A STOCHASTIC MODEL FOR A/D CONVERTERS

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Abstract

It is shown that a modified feedback measuring circuit for ADC-s can perform as a measuring system for ADC channel profile measurements (i.e. measurement of the stochastic behaviour). A mathematical model of the measurement is given. The measured data are used for constructing a stochastic model of the ADC, which provides us with information about ADC errors.

Introduction

Theoretically ADC-s are deterministic devices—the same input will always result in the same output code at an ideal ADC. But this is not the case at real converters—there are certain input values or rather input intervals where the output code can be different for repeated conversions. This effect can produce serious calculation problems during the evaluation of A/D conversion results, since theoretical examinations (e.g. the quantizing theorems [1]) consider ADC-s as deterministic devices. In this article we examine the effects of this stochastic behaviour.

The channel profile

The values of the switching points—the input values, where the output code changes take place—are well defined at ideal A/D converters, but not at the real ones. Figure 1. shows the behaviour of an ideal and a real ADC around the switching point of code M.

While at the ideal converters the output code belonging to an input value can be determined with absolute certainty even in the very neighbourhood of $V_{M_{id}}$ only probability values can be given for the occurrence of an output code at real ADC-s. We define V_{M_r}—the input value, where the occurrence of the two adjoining output codes are equally probable—as the effective switching point. (As the measurement results prove, the probability function is monotonous in the uncertainty interval, hence V_{M_r} is a well-defined value.) Generally $V_{M_{id}} \neq V_{M_r}$.

The stochastic behaviour can be characterised by the channel profile [2]. The channel profile belonging to the output code M is the probability of occurrence of M as a function of the input signal. Figure 2 [2] shows different possible channel profiles.

It can be seen, that the 50% point, V_{M_r} has different values at different channel profiles.

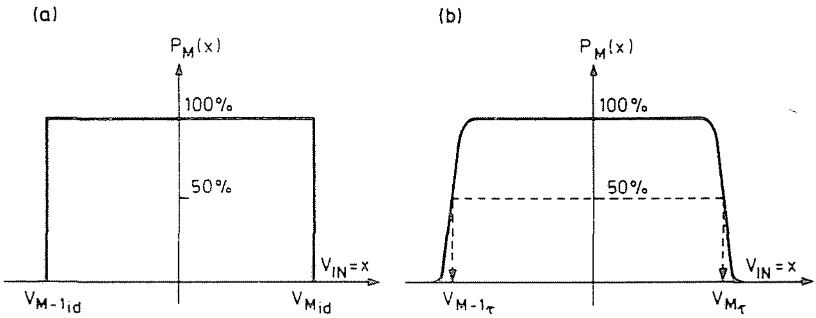

Fig. 1. Switching point of a) a theoretical ADC ($V_{M_{id}}$) and b) of a real one (V_{M_r}). Real ADC-s have uncertainty input intervals around the switching points, where the outcome of the output code decision is not deterministic. $P_M(x)$ shows the probability of the occurrence of code M

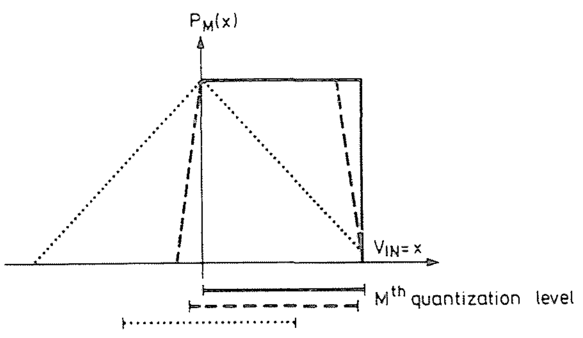

Fig. 2. Possible theoretical channel profiles

The setup of the measurement

Among the many ADC testing circuits only those measuring the values of the switching points can be made suitable for measuring the channel profile [3], [4]. We found, that the quickest of these is the measuring circuit with a feedback [5]. Figure 3 shows the modified version, which is already suited for measuring the channel profile. The difference between this and the one described in [5] is, that $|V_1^+|$ and $|V_2^-|$ are not necessarily equal here.

The system operates as follows: the controller sends the examined code M on one input of the comparator, while the ADC—which continuously converts its own input—is connected to the other one. The input signal of the ADC increases or decreases in accordance with the output of the comparator, because this signal connects the input of the integrator—aided by switch K—either to the positive V_1^+ or to the negative V_2^- voltage. As a result we will find a series of M and $M+1$ codes on the output of the ADC, supposing that the

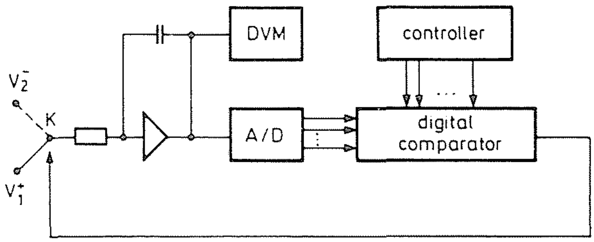

Fig. 3. Channel profile measuring setup

parameters are suitable for not producing other codes (i.e. the output of the integrator does not change more than the LSB of the ADC during one integration period). If the absolute values of the two voltages V_1^+ and V_2^- are equal, both codes will occur with a probability of 50%, and the mean value of the ADC input—measured by the DVM—will be the effective switching point, V_{M_r}.

