TRANSIENT ANALYSIS OF LOSSY TRANSMISSION LINE SYSTEMS BY DISCRETE FOURIER TRANSFORM

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Abstract

A method for transient analysis of multiconductor transmission line systems is presented. The propagation of the TEM mode which is perturbed into TM mode (because of the finite conductance of the conductors) is assumed. The system of the partial differential equations describing the model is solved analytically in the frequency domain. The spectrum obtained is transformed into the time domain by using the connection between the continuous- and discrete-time Fourier transform. The inverse Fourier transformation is carried out by using an FFT algorithm. The loads are linear, active n-terminal networks. The frequency dependence of the parameters and the loads of the transmission line system can be arbitrary. The results, obtained by the computer program (written on the base of the method) are compared with analytical results available in the literature. A numerical example is presented at the end of the paper.

Keywords: transient analysis, transmission line system, spectral analysis.

Introduction

Problems leading to the transient analysis of a transmission line system can be found in different fields of electrical engineering practice. For example the effect of different faults, switches or strokes of lightning are the focus of interest in high voltage multiphase transmission lines. An important class of filters can be described with the model presented in the field of microwave technique. By increasing the operational speed of digital circuits, the calculation of transient phenomena on the lines of printed circuits (RAZBAN (1987)) is a subject of interest. This problem can be treated as a circuit of transmission line systems. The analysis of such models can be carried out by an extension of the presented method. All the mentioned applications can be treated by the same algorithm. The only differences are in the method of calculation of the parameters and in the applied loads.

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Most of the solutions are obtained by approximating in the time domain, the z-domain, the frequency domain, or in the Laplace transform domain, given in literature (DOMMEL (1969); ROMEO and SANTOMAURO (1987); HUMPAGE et al (1980); BÁRDI et al (1985)). Another method combines the properties of the frequency- and time-domain approximation (DJORDEVIC et al (1986)). In this paper a frequency-domain analysis is presented.

The advantageous properties of the method presented, as compared to analyses in other domains, are the following. The frequency dependence of the parameters can be exactly and easily treated. If the terminals are linear circuits, then it is enough to know their measured or calculated transfer characteristics. All parameters and functions used in the analysis can be measured (they have physical meanings). An important property of the method presented can be seen if it is used for the analysis of transmission lines which are embedded in a telecommunication system, because a similar Fourier transform method is used for the solution applied to the analysis of telecommunication systems.

The investigated system (see *Fig. 1*) contains n conductors with arbitrary cross-sections and equal length, arranged parallel to each other and to the co-ordinate axis z (from $z = 0$ to $z = \ell$). The currents of the

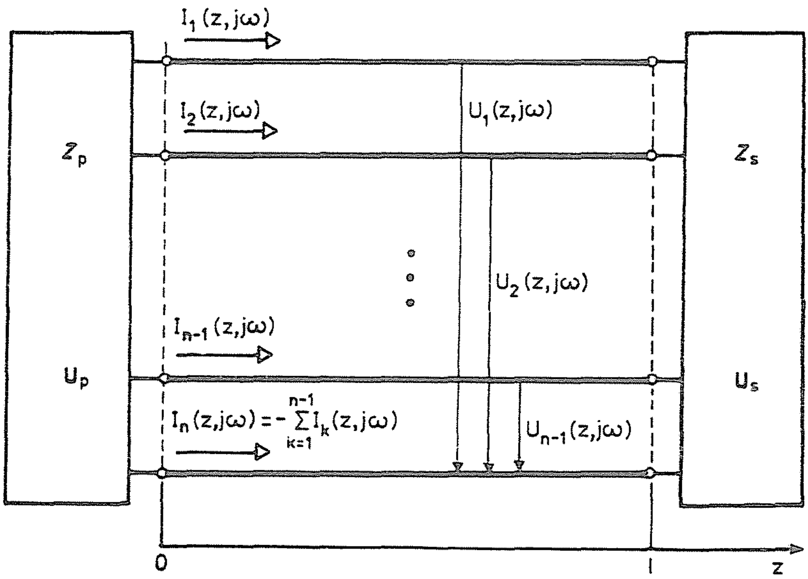

Fig. 1. The model of the problem

conductors (i_k, $k = 1, 2, \ldots, n$) satisfy the following equation at any plane perpendicular to direction z:

$$\sum_{k=1}^{n} i_k = 0. \tag{1}$$

In the case of high voltage transmission lines one of the conductors can be the earth.

Generally, the model can be described by a system of $2n-2$ partial differential equations (VÁGÓ (1970)):

$$\begin{aligned} -\frac{\partial}{\partial z}\mathbf{u} &= \boldsymbol{R}\,\mathbf{i} + \boldsymbol{L}\frac{\partial}{\partial t}\mathbf{i}\,, \\ -\frac{\partial}{\partial z}\mathbf{i} &= \boldsymbol{G}\,\mathbf{u} + \boldsymbol{C}\frac{\partial}{\partial t}\mathbf{u}\,, \end{aligned} \tag{2}$$

where $\mathbf{i}$ is the column vector formed by the currents of the conductors—the current of the n-th conductor is

$$-\sum_{k=1}^{n-1} i_k\,,$$

$\mathbf{u}$ is that of the voltages between the conductors and the reference surface. The reference surface is the n-th conductor (see *Fig. 1*). The column vectors are of order $n-1$. The Fourier transform of the variables will be used. The k-th elements of these vectors are the following:

$$\begin{aligned} (\boldsymbol{I})_k &= \mathcal{F}\{\,i_k(z,t)\,\} = I_k(z,j\omega) \\ & \qquad\qquad\qquad\qquad k = 1,\, 2,\, \ldots\, n-1, \\ (\boldsymbol{U})_k &= \mathcal{F}\{\,u_k(z,t)\,\} = U_k(z,j\omega) \end{aligned}$$

where $\mathcal{F}$ denotes the continuous-time Fourier transform. $\boldsymbol{R}$, $\boldsymbol{L}$, $\boldsymbol{G}$, $\boldsymbol{C}$ contain the parameters of the transmission line system. These are symmetrical, square matrices of order $n-1$. Their elements can be determined by means of the methods of electromagnetic field calculations (VÁGÓ (1970); WEI et al (1984); HARRINGTON and WEI (1984); VENKATARAMEN et al (1985)). In special cases the number of equations can be reduced (e.g. the system containing $n/2$ pair of conductors can be described by n equations)

The loads of the transmission lines can be arbitrary linear, active n-terminal networks (see *Fig. 1*). In the frequency domain these are described in the following manner:

$$\begin{aligned} -\boldsymbol{Z}_p\,\mathbf{I}_0 + \mathbf{U}_p &= \mathbf{U}_0\,, \\ \boldsymbol{Z}_s\,\mathbf{I}_\ell + \mathbf{U}_s &= \mathbf{U}_\ell\,, \end{aligned} \tag{3}$$

where $\mathbf{I}_0$, $\mathbf{U}_0$, $\mathbf{I}_\ell$, $\mathbf{U}_\ell$ are column vectors consisting of the elements of $\mathbf{I}$ and $\mathbf{U}$ at the points of $z = 0$ and $z = \ell$, respectively. $\boldsymbol{Z}_p$ and $\boldsymbol{Z}_s$ are square matrices of order $n-1$ representing the structure of the networks, $\mathbf{U}_p$ and $\mathbf{U}_s$ are column vectors of the same order determined by the sources of the terminals. It is assumed that the reference point of the voltages is the n-th terminal of the network. Eq. (1) is also considered.

Solution in the Frequency Domain

Fourier transforming Eq. (2) into the frequency domain, and eliminating $\mathbf{U}$ or $\mathbf{I}$, we get the following set of differential equations:

$$\begin{aligned} \frac{d^2}{dz^2}\mathbf{U} &= \boldsymbol{\Gamma}^2\,\mathbf{U}\,, \\ \frac{d^2}{dz^2}\mathbf{I} &= \left(\boldsymbol{\Gamma}^2\right)^*\,\mathbf{I}\,, \end{aligned} \tag{4}$$

where $\boldsymbol{\Gamma}^2 = (\boldsymbol{R} + j\omega\boldsymbol{L})(\boldsymbol{G} + j\omega\boldsymbol{C})$, and * denotes the transpose of the matrix. It was utilised that the matrices in Eq. (2) are symmetrical.

Eq. (4) can be solved analytically using the matrix functions of $\boldsymbol{\Gamma}^2$ (VÁGÓ (1970)).

$$\begin{aligned} \mathbf{U}(z,j\omega) &= e^{-\boldsymbol{\Gamma}z}\,\mathbf{U}^+(j\omega) + e^{\boldsymbol{\Gamma}z}\,\mathbf{U}^-(j\omega)\,, \\ \mathbf{I}(z,j\omega) &= \boldsymbol{Y}_0\left[e^{-\boldsymbol{\Gamma}z}\,\mathbf{U}^+(j\omega) - e^{\boldsymbol{\Gamma}z}\,\mathbf{U}^-(j\omega)\right]\,, \end{aligned} \tag{5}$$

where

$$\boldsymbol{Y}_0 = (\boldsymbol{R} + j\omega\boldsymbol{L})^{-1}\boldsymbol{\Gamma}$$

is the characteristic admittance matrix of the transmission line system.

Considering the Eq. (3) boundary conditions, the following expressions are obtained for $\mathbf{U}^+(j\omega)$ and $\mathbf{U}^-(j\omega)$:

$$\begin{aligned} \mathbf{U}^+ &= \boldsymbol{H}\mathbf{U}_p + \boldsymbol{K}\mathbf{U}_s\,, \\ \mathbf{U}^- &= \boldsymbol{D}\boldsymbol{H}\mathbf{U}_p + (\boldsymbol{D}\boldsymbol{K} + \boldsymbol{F})\,\mathbf{U}_s\,, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{F} &= \left[(\boldsymbol{Z}_s\boldsymbol{Y}_0 + \boldsymbol{E})\,e^{\boldsymbol{\Gamma}\ell}\right]^{-1}\,, \\ \boldsymbol{D} &= \boldsymbol{F}\left[(\boldsymbol{Z}_s\boldsymbol{Y}_0 - \boldsymbol{E})\,e^{-\boldsymbol{\Gamma}\ell}\right]\,, \\ \boldsymbol{H} &= \left[\boldsymbol{Z}_p\boldsymbol{Y}_0 - \boldsymbol{Z}_p\boldsymbol{Y}_0\boldsymbol{D} + \boldsymbol{E} + \boldsymbol{D}\right]^{-1}\,, \\ \boldsymbol{K} &= \boldsymbol{H}\left[(\boldsymbol{Z}_p\boldsymbol{Y}_0 - \boldsymbol{E})\,\boldsymbol{F}\right]\,, \end{aligned}$$

and $\boldsymbol{E}$ is the unit matrix.

