ON THE SOLVABILITY OF THE NULLATOR-NORATOR PAIRS NETWORK

I. PÁVÓ

Research Group on Theory of the Automata
of the Hungarian Academy of Sciences

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Abstract

The paper deals with the unique solvability of a nullator-norator pairs network consisting of RLC elements and source generators. After defining the kernel of a network the normal, inverse normal, distinguished and reactance trees of the network graph are introduced. Setting out from [6] necessary and sufficient conditions are given for the unique solvability. A topological formula is introduced from which many sufficient conditions of the unique solvability can be obtained as algebraic equations between the parameters of the RLC network elements. The results of the paper are illustrated by examples. Finally, a block scheme is presented for the examination of the unique solvability by computer technique.

Introduction

One of the methods for calculating an electronic network is based on the analysis of its nullator-norator pairs network model. For example let us consider the three network on Fig. 1 consisting of an ideal transistor and some RLC elements. Although each network has the same elements the results of the analysis are very different. Modelling the ideal transistor by a nullor on Fig. 2 one can directly see that the network model (a) has an unique solution and the voltage of the capacitor and the current of the inductor can be chosen as independent state variables. Changing the position of the elements R_3L connected paralelly and the elements R_2C connected in series we have the network (b) which can not be solved. Modifying the position of the resistance R_2 in the network (b) we get the network (c) which has also a unique solution and the only state variable is the current of the inductor.

However, the analysis of a more complicated network (for example, which consists of a great number of the elements) can not be performed simply in general. Before doing the concrete analysis one has to check the unique solvability, to estimate the order of the complexity of the network and to choose a possible set of the independent state variables. In case of RLC network the condition of the solvability is the existence of a normal tree. The voltages of the capacitors belonging to the normal tree and the currents of the inductors not belonging to the normal tree often form a possible set of the independent state variables for the analysis (see [3]).

In this paper we define the notion of the normal tree for the nullator-norator pairs network as a generalization of the normal tree of RLC network. We are going to show that similar theorems hold for a normal tree in the nullator-norator pairs network as in case of RLC network. Necessary and sufficient conditions are given for the unique solvability of the nullator-norator pairs network. The application of these conditions will be presented through many examples. In the end we give a computer implementation for the application of our results.

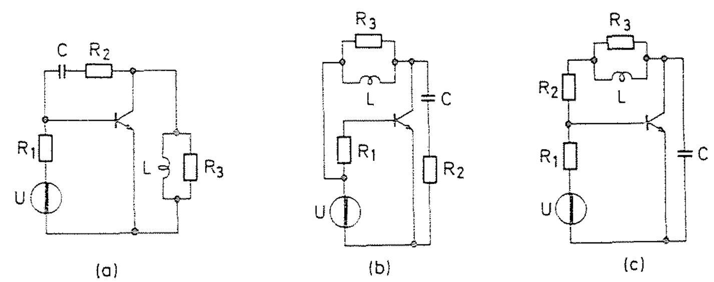


Fig. 1

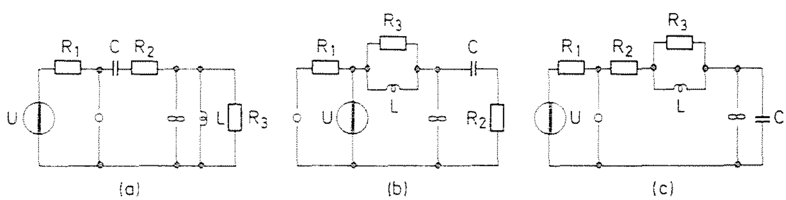


Fig. 2

Notations and definitions

Let us consider a nullator-norator pairs RLC network. Without loss of the generality we can assume that the network graph is connected. Denote by U, I, R, L, C, A and B the sets of the source voltage, source current generators, resistors, inductors, capacitors, nullators and norators of the network, respectively. We denote the set of the network elements and the corresponding subgraph of the network graph by the same symbol if this notation does not result in misunderstanding. If R_i, L_j and C_k are the elements of R, L and C, respectively, denote by r_i, l_j and c_k the parameters of these elements, respectively.

