A NEW PWM INVERTER WITH MINIMUM TOTAL HARMONIC DISTORTION

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Received June 1, 1987
Presented by Prof. Dr. Gyula Retter

Abstract

Drives with a wide range of variable speed using induction motors have various applications. By the great progress in microelectronics today digital control of such drives became a reality. PWM inverter is one of the best techniques used in power inverters to obtain controllable ac output.

There are two main methods of PWM signal generation either conventional (analogue) or digital. Many authors obtained optimum PWM signal with optimum harmonic content while other proposed PWM signal by eliminating low order harmonics.

This paper introduces new PWM inverter signals based on binary programming as a good method for optimal PWM inverter.

By comparing the proposed signal with other optimum signals presented by different authors we found that the proposed one has superior feature than the others. The comparison based on computing the total harmonic distorion (T.H.D.) for most well known techniques and between the proposed one, taking into consideration the number of commutation per half cycle "N". The comparison shows that the proposed technique give minimum T.H.D.

Introduction

With the rapid progress of micro- and power electronics technology and commercially available high rating power transistors, all these changes make the PWM inverter a powerful system among the modern electrical drives. It has been shown by several authors that the distortion minimization (DM) strategy demonstrates the best harmonic loss performance of any PWM strategy for a given inverter switching frequency. The DM strategy offers the possibility of optimum motor and drive performance. Perhaps the most obvious problem associated with the application of the optimum DM technique is the mathematical derivation of optimum switching angles for a desired fundamental voltage and the corresponding harmonics amplitudes.

By using conventional PWM inverters, the output waveform contains some harmonics. Therefore their use is always accompanied by parasitic losses and acoustic noise problems.

Various authors discussed general methods for eliminating up to the n-th harmonics [1]—[2]. On the other hand the elimination of low order harmonics largely increased the presence of high order harmonics.

Recently digital techniques can produce very precise waveforms without any offset or drift problems encountered in analogue-based system [3].

In this paper a new PWM signal is introduced based on binary programming which is a very good tool for distortion minimization of both single phase and three phase inverter waveforms [4].

The proposed PWM signal has superior feature than the other PWM signals obtained by using either harmonic elimination of low order ones or by using conventional techniques. This signal has the advantage of minimum total harmonic distortion compared with the other well known techniques such digital and loss optimal PWM signals [5]—[6].

Distortion minimization strategy

In PWM inverter induction motor drives, the distortion minimization modulation strategy offers the possibility of optimum inverter and motor performance. There are various techniques to determine the optimum switching angles for a desired PWM signal.

Buja et al. [7] minimize harmonic loss factor as a function of the switching angles for a particular waveform, subjected to the constraint condition of a constant fundamental voltage. The optimization is implemented for waveforms with up to four switching per quarter cycle. They assume that the machine parameters are frequency independent and derive a simple harmonic distortion criterion which minimizes the range of fundamental voltages from zero up to the maximum six step value of $2V_{dc}/\pi$. The analysis is based on an integration of the inverter output voltage waveform to give an approximate current waveform. This current waveform approach then yields an analytical expression for the distortion factor in the single-phase case. However in the three-phase problem, they are forced to use a numerical integration procedure. Casteel et al. [8] use z transform techniques and numerical integration of the analytical derived current waveform to solve the problem of DM in case of resistive/inductive (R/L) loads. They show that the effect of load resistance make it necessary to alter the switching angles slightly from those given in Buja but the difference is only significant for large R/L ratios.

Results are presented for five switchings per quarter cycle but the solutions given are not truly optimum since the resultant DM curve does not possess the best overall loss factor over the full range of fundamental voltages.

Halász [9] has applied the Park vector approach, to the solution of the DM problem and attempts to optimize the locus of the machine flux vector so as to minimize harmonic losses. His results indicate that increased rotor losses due to skin effect do not significantly affect the optimum switching angles. De Buck et. al [10] use an indirect method to minimize a complex mathematical expression for loss factor which takes both skin effect and iron losses into account.

The expression for the loss factor is non-algebraic and the minimization, using the steepest hill gradient procedure is complicated and complex. Solutions for up to seven switchings per quarter cycle are presented. De Carli et al. [11] have used an integration method to evaluate the rms value of the inverter output current and hence estimate a distortion factor which is minimized by using a gradient technique.