Substituting Eq. (6) into Eq. (5), the Fourier transform of the time functions of the voltages and currents are obtained at any point along the transmission line system.

The Inverse Fourier Transformation

The task is to transform the voltage and current functions (at the space point of interest) into the time domain. This can be performed by using the connection between the continuous- and discrete-time Fourier transform (OPPENHEIM et al (1983); SIMONYI (1986)). The main concept of the method is the following: Let us assume $X(\omega)$ as the Fourier transform of the continuous-time function $x(t)$. $\tilde{x}(t)$ is the periodic extension of $x(t)$ with the time period of T_0 and it is defined in the following way:

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x\left(t - nT_0\right). \tag{7}$$

Let us assume that

$$x\left(kT\right) = x\left(t\right)|_{t=kT} \qquad k = 0,\, 1,\, 2,\, \ldots\,,$$

and $T_0 = NT$, where N is an integer. The following transformation pair can be written:

$$\begin{aligned} \mathcal{D}\left\{\tilde{X}(nT)\right\} &= \frac{1}{T}\tilde{X}\left(k\omega_0\right), \\ \mathcal{D}^{-1}\left\{\tilde{X}(n\omega_0)\right\} &= T\,\tilde{x}\left(kT\right), \end{aligned} \tag{8}$$

where

$$\tilde{X}(k\omega_0) = \sum_{r=-\infty}^{\infty} X(k\omega_0 - r\omega_s) \tag{9}$$

is the sampled value of the periodical extension (with ω_s the frequency period) of $X(\omega)$. Parameters N, T, T_0, ω and ω_s have to fulfil the following relations:

$$\begin{aligned} NT &= T_0\,, & N\omega_0 &= \omega_s\,, \\ \omega_s &= 2\pi/T\,, & \omega_0 &= 2\pi/T_0\,. \end{aligned} \tag{10}$$

Two of the five parameters can be chosen independently. In the Eq. (8) $\mathcal{D}$ means the discrete Fourier transform (DFT) which yields the relationship between periods of functions $\tilde{x}(nT)$ and $\tilde{X}(k\omega_0)$. The definition of the $\mathcal{D}$ and $\mathcal{D}^{-1}$ operation is as follows: Let us assume $y(n)$ as a periodic discrete-time function with a period of N, and $Y(n)$ as its Fourier transform pair:

$$
\begin{aligned}
\mathcal{D}\{y(n)\} &= Y(k) = \sum_{n=0}^{N-1} y(n) W_N^{kn}, \\
\mathcal{D}^{-1}\{Y(n)\} &= y(k) = \frac{1}{N}\sum_{n=0}^{N-1} Y(n) W_N^{-kn},
\end{aligned} \tag{11}
$$

where $W_N = e^{-j2\pi/N}$. Eq. (11) can be evaluated with a fast Fourier transform (FFT) algorithm. The above relation can be followed expressively in *Fig. 2*

The determination of the exact value of function $x(t)$ at $t = nT$ $(n = 0, 1, 2, \ldots N-1)$ from its Fourier transform $X(\omega)$ by the $\mathcal{D}^{-1}$ operator is possible if one period of functions $\tilde{X}(k\omega_0)$ and $\tilde{x}(nT)$ is equal to $X(k\omega_0)$ and $x(nT)$, respectively, i.e.:

$$x(nT) = \tilde{x}(nT) \qquad (n = 0, 1, \ldots N-1), \tag{12}$$

$$X(k\omega_0) = \tilde{X}(k\omega_0) \qquad (k = 0, 1, \ldots N-1). \tag{13}$$

This condition is tried to be fulfilled by assigning the proper values to the two independent parameters in Eq. (10). It is evident that Eq. (12) and Eq. (13) cannot be fulfilled at the same time, because of theoretical reasons. For example, a time function of finite duration has a Fourier transform which is not bandlimited in the frequency domain and vice versa. But in most cases the above conditions can be approximated fairly well. Furthermore it is assumed that the excitations are Fourier transformable with finite energy. In this case, if the system is lossy (like any real system), then Eq. (12) and Eq. (13) can be approximated with an arbitrarily small error by increasing the value of N.

Taking the above considerations into account an algorithm can be outlined for the inverse Fourier transformation of the functions obtained in Eq. (5).

1. Let us calculate the values of the Fourier spectrum at some points of ω. Knowing these results the approximate value of the bandwidth of the signal can be determined. This means that one of the two independent parameters appearing in Eq. (10) is ω_s and its value is determined in the manner described above. The other independent parameter will be the number of samples (N). Now Eq. (13) is approximated by assigning a value to ω_s. The computer program used for the examples in this paper utilizes the absolute value of the functions to determine the bandwidth of the spectrum.

