PERIODICA POLYTECHNICA SER. EL. ENG. VOL. 33, NO. 5, PP. 263–272 (1989)

AN ADAPTIVE PID-CONTROLLER

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Received June 14, 1988.

Abstract

The adjustment of controllers to unknown processes is a time consuming and sometimes difficult task in industry. One possible solution for this problem is the usage of an adaptive controller which automatically determines its optimal parameters. This paper presents a new method to calculate the parameters of a PI(D)-controller from the measurement data of an unknown process for the single-input/single-output case.

Keywords: adaptive controller.

Introduction

A common task in process control is to adjust a PI(D)-controller to the unknown process. The tremendous progress in microprocessor technology allows the use of methods with high computational load in small applications. In recent years there appeared a large number of controllers with the ability of adapting themselves to the dynamics of the system under control (ASTRÖM, 1983; ISERMANN, 1987). This paper deals with a new algorithm to determine PI(D)-controller parameters for a linear process with real or slightly complex poles.

Basic concept of the controller design

The control loop for a single-input/single-output system can be seen in *Fig. 1.*

The dynamic behaviour of this system is determined by the closed-loop transfer function $F_c(s)$:

$$F_c(s) = \frac{R(s)G(s)}{1 + R(s)G(s)} = \frac{F(s)}{1 + F(s)} = \frac{Z(s)}{N(s)} \tag{1}$$

If $F_c(s)$ can be approximated by a dominant conjugate complex pole pair, $s = -\delta \pm j\omega$, we get the step-response of the closed-loop system as

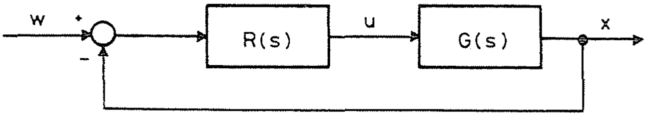

Fig. 1. Single-input/single-output control system

$$h_c(t) = 1 - e^{-\delta t}(\cos\omega t + \frac{\delta}{\omega}\sin\omega t) \tag{2}$$

The maximum overshoot of $h_c(t)$ is obtained for $t_{\max} = \frac{\pi}{\omega}$ as

$$\Delta h_c(t_{\max}) = h_c(t_{\max}) - 1 = e^{-\pi\delta/\omega}, \tag{3}$$

and the ratio of two consecutive extremal values of $\Delta h_c(t)$ is

$$\frac{\Delta h_c((k+1)t_{\max})}{\Delta h_c(kt_{\max})} = e^{-\pi\delta/\omega} \tag{4}$$

Both values in *Eqs.* (3) and (4) depend only on the proportion of the real part δ to the imaginary part ω of the poles s. Let us define these properties as a 'Damping ratio' $D = |\frac{\omega}{\delta}|$, which describes the behaviour of the resulting time function of a conjugate complex pole pair.

Our goal is to determine the controller in a way, that the following four conditions are fulfilled:

I.—the closed loop is stable, i.e. all poles $s = -\delta + j\omega$ have a negative real part $-\delta < 0$;

II.—one pole pair $s_0 = -\delta_0 \pm j\omega_0 = -\delta_0 + jD\delta_0$ has exactly the desired damping ratio $D = |\frac{\omega_0}{\delta_0}|$ and $|\delta_0|$ as large as possible;

III.—all other poles $s = -\delta + j\omega$ have a damping ratio not larger than $D = |\frac{\omega}{\delta}| \leq D = |\frac{\omega_0}{\delta_0}|$;

IV.—all other poles $s = -\delta + j\omega$ lie to the left of the pole pair $s_0 = -\delta_0 + j\omega_0 : -\delta \leq -\delta_0 < 0$.

Condition I. is obvious, conditions II. to IV. lead to a dominant conjugate complex pole pair $s_0 = -\delta_0 \pm jD\delta_0$ with $|\delta_0|$ being as large as possible without violating conditions III. and especially IV. Condition IV. guarantees that there are no poles in the system that produce a time behaviour decaying slower than $e^{-\delta_0 t}$.