By changing the ratio of the negative and positive integrator input voltages, the channel profile can be measured, as shown below.

Mathematical model for the measurement

Let $\left|\frac{V_2^-}{V_1^+}\right| = k$. Supposing that during one conversion period of the ADC—with the integrator input voltage V_1^+—its input increases with Δx, then—at the appropriate comparator output, and V_2^- as the input voltage of the integrator—the ADC input will decrease with $k \cdot \Delta x$. The procedure is the following:

Let us choose a random ADC input voltage in the uncertainty interval of the channel profile of code M: x_1. During the next conversion period the integrator output will either increase with a probability of $P_M(x_1)$, or decrease with a probability of $1-P_M(x_1)$, depending on the decision of the ADC. Thus the next input value will either be $x_1+\Delta x$ or $x_1-k \cdot \Delta x$, and the next decision takes place with this new input value.

In the following we show, that given a channel profile and a value k, with the above setup shown in Fig. 3. we can measure with good approximation a point of the channel profile determined by k. By changing the values of V_1^+ and V_2^-, the value of k can be changed, thereby the different points of the channel profile can be measured.

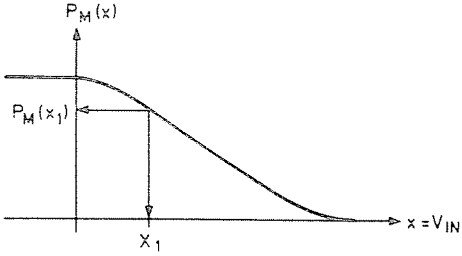

Fig. 4. Possible uncertainty interval of the channel profile. It can be seen, that choosing a random input value x_1 the probability of the output code being M is $P_M(x_1)$—and this is the probability of the integrator output being increased during the next conversion period. The output code will be $M+1$ with a probability of $1-P_M(x_1)$, and in this case the integrator output will decrease

According to the theory of Markov-chains the following is true: choosing any starting point in a finite interval with a probability function $P_M(x)$, that is 1 downwards the internal and 0 upwards it, and with a finite number of possible states there exist $p_0, p_1, \ldots p_n$ probabilities, that $\sum_{i=0}^{n} p_i = 1$, and the probability of being in the i-th state is p_i. (In other words: our model is a Markov-chain and it tends to a stationary distribution.)

It is to be remarked, that the finite number of states is a theoretical assumption, which cannot be fulfilled in reality, because of time and voltage uncertainties. But decreasing the value of Δx—and thus tending to a continuous Markov-chain—the error caused is negligible.

Supposing that k is an integer, our model fulfills the requirements of the above statement. By restricting k to integer values we can only measure some discrete points of the uncertainty interval of the channel profile, but this will provide us with enough information about the uncertainty interval. It can also easily be seen, that the uncertainty internal determines the whole channel profile. Supposing also, that Δx is not less, that the tenth of the uncertainty interval k must be not more than 10. (This was the smallest possible Δx value we could realize, because circuitry noises could falsify the results if more points were to be measured.)

According to Fig. 4 p_i can be determined from

$$p_i = p_{i-1} P_M(x_{i-1}) + p_{i+k}(1 - p_M(x_{i+k})) \qquad i = 1-k, \ldots, k \tag{1}$$

The reason for this is, that there are only two states from where the ADC input can arrive at the state x_1 during one conversion period (p_i is the probability of being in the state x_i):

a) from x_{i-1}, but only, when the integrator input is positive (the probability of this is $P_M(x_{i-1})$) and hence its output will change by exactly Δx, thus arriving at x_i and

b) from x_{i+k}, when the integrator input is negative, hence changing $k \cdot \Delta x$ and arriving at x_i once again. Let us denote with x_0 the state, where the uncertainty interval begins ($P_M(x_0)=1$ and $P_M(x_1)<1$), so the smallest possible state to be reached is x_{1-k}, because downwards integration is not more possible, when $i<1$. Similarly, if x_k is the smallest state, where $P_M(x)=0$, then the highest possible state to be reached is x_k.