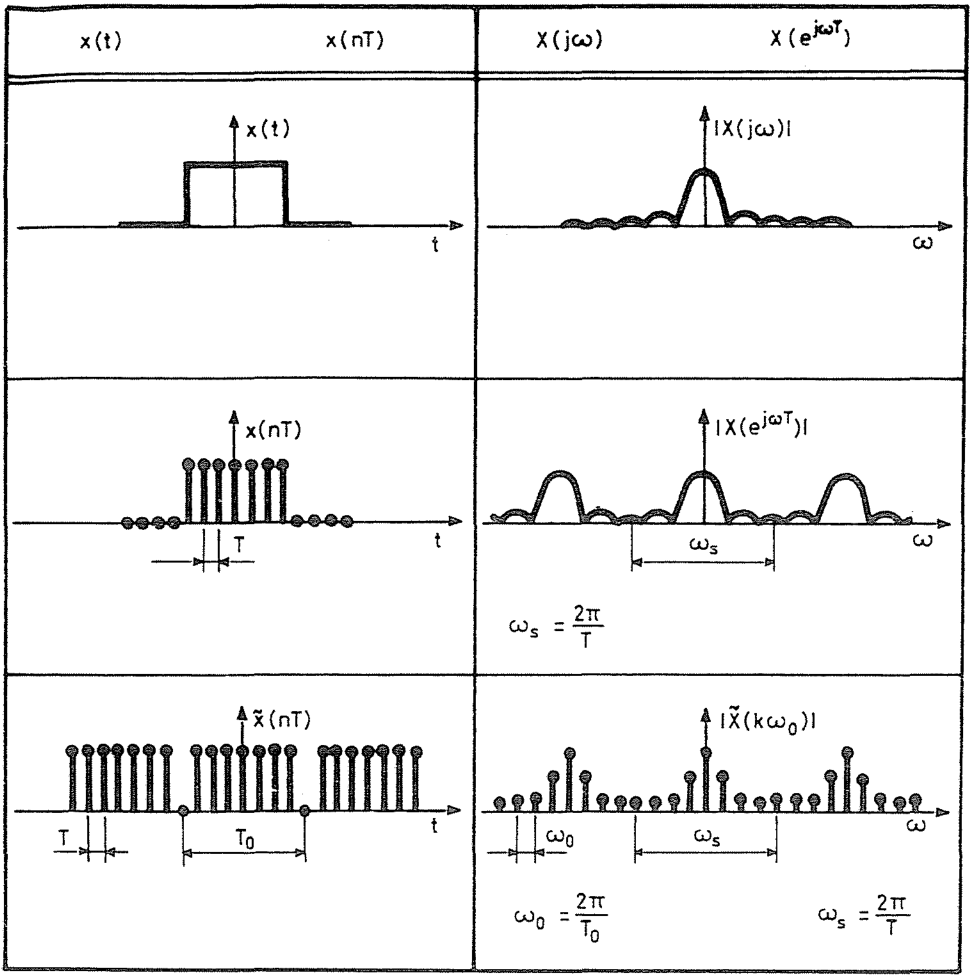

Fig. 2. An example for the connection between continuous- and discrete-time Fourier transform

2. Let us choose a value for N ($N = 2^m$, where $m = 1, 2, \ldots$) is based on an estimation of the duration of the transient response.

3. Let us calculate the values of the Fourier spectrum at the sampling points (here $X(\omega) = \overline{X}(-\omega)$ is used, where the overbar denotes complex conjugation).

4. Let us perform the $\mathcal{D}^{-1}$ operation (by an FFT algorithm).

5. Let us check the approximation of Eq. (12). If the condition

$$x(t) \cong 0 \qquad T_0 - \Delta t < t < T_0 \tag{14}$$

is fulfilled with a small error (where $\Delta t > 2\tau$ and τ is the longest line transit time calculated with the condition that the elements of the transmission line system are considered as individual loss-free transmission lines), it can be said that there is a negligible time-domain aliasing, i.e. Eq. (12) is fulfilled approximately. If there is a considerable aliasing, then new sampling points have to be chosen between the old ones (i.e. the value of N is multiplied by two) and the procedure is continued from point 3.

If the transmission line system is reasonably well matched, the duration of the transient response is short. In this case fewer samples are necessary than in case of extreme terminals (short- or open-circuit ports), if the approximated bandwidth is the same in both cases. If the terminals are of an extreme property, it is better to use the method presented by DJORDEVIC et al (1986), because only short duration Green's functions are calculated (for this purpose the above algorithm is useful), and the transient voltages and currents appearing in the case of arbitrary terminals are determined by a convolution in the time domain. In most of the practical applications the terminals are close to the well-matched case, so the above algorithm can be used without change.

The advantage of the presented algorithm is that the increase of the numbers of samples does not cause any numerical difficulty, because the value of any sample is independent of the others (they depend on the parameters only). This means that the increase of the value of N causes only an increase in computation time. Another important property of this method is that the parameters can be arbitrary functions of ω, because only their numerical values (at a given value of ω) are used. For this reason the phenomena of skin-effect or the impedance of the earth can be handled easily.