Let $R \subseteq \mathcal{R}$, $L \subseteq \mathcal{L}$ and $C \subseteq \mathcal{C}$ arbitrary (possible empty) subsets. The set $R \cup L \cup C$ will be denoted by K and called the kernel of the network, if both $\mathcal{U} \cup K \cup \mathcal{A}$ and $\mathcal{U} \cup K \cup \mathcal{B}$ are complete trees of the network graph. Let the name of the kernel $R \cup C$ be capacitance kernel, while the kernel $R \cup L$ is called inductance kernel, the common name of theirs is reactance kernel.

Let $\bar{L}$ be the complementer set of L with respect to $\mathcal{L}$. The number $|\bar{L}|+|C|$ is the degree of K and denoted by deg K. It is obvious that $0 \leqq \deg K \leqq |\mathcal{L}|+|\mathcal{C}|$, moreover deg $K=0$ if and only if K is inductance and deg $K=|\mathcal{L}|+|\mathcal{C}|$ if and only if K is capacitance. The kernel of the network is called maximum one if its degree is maximum and minimum one if its degree is minimum in the network. The common name of the maximum and minimum kernels is extreme kernel.

Consider the set of the kernels of the network denoted by $\mathcal{M}$. We classify the elements of $\mathcal{M}$ in the following manner: Let K_1 and K_2 belong to the same class of $\mathcal{M}$, if deg $K_1 = \deg K_2$ holds and vice versa. The subset $\mathcal{M}(K)$ of $\mathcal{M}$ is the class of $\mathcal{M}$ represented by K if $K \in \mathcal{M}(K)$. K is a distinguished kernel if the class $\mathcal{M}(K)$ has exactly one element.

Further on we name the tree $\mathcal{U} \cup K$ of the network graph after K. In this way we can speak about maximum or normal, minimum or inverse normal, extreme, inductance, capacitance, reactance, distinguished trees. These are all subsets of the network graph. Observe that in case $|\mathcal{A}|=|\mathcal{B}|=0$ the nullator-norator pairs RLC network becomes RLC network, and the normal tree of this special nullator-norator pairs RLC network is equivalent with the normal tree of RLC network known from the literature.

After collapsing the endpoints of all the voltage generators in the network, K corresponds to a common k-tree of the network graph, where $k=|\mathcal{A}|+1=|\mathcal{B}|+1$. So we can order a sign to each kernel of the network. Let sgn K be the sign ordered to K as common k-tree relative sign in accordance with reference [5]. The formula

$$\sum_{K_n \in \mathcal{M}(K)} \operatorname{sgn} K_n \cdot \frac{\prod\limits_{C_k \in K_n} c_k}{\prod\limits_{R_i \in K_n} r_i \cdot \prod\limits_{L_j \in K_n} l_j}$$

is the formula generated by K and denoted by f_K. It is clear that each kernel of the network generates a formula and formulas which are generated by kernels belonging to the same class are equivalent.

Theorems for the unique solvability

Theorem 1. If the nullator-norator pairs RLC network has a unique solution then there exists a normal tree $\mathcal{U} \cup K$ of the network graph. The order of the complexity of the network is at most deg K.

Theorem 2. The nullator-norator pairs RLC network with arbitrary parameters (i.e. all the parameters are general variables) has a unique solution if and only if there exists a normal three $U \cup K$ of the network graph. The order of the complexity of the network is exactly deg K. The voltages of the capacitors belonging to the normal tree and the currents of the inductors not belonging to the normal tree form a possible complete set of the independent state variables for the analysis.

Theorem 3. The nullator-norator pairs RLC network has a unique solution if and only if there exists a kernel K of the network such that the formula f_K generated by K is not zero.

Corollary 1. A sufficient condition of the unique solvability of the network is the existence of a distinguished tree of the network graph (for example the network graph has exactly one normal, inverse normal, capacitance or inductance tree).

Corollary 2. A necessary and sufficient condition of the unique solution of RLC network is the existence of a normal tree of the network graph.

Knowing these theorems we can easily discuss the network models on Fig. 2 as follows.

Since the set $\{R_2, C\}$ is the only kernel of the network (a) $\{U, R_2, C\}$ is the only capacitance tree. According to Corollary 1, the network (a) has a unique solution. As deg $\{R_2, C\} = 2$ the order of the complexity is at least 2, in this case exactly 2. Taking into account that $\{U, R_2, C\}$ is normal tree as well, we have the possibility to choose the set of the independent state variables as it was mentioned in the introduction.

The network (b) has no kernel because, by Theorem 1, it has no a unique solution.