The above equations form a homogeneous linear equation system:

$$\begin{bmatrix}
-1 & 0 & 0 & \cdots & 1-P_M(x_1) & & & & & & & & & \\
0 & -1 & 0 & \cdots & & 1-P_M(x_2) & & & & & & & & \\
0 & 0 & -1 & \cdots & & & 1-P_M(x_3) & & & & & & & \\
\vdots & \vdots & \vdots & & & 0 & & \ddots & & & & & & \\
 & & & & 0 & -1 & 0 & & & & & & & \\
 & & & & & 1 & -1 & 0 & & & & & & \\
 & & & & & & 1 & -1 & 0 & & & & & \\
 & & & & & & & 1 & -1 & 0 & & & & \\
 & & & & & & & & 1 & -1 & 0 & & & \\
 & & & & & & & & 1 & -1 & 0 & & & \\
 & & & & & & & & & \ddots & \ddots & \ddots & \ddots & \\
 & & & & & & & & & & & 1 & -1 & 0 \\
 & & & & & & & & & & & & 1 & -1 \\
 & & & & & & & & & & & & & 1
\end{bmatrix}
\begin{bmatrix}
p_{-k+1} \\ p_{-k+2} \\ \\ \\ p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ \vdots \\ p_{k-2} \\ p_{k-1} \\ p_k
\end{bmatrix} = 0 \qquad (2)$$

Solving this equation system for certain k values, we can determine the p_i probabilities belonging to the given k channel profile and code M from the general solution of the equation system, using that $\sum_{i=1-k}^{k} p_i = 1$. It follows from the theory of Markov-chains, that the solution exists and is unique. Out of these p_i values—after a large number of measurements—the following mean value can be formed:

$$M_k(x) = \sum_{i=1-k}^{k} p_i x_i \,. \qquad (3)$$

We state, that with this above mean value we can measure the points of the channel profile as a function of k at certain channel profiles (see next paragraph). At other channel profiles these points will be obtained with a good approximation. To a given k there belongs the $\frac{100}{k+1}\%$ point of the channel profile (i.e. the probability of the occurrence of code M in this point will be $\frac{100}{k+1}$). It can easily be seen that—by reflecting the channel profile on the normal axis of the uncertainty interval—the $(100-\frac{100}{k+1})\%$ points can be measured, if k is a reciprocal value of the integers, because of the symmetry.

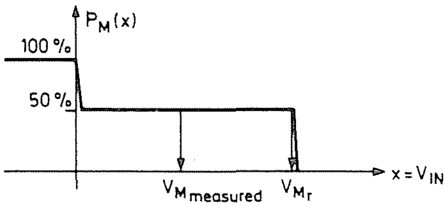

Fig. 5. Theoretical channel profile causing maximal distortion at our measurement. The reason for this distortion is that the measuring procedure builds a weighted mean of the channel profile as a result of equation system (2)

Using this model, the results of the measurements can be predicted for any given channel profile by solving the equation system (2). It was solved for many different channel profiles by the aid of a computer and the results are the following:

— If the channel profile is linear in the uncertainty interval, the obtained result of the measurement shows the points of the channel profile without any predetermined error (the distortion is 0). The result is independent of the initial value and so of the x_i values.
— At non-linear channel profiles a distortion will occur at some or at all measured points depending on the shape of the profile. We can see an extreme case in Fig. 5 where the distortion is nearly equal to half of the uncertainty interval.

Our measurements on AD7570 SAR-type ADC-s (see also [6]) proved, that real channel profiles belong with good approximation to the first, linear-type group (with possible small deviations tending to a very flat Gauss-distribution), and thus our measuring setup measures the real channel profile with a good approximation.

The difference between the realisation and the model is, that the input value of the ACD will not always change by exactly Δx or $k\cdot\Delta x$ during one conversion period, so there are an infinite number of possible states here, but—as we have seen, that the initial value and the set of x_i-s does not influence the results of the measurement at the linear-type channel profiles—this drift will not cause an error in the measurement.

Results

The setup is suitable for measuring other ADC properties, too. Thus the measurements are not restricted to the channel profile measurement only. A Hewlett-Packard measuring system at the Department of Measurement and Instrument Engineering at the Technical University Budapest was used. A problem arose at the realisation of an accurate switch, which was constructed of star-connected FETs. We measured AD7570 10 bit ADC-s, Δx being 1/50 LSB. The value of the linearity errors (counted from the values of the 50% points) proved to be 0.2LSB$\pm$0.05 LSB, which is well under the specified 0.5 LSB. The channel profile is global, i.e. it is uniform at all switching points and its uncertainty interval is linear, with a Δw width of 0.1 LSB$\pm$0.05LSB. With the given Δx value and the proof in the previous paragraph, the channel profile measurement is accurate with the given error bounds.