Examples

Comparison of numerical results and analytical solutions

The computer program written on the basis of the above theory can calculate (as a special case) the transient phenomena of independent transmission lines. In this case the matrices in Eq. (2) are diagonal ones, and so the results can be compared with the analytical ones available in SIMONYI et al (1966), for one conductor-pair.

a) As a first example, let us consider the loss-free transmission line seen in *Fig. 3* with the following parameters:

$$\begin{aligned} L &= 0.64\,\frac{\mu\mathrm{H}}{\mathrm{m}}\,, & C &= 41.9\,\frac{\mathrm{pF}}{\mathrm{m}}\,, \\ & & R = G = 0\,, & \\ R_p &= 62\,\Omega\,, & L_s &= 3.2\,\mathrm{mH}\,. \end{aligned} \tag{15}$$

The excitation is a rectangular pulse signal with an amplitude of 50V and

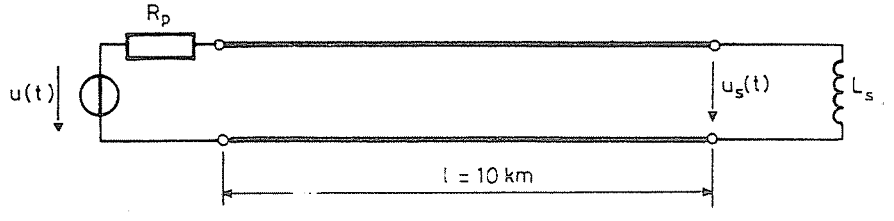

Fig. 3. The transmission line discussed in Example 1

its duration is $\tau = 51.8\ \mu$s (τ equals one line transit time of the transmission line). The analytical and computed results can be seen in *Fig. 4*. Practically the only difference is owing to the Gibbs' phenomenon at the points of discontinuity. 256 samples were taken from the spectrum. This means the application of an *FFT* with 512 samples.

b) As a second example, let us consider the transmission line seen in *Fig. 3* with the parameters given in Eq. (15), except $R = 4.4\,\frac{\mathrm{m}\Omega}{\mathrm{m}}$ and $G = 0.287\,\frac{\mu\mathrm{S}}{\mathrm{m}}$. In this case the equation $R/L = G/C$ is fulfilled (i.e. it is a distortionless transmission line). And thus the results can again be obtained analytically. The exact and the approximate solutions can be compared in *Fig. 5*. The number of samples is equal to that of example a).

Lossy transmission line system

Let us study the transmission line system seen in *Fig. 6*. It consists of 3 conductor-pairs with the indicated ports.

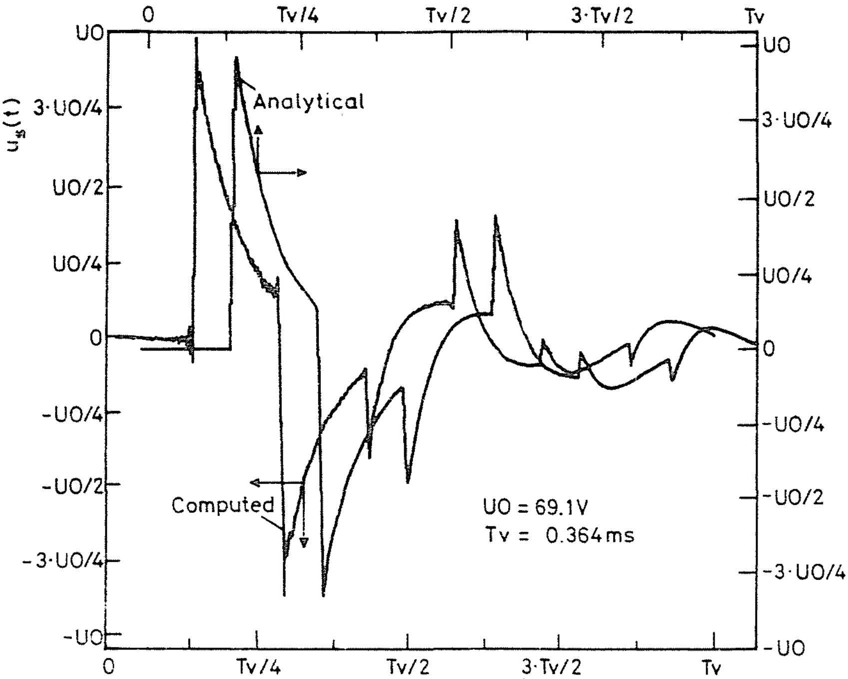

Fig. 4. The analytical and the computed voltage functions at the load of the transmission line shown in *Fig. 3*, if it is loss-free

The matrices describing the system are the following (see Eq. (2)):

$$\boldsymbol{L} = \begin{bmatrix} 644+L_i & 138 & 44.6 \\ 138 & 644+L_i & 138 \\ 44.6 & 138 & 644+L_i \end{bmatrix} \frac{\mathrm{nH}}{\mathrm{m}}$$

$$\boldsymbol{C} = \begin{bmatrix} 41.9 & -8.8 & -1.012 \\ -8.8 & 41.9 & -8.8 \\ -1.012 & -8.8 & 41.9 \end{bmatrix} \frac{\mathrm{pF}}{\mathrm{m}}$$

$$\boldsymbol{R} = \begin{bmatrix} R_i & 0 & 0 \\ 0 & R_i & 0 \\ 0 & 0 & R_i \end{bmatrix} \frac{\mathrm{m\Omega}}{\mathrm{m}}$$

$$\boldsymbol{G} = \begin{bmatrix} 118 & -24.8 & -2.85 \\ -24.8 & 118 & -24.8 \\ -2.85 & -24.8 & 118 \end{bmatrix} \frac{\mathrm{nS}}{\mathrm{m}}$$

where L_i and R_i are the parameters taking the skin-effect into account.