The next sections will explain, how these conditions can be satisfied with the help of the modified Nyquist stability criterion.

The Nyquist Stability Criterion

The Nyquist stability criterion (NYQUIST, 1932; FÖLLINGER, 1985) leads to a decision about the stability of the closed-loop system only through knowledge of the open-loop transfer function $F(s) = R(s)G(s)$.

This is achieved with the help of Cauchy's residue theorem. It states, that the contour integral around a closed curve C of a complex function $\frac{N'(s)}{N(s)}$ is equal to the number of zeros, z, minus the number of poles, p, within C multiplied by $2\pi j$:

$$\int_C \frac{N'(s)}{N(s)} \mathrm{d}s = (z-p)2\pi j \tag{5}$$

with

$$N'(s) = \frac{\mathrm{d}N(s)}{\mathrm{d}s}$$

The following equation also holds:

$$\int_{s_1}^{s_2} \frac{N'(s)}{N(s)} \mathrm{d}s = [\ln N(s)]_{s_1}^{s_2} = [\ln|N(s)| + j\arg(N(s))]_{s_1}^{s_2}$$

with

$$N(s) = |N(s)|e^{j\arg(N(s))}.$$

For $s_1 = s_2$ we get $\ln|N(s_1)| = \ln|N(s_2)|$, but the phases are not necessarily the same: $arg(N(s_1)) = arg(N(s_2))$ not necessarily holds. Therefore the contour integral equals the phase difference that we receive for one circulation on C:

$$\oint_C \frac{N'(s)}{N(s)} \mathrm{d}s = j \underset{C}{\Delta} \arg(N(s)), \tag{6}$$

Comparison of *Eqs.* (5) and (6) yields:

$$\underset{C}{\Delta} \arg(N(s)) = \underset{C}{\Delta} \arg(1 + F(s)) = (z-p)2\pi. \tag{7}$$

For the Nyquist stability criterion the curve C consists of the imaginary axis and a semicircle with radius r around the right s-half-plane (see *Fig. 2.*).

For real physical systems $F(s) = 0$ and $\Delta\arg(1 + F(s)) = 0$ on the semicircle of C for $r \rightarrow \infty$. Hence, to evaluate the contour integral, we

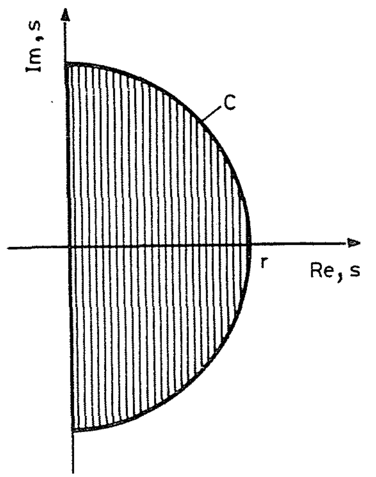

Fig. 2. Nyquist stability criterion: Integration curve C

have to calculate $\Delta\arg(1+F(s))$ only on the imaginary axis, that is for $-\infty \leq \omega \leq \infty$.

Using the symmetry of $N(j\omega)$ it is sufficient to obtain $\Delta\arg(1+F(j\omega))$ for $0 \leq \omega \leq \infty$ and for a stable open-loop and a stable closed-loop system ($p = z = 0$) *Eq.* (7) becomes:

$$\underset{\omega=0}{\overset{\infty}{\Delta}} \arg(1+F(jw)) \overset{!}{=} 0 \tag{8}$$

or

$$\underset{\omega=0}{\overset{\infty}{\Delta}} \arg(1+F(jw)) \overset{!}{=} \frac{\pi}{2}. \tag{9}$$

if we have an I-type controller with a pole $s = 0$. For the determination of $\underset{\omega=0}{\overset{\infty}{\Delta}} \arg(1+F(jw))$ it is convenient to move the curve $1+F(j\omega)$ by one unit to the left. This also changes the origin from where we count $\arg(1+F(jw))$ to point $(-1, j0)$. Now we can directly compute $F(j\omega) = R(j\omega)G(j\omega)$ and the phase difference $\underset{\omega=0}{\overset{\infty}{\Delta}} \arg(F(jw))$ observed from point $(-1, j0)$.