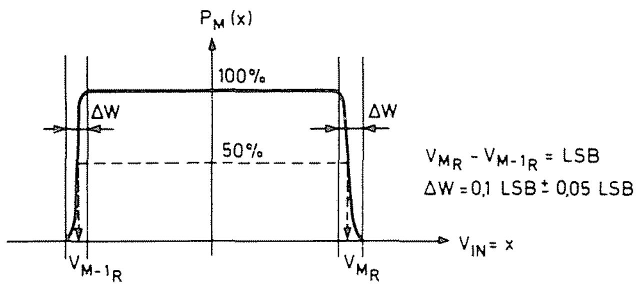

Fig. 6. Result of the channel profile measurement: the uncertainty interval is nearly linear

The ADC transfer characteristics and the channel profile were examined as a function of some environmental factors, too. The behaviour of one switching point as a function of temperature is shown on Fig. 7.

All switching points behaved similarly. So for the AD7570 the dependence upon temperature is 0.3LSB in the interval of [-5°C, $+60$°C]. This dependence offsets the transfer characteristics, but causes no distortion. Its behaviour is hysteresical, which is possibly caused by some huge time constants. The channel profile was not influenced.

The measurements were repeated several times with altering supply voltages, but these did not influence the results.

Influence of ADC errors on momentum estimations

On the basis of our results we tried to examine, that by using the analysed ADC for momentum estimations, how the results would match those of the quantizing theorems[1]. These theorems are strongly based on the fact, that the ADC is ideal. It is shown in [17], that even the measured linearity error of 0.2LSB can cause considerable errors in the momentum estimations. Therefore the linearity error of the ADC must be diminished to obtain reliable results. Several methods can be applied (e.g. [8], [9]). All of them use some sort of supplementary circuitry and the one described in [8] uses an averaging method. This is important when considering the effect of the channel profile. It is proven in [2], that if the channel profile is global, the equations resulting from the quantization theorem—determining the momentums of the input signal—remain valid. During the proof the fact is used, that the converter has no linearity error. Using the averaging method the converter will fulfil much better this requirement.

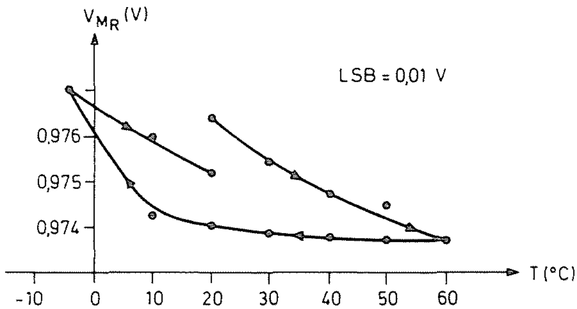

Fig. 7. Value of a switching point as a function of temperature. Similar hysteresical behaviour can be observed at all switching points

As our measurements showed that the channel profile of these converters is, in fact, global, we can state that diminishing the linearity error of such an ADC to near 0, the stochastic behaviour will not cause distortion when estimating the momentum. Variance, of course will alter by an additive constant, because of the quantization.

A stochastical model of the ADC

The ADC can cause the following errors during the realization of the quantization [10]:

— linearity errors
— stochastic behaviour of the output levels because of internal circuitry noises
— dependence upon environmental factors
— gain and offset errors
— frequency errors at high-speed input signals

The offset and gain errors are easy to compensate, and in general frequency errors are not dealt with. One environmental factor, temperature, causes an error which, being additive, can also be compensated. The linearity errors cause distortion, but with supplementary circuitry they can be diminished to small—in some cases negligible—values. The remaining—and related—factors all operate as a small amplitude dither superponing on the input signal, therefore they could be averaged out, and so will not cause distortion, but increase the variance.

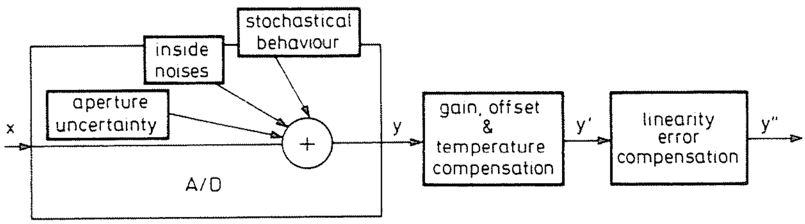

Fig. 8. Stochastic model for ADC-s

Linearity error compensation is perhaps the most serious problem when dealing with ADCs and is not a subject of this paper. It is true, however, that for obtaining reliable results it must be taken into consideration.

According to the above, we can state, that if the signal $x(t)$ satisfies the conditions of the quantization theorems, then the momentums of x can be determined from the quantized y'' signal. *Thus the ADC, completed with the devices shown in Fig. 8 does not cause distortion in the moment estimations* [11]. The whole or partial lack of these compensational devices —i.e. using of y or y' as output signal—results in a distorted output signal, that is to be used with further error examinations only.

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