Murphy et al. [12] show that the loss factor σ, as defined by Buja and Indri in [7], represents a good engineering approximation to overall machine harmonic losses. This loss factor has the advantage that it is determined uniquely from the PWM voltage waveform spectrum.

Recently a new analytical approach based on loss factor has been presented in [13]. This technique does not involve either numerical integration or the computation of large Fourier series expressions.

It's advantage is that it is direct, accurate and requires only the use of simple matrix operations. It is thus possible to implement this technique on small computer systems.

A novel advantage of the method is that it is possible to apply the same system equations both for a single-phase and three phase case as well.

Proposed PWM signal

By using binary programming [14] we check in each interval the value of the PWM signal if it is zero or unity subject to the following constraints:

— the fundamental harmonic (V_1) has to be maximum

— the harmonics from 3rd till 13th related to the fundamental are to be minimized ($\varepsilon_k = V_{2k+1}/V_1$)

There are two main procedures either for resistive load where

$$\varepsilon_k = \varepsilon = \text{const}$$

or for inductive load where

$$\varepsilon_k = (2k+1)\varepsilon, \quad \varepsilon = \text{const}$$

Since PWM signal is generally a periodic function for every 2π, so we divide this interval into 4S equal subintervals the width of each is ($\pi/2S$).

In this work we carry out this procedure for single phase PWM signals. Therefore we take $S = 2^J$ where J is an integer number to simplify the digital generation of PWM pulses. Therefore S takes the values 32 and 64, i.e. each subinterval is 1.406 25° and 0.703 125° respectively. Tables (1) and (2) show the results for resistive and inductive load conditions with different values of ε_k from arbitrary high values till the optimum solution.

Table 1

Resistive Load

S	32				64	
ε%	2.7	3	4	8	2.7	2.8
K \ N	13	13	11	13	15	15
3	4.518	0.921	2.127	8.9	2.64	1.721
5	2.47	0.384	3.089	2.65	0.776	0.63
7	1.53	2.128	3.858	0.19	2.294	1.909
9	1.53	2.63	0.83	9.0	0.998	1.708
11	2.97	1.064	2.049	6.67	1.989	0.1504
13	4.43	1.499	1.715	3.4	1.02	1.977
15	17.55	18.451	2.047	9.515	3.423	0.902
17	3.12	6.49	2.956	11.85	14.93	2.74
19	16.69	30.961	1.269	2.18	13.23	13.628
21	16.686	18.955	8.216	12.59	5.757	19.411
23	27.479	25.81	12.784	15.716	8.777	10.925
25	26.554	24.13	9.864	10.189	12.014	13.48
27	4.053	1.917	8.2	13.579	6.451	7.386
29	3.199	2.705	7.12	3.978	3.223	12.629
31	3.207	0.146	0.584	0.63	8.383	4.151
33	3.106	2.064	3.164	14.27	0.803	14.597
35	1.712	6.378	6.285	5.42	11.779	16.014
37	7.186	6.493	16.324	7.949	4.212	13.893
39	15.025	14.249	7.909	2.77	4.293	7.407
T. H. D.	0.5216	0.5714	0.2963	0.3634	0.3131	0.4282

To prove that the obtained PWM signal has lower harmonic content, comparison was made with the most popular PWM techniques. In conventional PWM techniques the harmonic content depends on the frequency ratio (m = carrier frequency/reference frequency). Calculation was made for T.H.D. for each technique up to the 39th harmonic to cover all the interesting harmonics ($m+2$, $2m+1$, $3m+2$, . . .) taking into consideration the number of commutations per half cycle "N".

where

$$\text{T.H.D.} = \frac{1}{v_1}\left(\sum_{k=2}^{39} (\text{kth harmonic component})^2\right)^{\frac{1}{2}}$$

Tables 3, 4 and 5 show the results of T.H.D. for natural sampling [15], regular sampling [16] (for different frequency ratio and modulation index "$A=1$") and loss optimal signal [10] (four cases) and optimum digital PWM signals [5] respectively.