The network (c) has all kernels $\{R_3, R_2\}$ and $\{L, R_2\}$, the latter is an inductance, thus $\{U, L, R_2\}$ is the only inductance tree. By Corollary 1 it has a unique solution. Since $\{U, R_3, R_2\}$ is a normal tree the only possible state variable is the current of L. Really, we can choose the state variable mentioned earlier.

Topological conclusions

During the analysis procedure the following properties of the network graph are often used:

if the nullator-norator pairs RLC network has a unique solution, then

(a) both $U \cup A$ and $U \cup B$ is circuitless, and neither $I \cup A$ nor $I \cup B$ contains cutset;

(We remark that in case of the network consisting of relation elements and nullator-norator pairs the property (a) follows from [1] as well.)

(b) there always exists a disjoint decomposition of $R \cup L \cup C$ into K and $\bar{K}$ such that we can classify the RLC elements of the network graph in two manner as follows:

I. the elements of $U \cup A \cup K$ are branches and the elements of $I \cup B \cup \bar{K}$ are chords, and
II. the elements of $U \cup B \cup K$ are branches and the elements of $I \cup A \cup \bar{K}$ are chords,

i.e., making the classification in any of these two manners the same RLC elements correspond to branches and to chords.

The property (b) is used in [7] for the solution of the network equations system. The properties (a) and (b) immediately follow from Theory 1.

We note that the application of these theorems slightly modifies the view of the classical network analysis. In the classical case the main tool of the analysis is the normal tree. Because of Theorem 2, the normal tree is the most important subgraph for the analysis whenever the parameters of the network elements are general variables (general case, [6]). But when some algebraic relations hold among the network parameters (non-general case, [6]) then the inverse normal, reactance, distinguished trees of the graph come into prominence during the analysis. Later this fact will be shown by some examples.

Proofs of the theorems

First of all we are going to show that if the nullator-norator pairs RLC network has a unique solution then there exists a kernel of the network. For this, let $\mathrm{L} \subseteq L$ and $\mathrm{C} \subseteq C$ be arbitrary (possible empty) subsets. We introduce the notations for the sets of the network elements as follows:

$$\begin{aligned} D &= U \cup \mathrm{L} \cup \mathrm{C} \\ H &= I \cup \bar{\mathrm{L}} \cup \bar{\mathrm{C}} \\ E &= H \cup B \\ F &= D \cup B \end{aligned} \tag{1}$$

where the overscript "—" means the complementer, as earlier. Let us write the system of the network equations in the following form:

$$\mathfrak{R}\begin{array}{c} \\ \\ \left[\begin{array}{cccccccc} D^{u} & E^{u} & A^{u} & R^{u} & R^{i} & A^{i} & F^{i} & H^{i} \\ \hline \multicolumn{4}{|c|}{\mathfrak{B}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \multicolumn{4}{|c|}{\mathfrak{Q}} \\ \hdashline 0 & 0 & 0 & \boxed{\mathfrak{R}^{-1}} & \boxed{-1} & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 \end{array}\right]\end{array} \cdot \begin{bmatrix} u_D \\ u_E \\ u_A \\ u_R \\ i_R \\ i_A \\ i_F \\ i_H \end{bmatrix} = 0 \quad (2)$$

In (2) u_D, u_E, u_A and u_R are vectors, components of which are voltages of network elements, defined by (1) and by A and R, while i_R, i_A, i_F and i_H are vectors, components of which are the currents of the network elements defined similarly, $\mathfrak{B}$ and $\mathfrak{Q}$ are circuit and cutset matrices based on the same fundamental tree of the network graph (or shortly, matrices belonging to each other), $\mathfrak{R}$ is a diagonal matrix non-zero entries of which are the parameters of the elements of R. In (2) $\mathfrak{R}$ means the submatrix under the dashed line. Symbols D^u, E^u, A^u, R^u, R^i, A^i, F^i and H^i denote the sets of the columns of the matrix, respectively. These latter symbols are used to denote sets of the columns of $\mathfrak{B}$, $\mathfrak{Q}$ and $\mathfrak{R}$ as well.

