-\frac{T}{2\tau_i}}} . \tag{25}$$

The transported charge during $T/2$ is:

$$Q_x = \int_0^{\frac{T}{2}} i_c(t)\, dt = V_r C(\infty) \left[1 + k_d \sum_{i=1}^{N} g_i \tanh \frac{T}{4\tau_i}\right] . \tag{26}$$

From Eq. (17) and Eq. (26) the measured value is:

$$C_m = C(\infty) \left[1 + k_d \sum_{i=1}^{N} g_i \tanh \frac{T}{4\tau_i}\right] . \tag{27}$$

The true value of the capacitance is:

$$C = C(\infty) \left[1 + k_d \sum_{i=1}^{N} g_i \frac{1}{1 + (\omega\tau_i)^2}\right] . \tag{28}$$

Supposing that C_m is the measured value at $f = 1/(\pi T)$ equivalent frequency the modelling error is given by:

$$E = \frac{C_m - C}{C(\infty)} = k_d \sum_{i=1}^{N} g_i \left[\tanh \frac{1}{2\omega\tau_i} - \frac{1}{1 + (\omega\tau_i)^2}\right] . \tag{29}$$

The equivalent frequency is calculated by minimization of the maximum of $|E|$.

Since the form of Eq. (28) is the same as Eq. (12), we can write independently of the dielectric:

$$|E| \leq k_d \max_{\omega} \left\{ \left| \tanh \frac{1}{2\omega\tau_{\mathrm{D}}} - \frac{1}{1 + (\omega\tau_{\mathrm{D}})^2} \right| \right\} \cong 6.6 \cdot 10^{-2} k_d . \tag{30}$$

The RC Discharge Method

Measuring a capacitor as modelled in *Fig. 1*, the V_c voltage of the whole capacitor changes during discharging, as follows:

$$V_c(t) = V_r \frac{R}{R_s + R} e^{-\frac{t}{R_d C_p}} \quad ; \quad R_d = \frac{R_p(R_s + R)}{R_s + R + R_p} , \tag{31}$$

assuming that L_s is negligible and R is the discharging resistor. The T_x time, while V_c reaches the αV_r level, is given by:

$$T_x = -R_d C_p \ln \alpha \left[1 + \frac{R_s}{R}\right] . \tag{32}$$

From T_x, on the basis of Eq. (19), the measured value of the capacitor is:

$$C_m \cong C_p \left[1 + \frac{R_s}{R}\right] \left[1 - \frac{R}{R_p} - \frac{R_s}{R_p}\right] \left[1 + \frac{R_s}{R} \frac{1}{\ln \alpha}\right] , \tag{33}$$

supposing that R_s/R and R/R_p are much smaller than one. The modelling error is given by:

$$E = \frac{C_m - C_p}{C_p} = \frac{R_s}{R} \left[1 + \frac{1}{\ln \alpha}\right] - \frac{R}{R_p} - \frac{R_s}{R_p} \left[2 + \frac{1}{\ln \alpha}\right] . \tag{34}$$

Choosing $\alpha = 1/e$, the first member in the expression of the error becomes zero. Introducing after HAGIWARA and SAEGUSA (1983), the $\omega = 1/(RC_p)$ equivalent angular frequency, Eq. (34) can be written:

$$E = -[\tan \delta_p + \tan \delta_s \tan \delta_p] . \tag{35}$$

Considering Eq. (35) we can see that this method can be used for precision capacitance measurement ($E < 10^{-4}$) if the capacitor under measurement is nearly free from losses. Since dielectrics having low loss factor usually do not depend on frequency, it is not worth to calculate the modelling error caused by the frequency dependence. Note that this kind of error is calculated in HAGIWARA and SAEGUSA (1983) in the case of a special dielectric model.

The Linear Charge-Discharge Method

Replacing the capacitor of the integrator by the model in *Fig. 1* and switching to the input a symmetrical square wave signal of V_r amplitude and T period time, the rising part of the V_I output signal will change as follows:

$$V_I(t) = V_r \left[\frac{R_s}{R} + \frac{R_p}{R} \left[1 - \frac{e^{-\frac{t}{R_p C_p}}}{\cosh \frac{T}{4R_p C_p}}\right]\right] , \tag{36}$$

supposing that the disturbing effect of L_s around the peaks of the output signal is negligible.

The T_x time while V_{I} gets through between $-\alpha V_r$ and $+\alpha V_r$ is given by:

$$T_x = R_p C_p \ln \frac{1+\frac{\alpha R}{R_s+R_p}}{1-\frac{\alpha R}{R_s+R_p}} \cong 2RC_p\alpha \left[1-\frac{R_s}{R_p}+\frac{\alpha^2}{3}\frac{R^2}{R_p^2}\right], \tag{37}$$

if R_s/R_p and R^2/R_p^2 are much smaller than one.

Choosing $T = RC_p$ and $\omega = 2\pi/T$ equivalent angular frequency, taking into account Eq. (20) and Eq. (37) the measured value is given by:

$$C_m = C_p \left[1-\tan\delta_s \tan\delta_p + \frac{4}{3}\pi^2\alpha^2\tan^2\delta_p\right], \tag{38}$$

where α is between 0 and 1/4 because of $T = RC_p$.