Table 2
Inductive Load

S	32	64	
ε%	0.8	0.25	0.35
K \ N	9	15	13
3	0.3707	0.1825	0.1232
5	1.025	0.6835	0.7683
7	2.8689	1.0776	0.9923
9	2.6468	2.2264	0.6941
11	4.8659	1.4679	0.5601
13	8.5411	2.0356	2.6273
15	6.0179	0.9718	5.4626
17	9.4305	9.3189	3.4297
19	11.6639	3.0085	20.9735
21	25.2792	2.961	16.4052
23	15.5818	18.3697	5.2456
25	2.2959	11.1811	14.384
27	4.3862	12.6624	4.5112
29	8.4037	10.599	11.6566
31	6.206	7.1292	10.0066
33	0.4464	12.5104	10.414
35	1.5449	8.4692	22.7417
37	0.3369	1.966	5.0098
39	2.5134	11.0772	1.8593
T. H. D	0.3739	0.3548	0.4364

Table 3
Natural sampling
A = 1

k \ m	11	13	15	17	19	21
3						
5						
7	1.716					
9	31.29	1.716				
11		31.29	1.716			
13	31.30		31.29	1.716		
15	1.92	31.29		31.29	1.716	
17	3.22	1.72	31.29		31.29	1,716
19		24.56	1.716	31.29		31.29
21	18.906	3.18		1.716	31.29	
23	18.87		0,2		1.716	31.29
25		18.9	3.18			1.716
27	0.997	18.9		0.2		
29	15.32		18.9	3.18		
31	6.89	2.69	18.9		0.2	
33	0.09			18.9	3.18	0.2
35	7.76	15.52	3.15	18.9		0.2
37	20.344	6.89	0.29		18.9	3.18
39	16.			3.18	18.9	
T. H. D.	0.608	0.598	0.519	0.519	0.519	0.444

Table 4

Regular sampling
A = 1

K \ m	9	12	14	15	18	21
3	0.0	0.0	0.469			0.2
5	0.3637					
7	27.020					
9	0.0	0.0				
10		28.356	0.69			
11	34.97			0.745		
			28.9			
13	3.579			29.112		
14		34.339				
15	17.884					
16			34.03		29.598	
17	23.735			33.902		0.985
18			3.49			
19	13.022			3.358		29.9
20			0.206		33.59	
21	0.0	0.0	0.04			
23	20.82	22.305	1.74			33.343
25	13.26	14.205	19.54	1.825		2.846
27	0.0	0.0	21.689			0.12
29	1.156		14.748	21.445		
31	3.283			14.967		
33	0.0	0.0	5.396			
34		10.53	0.1509			
35	10.771		0.693	5.25	20.877	0.076
36		11.307				
37	2.192			0.8146	15.48	20.19
38		2.236	14.975			
39	0.0			2.71		
T. H. D	0.61324	0.5409	0.5773	0.5226	0.5176	0.4922

PWM signal with minimum T.H.D.

From tables 1, 2, 3, 4 and 5 we pointed out the best signals which have minimum T.H.D. for each technique and put them into table 6.

It is obvious from table 6, that we obtain the best signal for resistive load when $S=32$, $\varepsilon\%=4$, $N=11$ and T.H.D. $=0.296$.

For inductive load we got the best signals which have minimum T.H.D. (0.354) at $S=64$, $\varepsilon\%=0.25$, and $N=15$. Both of these signals are shown in Fig. 1. for $S=32$ and 64.

The other PWM techniques mentioned in table 6 ensure only 0.5—1.2 T.H.D. values which are really far from the T.H.D. value of the proposed signals. Therefore the proposed signals are competitive to the other well known optimal techniques, for the same number of commutation.

Table 5

K \ N	Loss optimal (10) 5	5	5	9	Optimum Digital (5) 10
3					5.826
5				6	1.185
7			28	14	1.752
9					5.897
11	36	84	28	18	1.52
13	52	80		8	1.849
15					2.17
17	26	18	38	36	6.543
19	6	12	38	31	2.804
21					2.457
23	34	20	16	24	7.094
25	6	22	22	37	13.76
27					14.993
29	14	20	20	12	2.957
31	22	4	4	25	24.106
33					14.436
35	5	18	14	14	1.97
37	2	3	13	22	0.883
39					3.376
T. H. D	0.8131	1.24	0.7731	0.795	0.3767