Let $\mu(\mathfrak{B})$, $\mu(\mathfrak{Q})$ and $\mu(\mathfrak{R})$ denote the matrix matroids of $\mathfrak{B}$, $\mathfrak{Q}$ and $\mathfrak{R}$, respectively. Since $\mathfrak{B}$ and $\mathfrak{Q}$ belong to each other

$$\mu^*(\mathfrak{B}) = \mu(\mathfrak{Q}) \tag{3}$$

holds, where the star stands for the dual matroid.

Letting $b = E^u \cup A^u \cup R^u \cup R^i \cup A^i \cup F^i$ it follows from [6] that there exist $L \subseteq L$ and $C \subseteq C$ such that

$$\begin{gathered} b \text{ is the basis of the matroid} \\ ((G^u, \mu(\mathfrak{B})) \oplus (G^i, \mu(\mathfrak{Q})) \vee (G^u \cup G^i, \mu(\mathfrak{R})). \end{gathered} \tag{4}$$

In (4) G^u and G^i denote the sets of the columns of $\mathfrak{B}$ and $\mathfrak{Q}$, $G^u \cup G^i$ means the set of the columns of $\mathfrak{R}$, $\vee$ is the symbol of the sum, $\oplus$ of the direct sum of matroids. Further on we suppose that L and C in (2) are such that b defined above fulfils (4).

It follows from the definitions of $\mathfrak{B}$ and $\mathfrak{Q}$ that there exists (at least) one disjoint decomposition of b into b_1', b_1'' and b_2 such that

$$\begin{aligned} &b_1' \text{ is basis of } (G^u, \mu(\mathfrak{B})), \\ &b_1'' \text{ is basis of } (G^i, \mu(\mathfrak{Q})) \text{ and} \\ &b_2 \text{ is basis of } (G^u \cup G^i, \mu(\mathfrak{R})). \end{aligned} \tag{5}$$

It is clear, from (2) that $(G^u \cup G^i, \mu(\mathfrak{N}))$ is a graph matroid illustrated on Fig. 3.

Let $R \cup R$ be defined by the formula

$$\{R_j | R_j^u \in b_2 \cap R^u,\ R_j \in R\} \tag{6}$$

It immediately follows from (6) that $b_2 = R^u \cup \bar{R}^i \cup A^u \cup A^i$.

Taking into account (5) and (2) we obtain

$$b'_1 = \bar{R}^u \cup E^u \quad \text{and} \quad b''_1 = R^i \cup F^i. \tag{7}$$

Because of (1), we can write (7) in the form

$$b'_1 = \bar{R}^u \cup I^u \cup \bar{L}^u \cup \bar{C}^u \cup B^u \quad \text{and} \quad b''_1 = R^i \cup U^i \cup L^i \cup C^i \cup B^i. \tag{8}$$

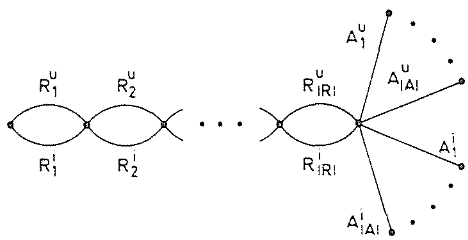


Fig. 3

Introducing the notation $K = R \cup L \cup C$, from the right-hand side of (8) it follows that $U \cup K \cup B$ is a complete tree while from the left-hand side of (8) $U \cup K \cup A$ is a complete tree of the network graph. With this the first part of the Theorem is proved. The second part is obvious (see [3]).

Studying the proof of Theorem 1 it comes to light that if we evaluate the determinant of the system equations (2) by Laplace theorem according to the rows of $\mathfrak{N}$, then we have a non-zero term. This term is the product of parameter reciprocals of those elements of R, with sign $+$ or $-$, which belong to the kernel K. It follows from this that if the parameters of the network elements are arbitrary (i.e. general variables) then the value of the determinant of (2) is a polinom of at least one term so it is possible the hibrid immittance description for the nullator-norator pairs network as well, according to [6]. Using the procedure given in [6] we can obtain the state equations system for the network in question, analogously. So we have the proof of Theorem 2.

If the parameters of the network are non-general variables, then the existence of the normal tree of the network graph does not guarantee the unique solvability. By all means, each kernel produces a term in the determinant of (2) which is the product of its resistor parameter reciprocals and the determinant is the algebraic sum of these products with respective all kernels of the network. If this sum is not zero (and this is already an algebraic relation among the parameters), the network has no unique solution, in general, because, for the unique solution, it needed the solution of the system of the hibrid immittance equations as well. This fact produces a further algebraic relation among the parameters, in which the parameters of the inductors and capacitors can occur.