To apply the criterion, we have to plot the Nyquist curve $F(j\omega)$ for $0 \leq \omega \leq \infty$ (see *Fig. 3.*) and check if it encloses point $(-1, j0)$, which means: does it intersect the negative real axis to the left or to the right of point $(-1, j0)$?

In the latter case, the closed loop is stable and has no poles in the right s-half-plane. If the Nyquist curve runs exactly through point $(-1, j0)$, the closed loop has a complex conjugate pole pair on the curve C which is the imaginary axis $s = j\omega$.

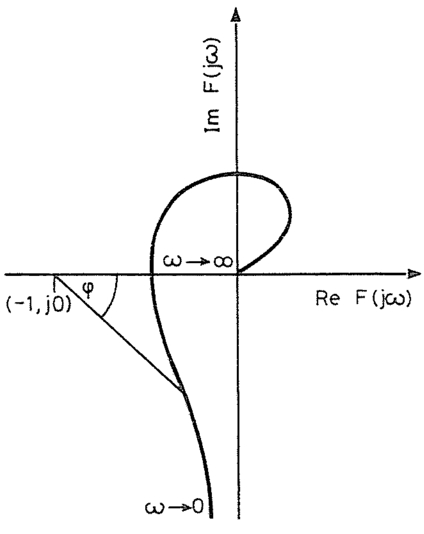

Fig. 3. Nyquist curve $F(jw)$, $\phi = \arg(1 + F(jw))$

So the critical point $F(j\omega_c)$ of the Nyquist curve is the intersection with the negative real axis. For this point the following equality must hold:

$$\arg(R(j\omega_c)G(j\omega_c)) = -\pi \tag{10}$$

and thus the absolute value

$$|R(j\omega_c)G(j\omega_c)| \begin{cases} < 1 \\ = 1 \\ > 1 \end{cases} \tag{11}$$

determines, if the closed-loop is stable or not.

These two formulas (10), (11) for the critical point will be used to obtain the controller parameters with the help of the extended Nyquist criterion described in the next paragraph.

The Extended Nyquist Criterion

The idea of calculating a contour integral around a closed curve to detect poles of a complex function is of course not limited to the right s-half-plane. For the controller design according to the conditions in the previous chapters we use an extended integration curve C' which includes the right s-half-plane to obtain a stable system and additionally those regions in the left s-half-plane, where $|\omega| \geq |D\delta|$ (see *Fig. 4.*).

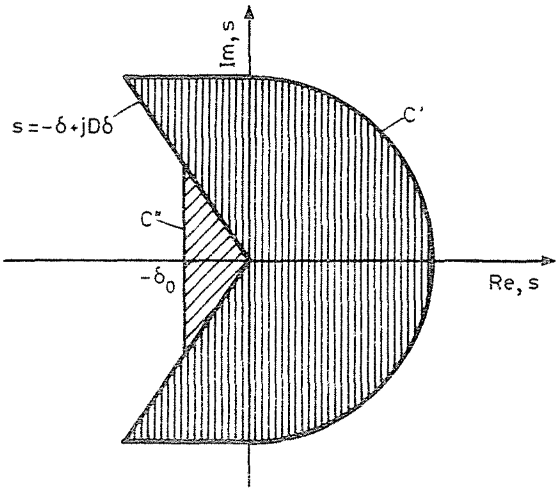

Fig. 4. Extended Nyquist criterion: Integration curves C',C''

We can now perform the calculations corresponding to the conventional Nyquist criterion and get similar results. Function $F(s)$ has to be calculated for $s = -\delta + jD\delta$ with $0 \leq \delta \leq \infty$, function $F(s) \rightarrow 0$ for radius $r \rightarrow \infty$ has no contribution to the phase shift. If there should be no closed loop poles in the shaded area of *Fig. 4.* the phase difference has to be:

$$\mathop{\Delta}_{\delta=0}^{\infty} \arg(1 + F(-\delta + jD\delta)) \stackrel{!}{=} 0 \tag{12}$$

or

$$\mathop{\Delta}_{\delta=0}^{\infty} \arg(1 + F(-\delta + jD\delta)) \stackrel{!}{=} -\tan D + \pi \tag{13}$$

if we have an I-type controller with a pole $s = 0$.