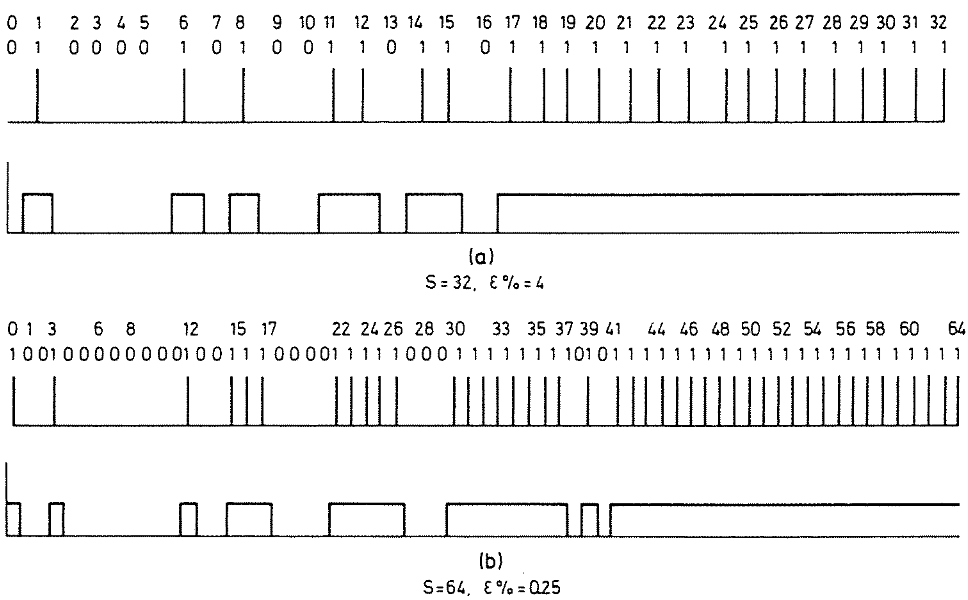

Fig. 1. Proposed PWM signals

Table 6

Harmonic order k	Natural Sampling		Regular Sampling		Optimum Digital inverter (5)	Loss optimal (10)		Proposed PWM signals			
								S 32	64	32	64
								$\varepsilon\%$ 4	2.7	0.8	0.25
	N: 15	21	15	21	10	9	5	11	15	9	15
3				0.2	5.83			2.13	2.64	0.37	0.183
5					1.18	6		3.09	0.78	1.03	0.68
7					1.75	14	28	3.8	2.29	2.87	1.08
9					5.90			0.88	0.99	2.65	2.2
11	1.716		0.74		1.52	18	38	2.05	1.98	4.86	1.47
13	31.29		29.11		1.85	8		1.72	1.02	8.54	2.03
15					2.17			2.05	3.42	6.02	0.97
17	31.29	1.716	33.9	0.98	6.54	36	38	2.96	14.92	9.4	9.3
19	1.716	31.29	3.36	29.9	2.80	31	38	1.27	13.23	11.66	3
21					2.46			8.22	5.76	25.28	2.69
23	0.2	31.29		33.34	7.09	24	16	12.78	8.78	15.58	18.37
25	3.18	1.716	1.83	2.84	13.76	37	22	9.86	12.00	2.3	11.18
27				0.12	14.99			8.2	6.4	4.39	12.66
29	18.9		21.4		2.96	12	20	7.12	3.2	8.4	10.6
31	18.9		14.96		24.10	25	4	0.58	8.4	6.2	7.13
33		0.2			14.4			3.16	0.8	0.44	12.5
35	3.15	0.2	5.25	0.076	1.97	14	14	6.29	11.78	1.5	8.47
37	0.29	3.18	0.81	2.19	0.88	22	13	16.32	4.2	0.33	1.69
39			2.71		3.38			7.9	4.29	2.5	11.07
T. H. D	0.519	0.444	0.52	0.49	0.376	0.795	0.77	0.296	0.31	0.374	0.354

Conclusion

From the results of the investigations of natural sampling, regular sampling, other optimum techniques and from the proposed PWM signals taking into account the number of commutations per half cycle, we can drawn the consequences that the proposed PWM signal give a better results than the other techniques. Therefore these proposed signals give high quality performance for PWM inverter fed induction motor drives. Such drives have the advantage of increasing economical properties.

Acknowledgement

The author would like to acknowledge Prof. Sándor Halász Budapest Technical University for his valuable suggestions and useful discussions during this work.

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