From Eq. (38) the modelling error caused by the DC losses is:

$$E = \frac{C_m - C_p}{C_p} = -\tan\delta_s \tan\delta_p + \frac{4}{3}\pi^2\alpha^2\tan^2\delta_p\,. \tag{39}$$

Eq. (39) shows that the Linear Charge-Discharge Method is not so sensible to the parallel conductance of the capacitor than the RC Discharge Method.

Replacing the capacitor in the integrator by the model in *Fig. 3*, the rising part of V_{I} will change as follows, if $k_d \ll 1$:

$$\begin{aligned} V_{\mathrm{I}}(t) &= V_r \left[\frac{k_d}{RC(\infty)}\sum_{i=1}^{N} g_i\,\tau_i + \frac{t}{RC(\infty)}(1-k_d)\right. \\ &\quad \left. - \frac{k_d}{RC(\infty)}\sum_{i=1}^{N} g_i\,\tau_i \frac{e^{-\frac{t}{\tau_i}}}{\cosh\frac{T}{4\tau_i}}\right]. \end{aligned} \tag{40}$$

Replacing $\exp(-t/\tau_i)$ by the first two members of its Taylor expansion around $t_0 = \alpha RC(\infty)/(1-k_d)$, T_x can be approached by:

$$T_x \cong 2\alpha RC(\infty)\left[1+k_d\right] - 2k_d \sum_{i=1}^{N} g_i\,\tau_i \frac{\sinh \alpha\frac{RC(\infty)}{\tau_i}}{\cosh\frac{T}{4\tau_i}}\,. \tag{41}$$

From Eq. (41) the measured value is:

$$C_m = C(\infty)\left[1+k_d - \frac{k_d}{\alpha RC(\infty)}\sum_{i=1}^{N} g_i\,\tau_i \frac{\sinh \alpha\frac{RC(\infty)}{\tau_i}}{\cosh\frac{T}{4\tau_i}}\right]. \tag{42}$$

Comparing Eq. (41) with the correct value of C the estimation of the modelling error is:

$$E = \frac{C_m - C}{C(\infty)} = k_d \sum_{i=1}^{N} g_i \left[\frac{(\omega\tau_i)^2}{1+(\omega\tau_i)^2} - \frac{\tau_i}{\alpha RC(\infty)} \frac{\sinh \alpha \frac{RC(\infty)}{\tau_i}}{\cosh \frac{T}{4\tau_i}} \right] . \quad (43)$$

Choosing $T = RC \cong RC(\infty)$ (if $k_d \ll 1$), and $\omega = 2\pi/T$ the form of Eq. (43) is the same as Eq. (12). That means we can write , independently of the dielectric model:

$$|E| \leq k_d \max_{\omega} \left\{ \left| \frac{(\omega\tau_D)^2}{1+(\omega\tau_D)^2} - \frac{\omega\tau_D}{2\pi\alpha} \frac{\sinh \frac{2\pi\alpha}{\omega\tau_D}}{\cosh \frac{\pi}{2\omega\tau_D}} \right| \right\} . \quad (44)$$

Eq. (44) is minimal if $\alpha \cong 0.197$. Then the value of the error is:

$$|E| \leq 9 \cdot 10^{-3} k_d . \quad (45)$$

Eq. (43) is true if $k_d \ll 1$. The modelling error can be calculated with higher accuracy by a simulation program. Comparing Eq. (43) with this more accurate value of the error we can say that the error of Eq. (43) is less than 10 % if $k_d \leq 10^{-2}$, and the Debye model remains the pessimistic model even if $10^{-2} < k_d \leq 1$ but in this case we get higher upper bound for the error:

$$|E| \leq 7.1 \cdot 10^{-2} k_d . \quad (46)$$

Conclusions

Investigating the results of the previous calculations the following facts can be laid down:

1. Considering the modelling errors the Switched-Capacitor Method and the Linear Charge-Discharge Method are suitable to measure capacitances with high accuracy even if the capacitors are non-ideal. The RC Discharge Method gives accurate result only if the capacitor under test is nearly ideal.

2. In spite of using nonsinusoidal measuring signals the measured values can be interpreted as capacitance at an equivalent frequency.

3. The Debye model is suitable for calculating the upper bound of modelling error caused by the frequency dependence of capacitor in the case of the Switched-Capacitor Method and the Linear Charge-Discharge Method.

Since the sum of the measurement error and the modelling error determines the accuracy of an instrument, to choose the optimal method, we have to compare the measurement error as well. This comparison after (MATSUMOTO et al (1987); HAGIWARA and SAEGUSA (1983); ZOLTÁN and SZENN (1985)) shows that the realization of the Linear Charge-Discharge Method has the lowest measurement error ($2 \cdot 10^{-2}\%$ absolute and $10^{-3}\%$ comparison error at typical measuring time of 20ms) and using more precise components the measurement error can be further reduced.

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