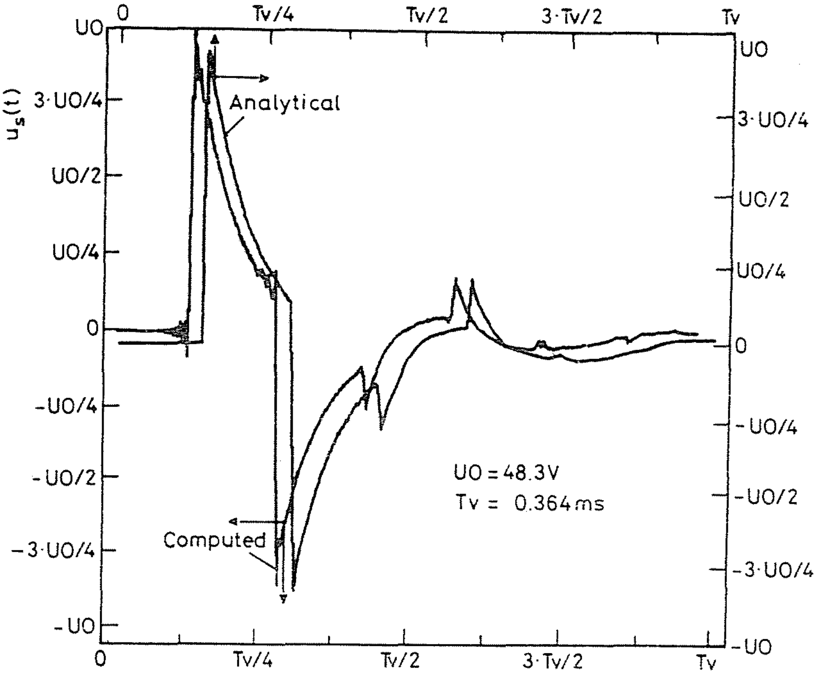

Fig. 5. The analytical and the computed voltage functions at the load of the transmission line in *Fig. 3*, if it is lossy but distortionless

Their values are approximated in the following manner (SIMONYI et al (1966)):

$$\left.\begin{aligned} L_i &= \tfrac{1}{\omega}\, x^2\, R_0 \\ R_i &= (1 + \tfrac{1}{3}x^4) R_0 \end{aligned}\right\} \quad \text{if } x < 1\,,$$

$$\left.\begin{aligned} L_i &= \frac{1}{\omega}\,(x - \frac{3}{64x})\, R_0 \\ R_i &= (x + \frac{1}{4} + \frac{3}{64x})\, R_0 \end{aligned}\right\} \quad \text{if } x > 1\,, \tag{16}$$

where $R_0 = 1.4\,\frac{\mathrm{m}\Omega}{\mathrm{m}}$ and $x = 189.2\sqrt{\omega}$, *where* ω is given in $\frac{\mathrm{Grad}}{\mathrm{s}}$.

The excitation can be seen in *Fig. 7* (the duration of the rectangular pulse signal is the line transit time calculated if the dissipation and the coupling between the lines are neglected). *Figs. 8–10* show the transient voltage functions of the members of the transmission line system at the loads (a) and at the point of z = 4km (b) (see *Fig. 6*). 256 samples were taken from the spectrum. The required CPU time was 12.5 minutes (without plotting the figures) on an IBM XT compatible personal computer.