We must plot the Nyquist curve $F(s = -\delta + jD\delta)$ again for $0 \leq \delta \leq \infty$ and check, if it encloses point $(-1, j0)$. If the curve intersects the negative real axis to the right of $(-1, j0)$, then the closed-loop has no poles in the shaded area of *Fig. 4.*, if it intersects to the left, the closed-loop has poles in the forbidden area. If the curve crosses the real axis exactly at $(-1, j0)$, the closed-loop has a complex conjugate pole pair s_0 on the lines $s = -\delta \pm jD\delta$, and this is what we intended to obtain from condition II. and III. The point of interest is again at $(-1, j0)$, and we can make use of *Eqs.* (10) and (11) to apply the extended Nyquist criterion.

To fulfil condition IV. it is necessary to check, if there are no other poles in the triangle between $s_0 = -\delta_0 \pm jD\delta_0$ and the origin s = 0. This can again be done with a modified integration curve C'' for the Nyquist-criterion which theoretically consists of line $s = -\delta_0 + j\omega$ for $-\infty \leq \omega \leq \infty$ and a semicircle around the right s-half-plane (*see Fig. 4.*). However, since we can be certain that there are no poles in the shaded area of *Fig. 4.*, this Nyquist curve $F(s = -\delta_0 + j\omega)$ has to be calculated only for $0 \leq \omega \leq D\delta_0$. If it doesn't enclose the point $(-1, j0)$, condition IV. is satisfied and there are no poles nearer to the imaginary axis as the pole-pair $s_0 = -\delta_0 \pm jD\delta_0$.

Determination of the optimal controller parameters

This section describes the necessary steps to calculate the optimal controller parameters according to the earlier stated conditions. The process is assumed to be identified, so the Laplace-Transform $G(s)$ is available (see next chapter).

In order to reduce the number of the PID-controller parameters, we choose $T_N = 4T_V$, which results in a double zero at $s = -1/T_R$:

$$R(s) = K_R(1 + \frac{1}{T_N s} + T_V s) = P\frac{(1 + T_R s)^2}{s} \tag{14}$$

with

$$T_R = 2T_V = \frac{T_N}{2} \text{ and } P = \frac{K_R}{T_N} = \frac{K_R}{4T_V}.$$

At first, we have to apply the extended Nyquist criterion with integration curve C' to the function $R(s)G(s)$ and find point $s_0 = -\delta_0 + jD\delta_0$ for which *Eq.* (10) holds. Inserting (14) in (10) we get:

$$\arg(R(s)G(s)) = \arg(R(s)) + \arg(G(s)) =$$

$$= \arg(P) - \arg(s) + 2\arg(1 + T_R s) + \arg(G(s)) =$$

$$= 0 - \tan D + 2 \arctan \frac{T_R D\delta}{1 - T_R \delta} + \arg(G(s)) \stackrel{!}{=} -\pi. \qquad (15)$$

The controller gain P makes no contribution to $\arg(R(s)G(s))$, so the solution of *Eq.* (15) is independent of P and the only parameter is the controller time-constant T_R. System $G(s)$ essentially consists of poles in the left s-half-plane, so $\arg(G(s))$ is negative. The solution of *Eq.* (15) with the smallest $\delta_0 \neq 0$ is therefore obtained for $T_R = 0$. If we then increase T_R and solve *Eq.* (15) we will have a new solution for δ which is larger than the δ_0 obtained for $T_R = 0$. To fulfil condition II. (δ_0 as large as possible), we have to repeat this procedure — by increasing T_R and calculating δ_0 — until the new δ_0 is not larger than the previous one. To get a closed-loop pole pair at $s_0 = -\delta_0 \pm jD\delta_0$ we then have to determine the controller gain P in a way, that $|G(s_0)R(s_0)| \stackrel{!}{=} 1$ (compare with *Eq.* (11)). In fact, the gain P must be calculated for each value of T_R, because it is needed to check condition IV. with the Nyquist criterion and integration curve C''. If we detect a pole in C'' then T_R has to be reduced to the last value which satisfied condition IV.