To prove Theorem 3 let us assume the existence of the kernel K together with f_K generated by K differing from zero. Write the differential equations for the inductors and capacitors of the network in the form

$$u_L = \mathfrak{L}\dot{i}_L \qquad i_C = \mathfrak{C}\dot{u}_C \tag{9}$$

where u_L and u_C are the voltages while i_L and i_C are the currents of the inductance and the capacitance elements, respectively. $\mathfrak{L}$ and $\mathfrak{C}$ are diagonal matrices which consist of the parameters of the inductors and capacitors (mutual reactance elements are not allowed).

After Laplace transforming the equations (2) and (9) and a suitable permutating the columns of the matrices it can easily be seen that the system of the equations obtained for the Laplace transforms of the voltages and currents of the network elements in this way is similar to (2). At present the role of the matrix $\mathfrak{R}^{-1}$ is taken over by the diagonal matrix $\mathfrak{Y}$ non-zero entries of which are the operator admittances of the RLC elements. Taking into account the initial conditions the right-hand side of the system of the Laplace transform equations can differ from the zero vector, in general. Repeating the procedure applied in the proof of Theorem 1 we obtain that a kernel of the network produces an edge admittance product in the determinant of the Laplace transform system. More precisely, if r_i, l_j, and c_k are the parameters of R_i, L_j and C_k, then $K = R \cup L \cup C$ produces the following term in the determinant

$$\pm \frac{\prod_{C_k \in K} c_k}{\prod_{R_i \in K} r_i \cdot \prod_{L_j \in K} l_j} s^{\deg K - |L|} \tag{10}$$

where "s" denotes the complex frequency.

Taking into account all elements K_n of $M(K)$, we get that the coefficient of s of degree $\deg K - |L|$ can be written as follows:

$$\sum_{K_n \in M(K)} \pm \frac{\prod_{C_k \in K} c_k}{\prod_{R_i \in K} r_i \cdot \prod_{L_j \in K} l_j} \tag{11}$$

In (11) the sign of a term is determined by the value of the corresponding minors in $\mathfrak{B}$ and $\mathfrak{Q}$.

Fix the signs in (11) according to sgn K_n. Now (11) becomes f_K of non-zero value and, because of [5], the node admittance determinant of the network differs from zero. So it is possible the unique calculation of the node potentials of the network. Therefore the voltages of all network elements can be calculated uniquely, moreover the currents of the RLC elements are uniquely obtained. As $U \cup K \cup B$ is a complete tree of the network, the currents of the voltage generators and the norators are uniquely determined as well. By this the sufficiency of the condition of the theorem is proved. The necessity of the condition follows from this fact that the systems of (2) and of the node potential equations of the network have simultaneously a unique solution.

Observe that if K is distinguished, (11) has exactly one term, from which Corollary 1 follows. In case of RLC network the relative signs of the terms in (11) are the same, so Corollary 2 is also right.

Remark. During the proof of Theorem 3 we unspokenly assumed that all voltage generators of the network can be regarded as Thevenin generators. This assumption holds in case of practical networks, really. If it were still a voltage generator in the network model which is not a Thevenin one, connect a resistor of parameter r_0 in serial with it. For this modified network the earlier procedure can be performed and an algebraic relation can be obtained among r_0 and the original parameters. Letting $r_0 \to 0$ we may obtain a sufficient condition for the unique solvability.

We note that, by Theorem 3, many sufficient conditions can be produced of the uniquely solvability. The number of the possible conditions is exactly the number of the classes of M. For a concrete analysis we can choose an optimal condition from some kind of aspects. For example a condition which can be simply checked or which is the shortest, etc.

Applications

In this section the symbol of a network element also means its parameter.

Example 1. Let us consider the network with an (ideal) operational amplifier and its nullator-norator pairs model on Fig. 4. The network model has only one kernel (the resistance R_2) hence, by Theorem 3, the unique solution is guaranteed.

Example 2. Connect a serial RC term to the input and connect the inductor L to the output of the gyrator of parameter α, see in Fig. 5. Assume that the initial voltage of C is non-zero and examine whether the current of the inductor is uniquely determined.