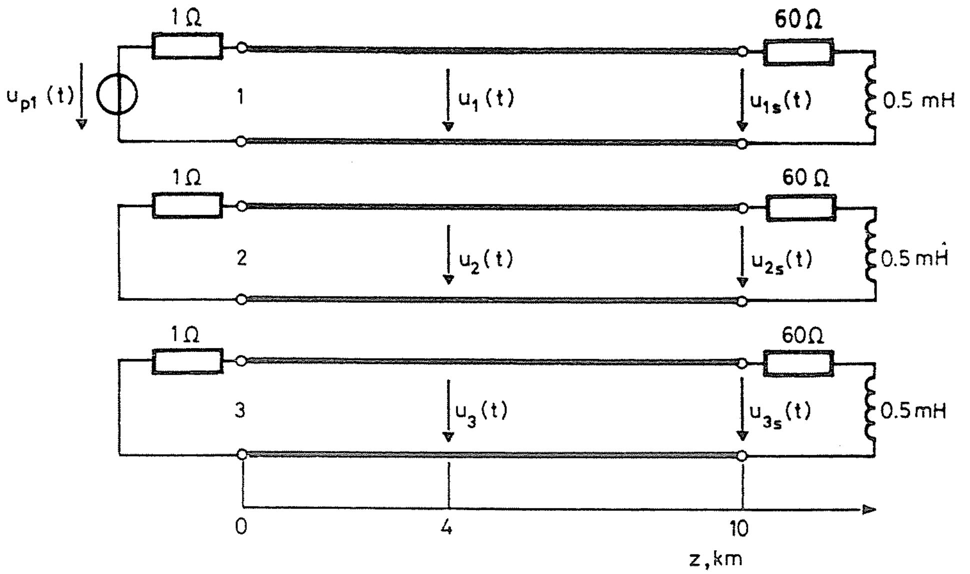

Fig. 6. The transmission line system discussed in Example 2

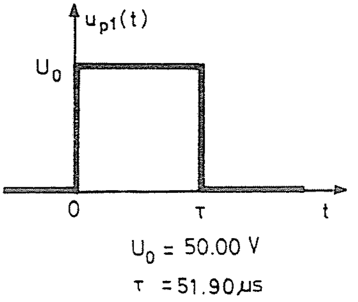

Fig. 7. The time function of the excitation

Conclusion

A method for computing the transient phenomena of a transmission line system was presented, where the propagation of the quasi-TEM mode was assumed. The loads can be arbitrary linear, active n-terminal networks and they (like the other parameters of the system) can depend on the frequency. The excitation must be a Fourier transformable signal with finite energy. If the mentioned conditions are not fulfilled the method can be used by completing it with the theory suggested by DJORDEVIC et al (1986).

It has been shown that the required CPU time for the calculation is reasonable if there are no extreme loads (short- or open-circuit ports). In

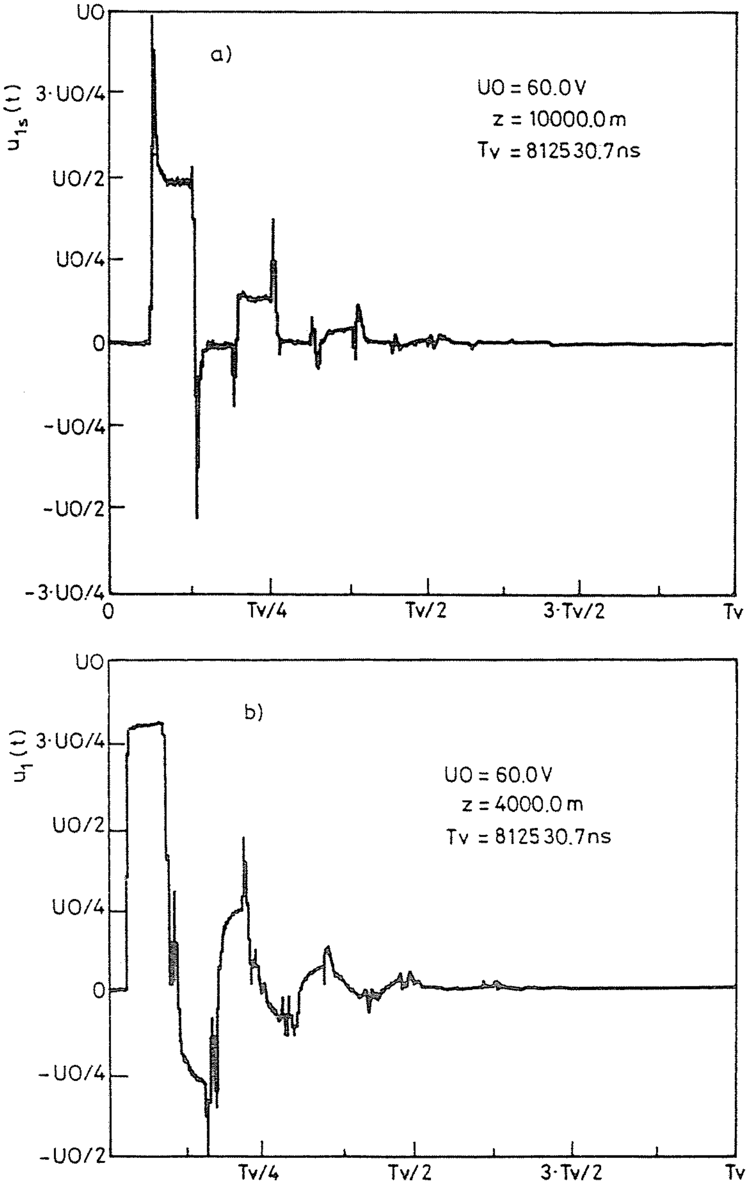

Fig. 8. The computed voltage functions of Line 1 (see *Fig. 6*)