The steps just described lead to a recursive procedure for determining the controller parameters P and T_R:

— The initial values for δ_0 and P are found with $T_R = 0$;
— repeat
— increase T_R and solve *Eq.* (15) to find new δ_0;
— determine P such, that $s_0 = -\delta_0 \pm jD\delta_0$ is a closed-loop pole pair until

a) the new δ_0 is not larger than the previous one;

or

b) there is a pole within the integration curve C'';

— the last parameters T_R and P that did not violate *a*) or *b*) and are the optimal parameters according to conditions I. to IV.

System identification

If a controller is to be adapted to an unknown process, it is necessary to determine somehow the dynamic behaviour of this process. Most of the available controllers are based on linear mathematical system-models such as the transfer function $G(s)$, derived from the differential-equation of the process:

$$G(s) = \frac{\sum_{j=0}^{n} b_j s^j}{\sum_{i=0}^{n} a_i s^i} \tag{16}$$

The parameters a_i and b_j are estimated from measurement data, for example with the recursive least-squares method. A problem of these parametric models is to determine n, because the result of the parameter estimator very much depends on this value. In general, n does not have a distinct value, but has to be calculated approximately from the measurement data. A similar problem arises, if the process contains a dead-time. To avoid these difficulties, we use a nonparametric model of the process: its impulse response $g(t)$. We do not have to cope with order n or coefficients a_i, b_j, they are implicitly included in the samples of $g(t)$.

There are basically two ways to calculate $g(t)$ from the measured signals $u(t)$ and $x(t)$.

Deconvolution in the frequency-domain

From *Eq.* (16) we get

$$g(t) = \mathcal{F}^{-1} \frac{\mathcal{F}\{x(t)\}}{\mathcal{F}\{u(t)\}} \tag{17}$$

This method can be implemented using the Fast-Fourier-Transform, so the number of calculations is relatively low. However, a disadvantage is, that $\mathcal{F}\{u(t)\}$ must not be zero for all frequencies. Since $u(t)$ is the measured output of the controller R, this cannot be guaranteed, in general.

A possible way around this problem is to calculate $g(t)$ in the time-domain.

*Deconvolution in the time-domain (*NAHI*, 1969)*

The equation corresponding to (16) in the time-domain is the convolution integral:

$$x(t) = \int_{-\infty}^{\infty} g(\tau) u(t-\tau) \mathrm{d}\tau, \tag{18}$$

or for discrete signals:

$$x(k) = \sum_{l=-\infty}^{\infty} g(l)u(k-l) = \sum_{l=0}^{N-1} g(l)u(k-l) \tag{19}$$

with $g(l) = 0$ for $l < 0$ and $l \geq N$.

Introducing error $e(k)$ we get

$$x(k) = \sum_{l=0}^{N-1} g(l)u(k-l) + e(k),$$

or in matrix notation:

$$\mathbf{x} = \mathbf{U}\mathbf{g} + \mathbf{e} \tag{20}$$

If we take $M > N$ samples of $u(k)$ and $x(k)$ we can formulate a least-squares estimator to minimize $\mathbf{e}^{\mathrm{T}}\mathbf{e}$ and obtain an estimate $\hat{\mathbf{g}}$ for $\mathbf{g}$:

$$\hat{\mathbf{g}} = (\mathbf{U}^{\mathrm{T}}\mathbf{U})^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{x} \tag{21}$$

It makes no difference for the estimation of $\hat{\mathbf{g}}$ if the signals $u(t)$ and $x(t)$ are measured in the open-loop or in the closed-loop system.

From *Eq.* (21) we finally calculate the Laplace transform

$$G(s) = T \sum_{k=1}^{N} \hat{g}(k)e^{-skT} \tag{22}$$

which is needed to determine the controller parameters as described in the previous section.

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