The network model in question can be seen on Fig. 5 as well. As the number of the vertices of the network graph is 7 and the number of the nullator-norator pairs is 3 each kernel of the network consists of exactly three elements. Observe that R can not belong to a capacitance kernel thus the only capacitance kernel may be $\{C,\alpha,\alpha\}$ and, in fact, it is a kernel. It follows from Theorem 3 that the current of the inductor is uniquely determined.

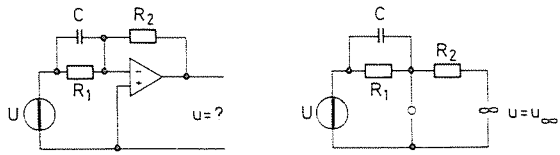


Fig. 4

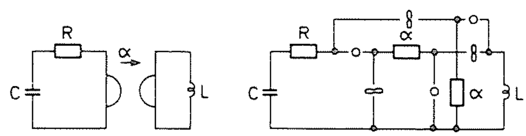
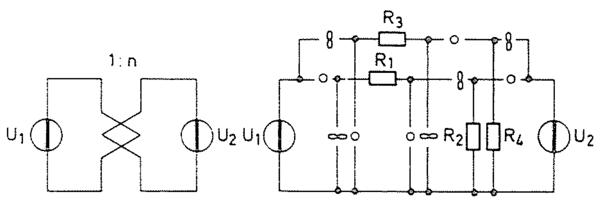


Fig. 5



$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = n$$

Fig. 6

Example 3. Connect the voltage generator U_1 and U_2 to the input and to the output of an ideal transformer illustrated in Fig. 6. Using the nullator-norator pairs model of the transformer appearing in [7] we obtain the network model, see Fig. 6. Because none of the resistors can belong to a kernel of the network, the network has no kernel therefore it can not be solved (see Theorem 1). If we exchange the voltage generator U_2 for a current generator, the only kernel $\{R_2,R_4\}$ of the network guarantees the unique solution. This example is a classical one of the network analysis. But it is not trivial that the relation among the parameters of the network elements defined by the ideal transformer does not influence the discussion and that the obtained result can be extended for negative impedance converters, too.

Example 4. The network model on Fig. 7 has no unique solution because of a set of the norators (nullators) is a cutset of the network graph (see the topological conclusion (a)). It immediately follows from the concrete analysis that the voltages of the norators are undetermined but their sum is $U\frac{R_2}{R_1}$ in any case. However the voltages and the currents of all further network elements are uniquely determined. If we complete the network by the element Z according to Fig. 7 we get a new network which has a unique solution. The original network can be regarded as the model of a controlled generator introduced in [2] the input of which is isolated from the output.

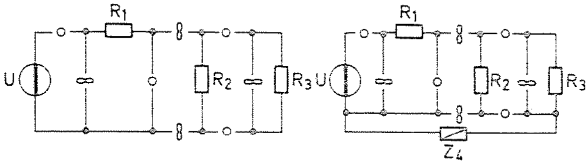


Fig. 7

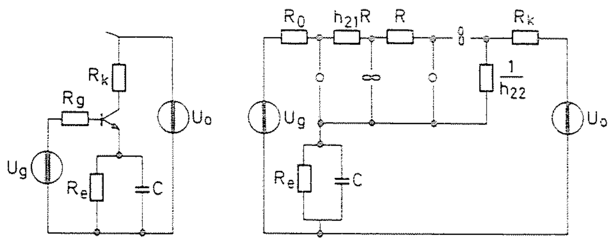


Fig. 8

Example 5. Consider the transistor amplifier network on Fig. 8. Taking into account the nullator-norator pairs model of hybrid parameters of the transistor we can construct the model of the practical network, see Fig. 8. Following [7] we assume that $h_{12}=0$ and use the notation $R_o=R_g+h_{11}$.

The network graph has no distinguished tree. But we can easily find all of its capacitance kernel. It is trivial that each kernel has 3 elements. After choosing C we can not choose R_e and R_o as the elements of the kernel, so we take into consideration only $h_{21}R$, R, R_k and $\frac{1}{h_{22}}$. Because of the situation of the norators, the pairs $\{R, R_k\}$ and $\left\{R, \frac{1}{h_{22}}\right\}$ can not be choosen moreover the pairs $\{R, h_{21}R\}$ and $\left\{R_k, \frac{1}{h_{22}}\right\}$ are also excluded because of the nullators and the voltage generator U_o, respectively. Thus the possible kernels are $\{C, h_{21}R, R_k\}$ and $\left\{C, h_{21}R, \frac{1}{h_{22}}\right\}$ and, really, they are all the capacitance kernels. These two kernels form a class and the formula generated by them is $\frac{C}{h_{21}RR_k}+\frac{Ch_{22}}{h_{21}R}$. By Theorem 3 the network has a unique solution unless $1+h_{22}R_k=0$ holds.