a) at the load

b) at point $z = 4$ km

such cases it is very useful to again complete the theory with that indicated

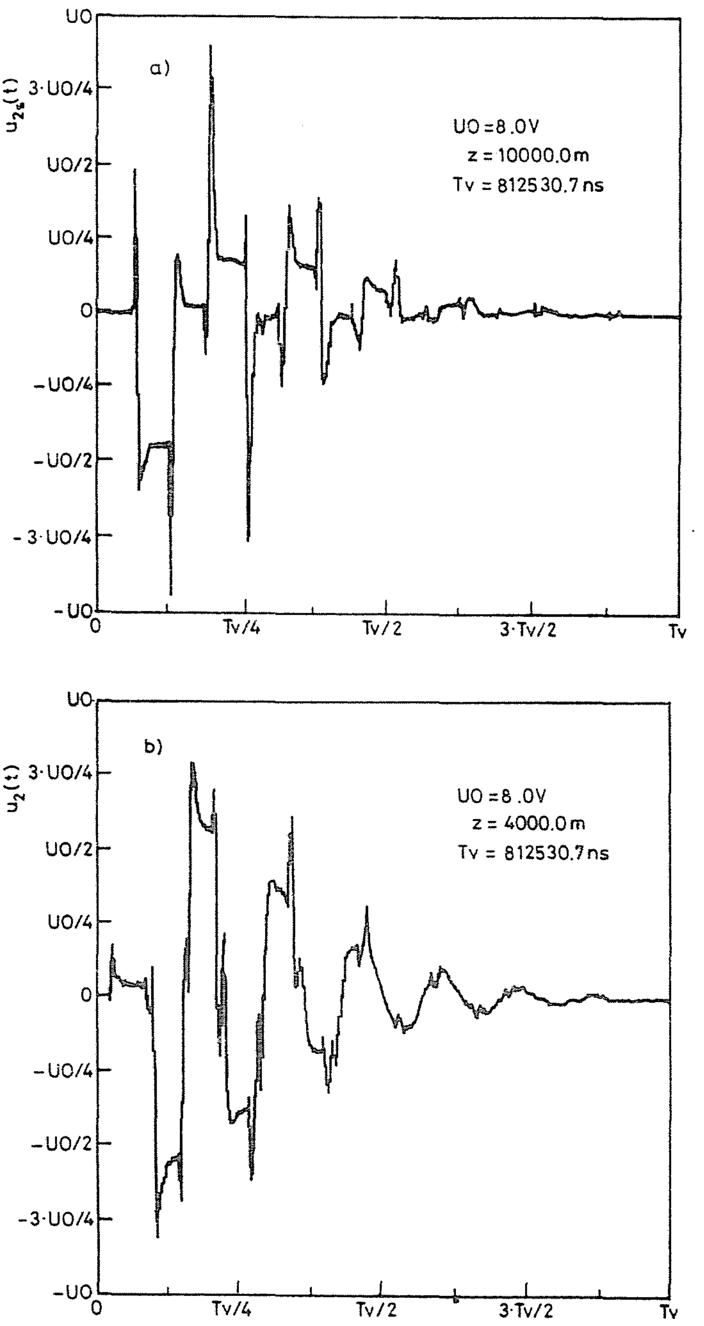

Fig. 9. The computed voltage functions of Line 2 (see *Fig. 6*)

a) at the load

b) at point $z = 4$ km

by DJORDEVIC et al (1986).

The examples presented demonstrate that the only appreciable error in the approximation is caused by Gibbs' phenomenon at the points where the time-functions are discontinuous.

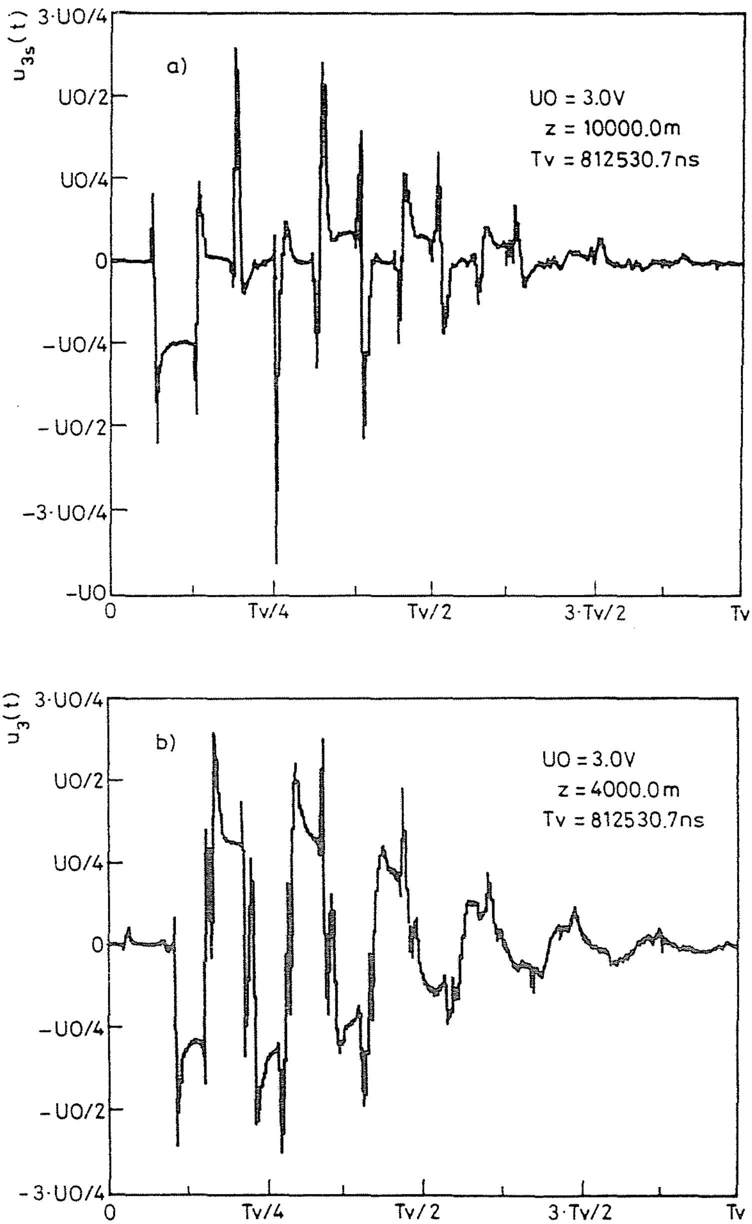

Fig. 10. The computed voltage functions of Line 3 (see *Fig. 6*)
a) at the load
b) at point $z = 4$ km

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