Example 6. Consider the network with a current generator controlled by a current on Fig. 9 (see also in [6]). The nullator-norator pairs network model constructed by using [7] is on the right-hand side of Fig. 9.

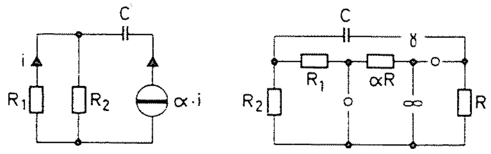


Fig. 9

As C is connected in serial with a norator any of the kernels must contain C. Other elements of a kernel form a pair of the resistor elements. Therefore we can consider six pairs of elements. Now $\{R_1, R_2\}$ and $\{R, \alpha R\}$ can not be suitable pairs for the kernel because of the nullators, while $\{R_2, R\}$ must be excluded because of a norator. We obtain $\{R_1, R, C\}$, $\{R_1, \alpha R, C\}$ and $\{R_2, \alpha R, C\}$ as possible kernels, and they are kernels, in fact. As these kernels form a class, the network has a unique solution unless

$$-\frac{C}{R_1R}+\frac{C}{R_1\alpha R}+\frac{C}{R_2\alpha R}=0 \text{ holds, i.e. unless}$$

$\alpha=1+\frac{R_1}{R_2}$ holds. At present this condition is also necessary because these kernels are all kernels of the network.

Example 7. Consider the network on Fig. 10 consisting of 3 operational amplifiers together with its nullator-norator pairs model. It is clear, that the number of the elements of each kernel is 3. Since there can not exist kernels containing either the two capacitors or any two of the inductors the degree of a kernel may be 2, 3 or 4.

Let us try to produce all kernels of degree 3. It deg $K=3$ then either K consists of purely resistors or it is mixed, i.e. contains a resistor an inductor and a capacitor. K can not contain R_1 and R_9 simultaneously because of the norators and $\{R_3, R_4\}\not\subset K$ because of the nullators, so the first case is impossible. It remained the mixed case. From the situation of the inductors, $C_6\notin K$ follows therefore only $C_2\in K$ may hold. Observing that after choosing C_2 only the resistor R_4 can be choosen we obtain that the only 3-degree kernel is $\{C_2, R_4, L_8\}$. As this kernel is distinguished, the network has a unique solution.

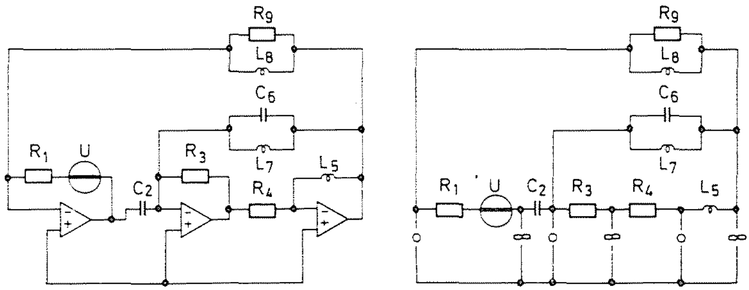


Fig. 10

Computer implementation

The examples given in the earlier section show the surprising fact that it is possible to examine the unique solvability of the network in a purely combinatorical way. However, in case of a more complicated network this discussion may be very difficult. The problems are to decide whether there exists a kernel of the network, to give a sufficient condition for the solvability and to choose a normal tree for the numerical analysis.

By all means, the examination of the unique solvability is a combinatorical procedure and we propose the block scheme on Fig. 11 for constructing a complex computer program of this procedure. By such a program, all kernels of the network can be produced. If the set of the kernels is empty then the network has no unique solution (see Theorem 1). Otherwise let $M(K)$ be a class represented by K and f_K the formula generated by K. Because of Theorem 3 the network has a unique solution unless $f_K=0$. At the end of the program a sufficient condition is obtained together with a normal tree of the network for the numerical analysis.

The kernels of the network are sourced by a tree generation method given in [4]. First all the complete trees F_i of $G\setminus(I\cup B)$ containing all the nullators and voltage generators are produced, where G means the network graph. F_i determines a kernel if the graph G^i_∞ is circuitless where G^i_∞ arises from $F_i\setminus A$ after collapsing the endpoints of all norators. The procedure can be followed in the left-hand side of Fig. 11.

The rest of the procedure is summarized on the other side of Fig. 11. Namely, the set of the kernels generated earlier is decomposed into the classes according to the degrees of the kernels. Now one of the maximum kernels is chosen, denoted by $K_{\sigma\max}$, for a normal tree. According to [5] the relative sign of each kernel of $M(K)=M_{\sigma_0}$ with minimum $|M_{\sigma_0}|$ is determined. The network has a unique solution unless $f_K=0$ and $U\cup K_{\sigma\max}$ is a possible normal tree for the analysis.

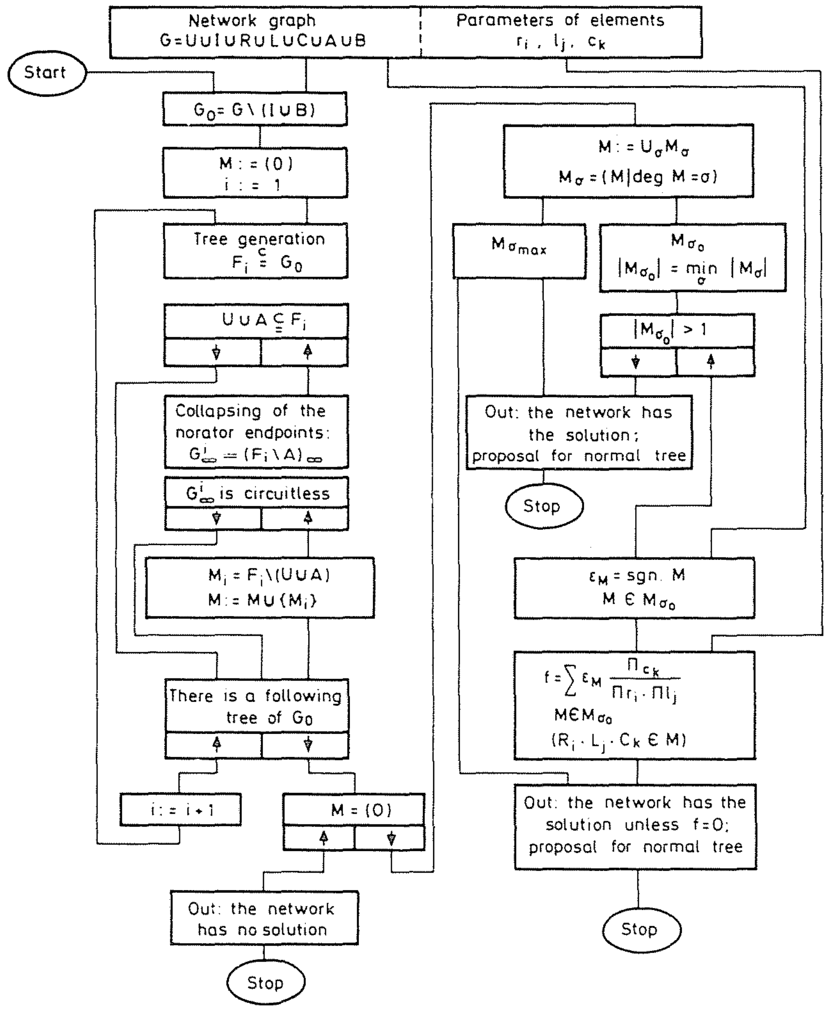


Fig. 11

We remark if one of $M(K)$s has only one element then the network has a unique solution independently of the parameters (see Corollary 1). This case is also handled in the block scheme. Observe that the program in question produces a shortest formula for a sufficient condition of the unique solution. Of course, it is possible to obtain further conditions mentioned earlier for the practical analysis. The check of the condition generated in this way is not a task of this program. Finally we note that this program requires the dates (i.e. the network parameters) only at the end of the procedure so the program can be regarded as the first step of a full numerical analysis of the nullator-norator pairs RLC network.

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Dr. Imre PÁVÓ H-6720 Szeged Somogyi u. 7.