NON-NEWTONIAN LUBRICANTS IN JOURNAL BEARINGS

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Abstract

For moderately loaded bearings, we show how the viscometric functions are introduced, most simply, into the methods of hydrodynamic theory of lubrication. The method of irreversible thermodynamics is proposed to estimate the viscometric functions and some relations between them.

Introduction

In the rapidly developing lubrication technique, more and more modern lubricants are used, which can not be regarded as Newtonian fluids. On the other hand, market requires designers to pay more and more attention to the dynamic behaviours of lubricants. One of the obstacles of these efforts is that the lubrication theory is based mainly on Newtonian fluids, and the theory of non-Newtonian lubricants is not developed enough. One of the difficulties of the development of the latter is the great variety of the present theories for non-Newtonian fluids making the process of finding the way uncertain. On the base of physical, mechanical or other considerations, a lot of different and hardly comparable material equations have been proposed, which give account, more or less correctly, of the behaviour of some non-Newtonian fluids. Their quantitative confidence is problematic, because they deviate from each other and no principle has been known to decide what method leads to goal in a practical case. We often do not know how the conditions of these or those abstract physical methods are satisfied in the case of a particular lubricant. The next difficulty is that the material equations motivated by one or other theory are too complicated to be employed directly. So it is necessary to bring some simplification, most conveniently, those customary in the hydrodynamic theory of lubrication. At the complication of the theoretical and practical problems, any general principle is of real help, even if it means a little step toward the complete solution of the mentioned problem. The theory of viscometric flow of simple fluids gives us a general principle [1]. The success of the simplification in the hydrodynamic theory of lubrication assures applicability for the viscometric functions. From practical point of view, non-equilibrium thermodynamics also gives help when estimating possible forms of the viscometric functions. The linear Onsager, within its limits, offers a relatively simple mathematical approximation of viscometric functions. Furthermore, it is convenient that the application of thermodynamic methods does not require deep, thorough knowledge of the molecular structure of the lubricant at hand. In the following, we would like to show that the application of these two methods promises considerable results, at least in the case of moderately loaded bearings. We show that in the case of 360-rotation-angle long hydrodynamical bearings, the normal stress functions play no role in calculations. Moreover, for moderate loads, the theory based on Newtonian lubricants can be employed directly, only the viscosity should be changed to differential viscosity, and the well known results change but little.

The flow in the clearance

At the first approximation, the sliding pair is considered parallel and plane. One of them slides on the other and the relative speed is parallel to them. Choose a Cartesian coordinate system, the z-axis perpendicular to the planes, the origin on the plane in rest, the x-axis directed with the velocity of the sliding plane. We have the velocity field for the lubricant in vector form:

$$\mathbf{V} = u(z)\mathbf{i} + v(z)\mathbf{j} \tag{1}$$

or in coordinates:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= u(z) \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= v(z) \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= 0 \end{aligned} \tag{1a}$$

The boundary conditions read:

$$\begin{aligned} u(0) &= 0 \qquad u(d) = u_0 \\ v(0) &= 0 \qquad v(d) = 0, \end{aligned} \tag{2}$$

where d is the width of the clearance, u_0 the relative velocity.

The function $v(z)$ is not equal to zero, because of the pressure gradient. In the case of long bearings, however, it can be neglected. Now we show that the flow described by Eq. (1) is viscometric. To do this, we determine the trajectory functions of a particle. We consider the Eq. (1a) as differential equations. Their solution is:

$$\begin{aligned} x &= x_0 + u(z_0)(t - t_0) \\ y &= y_0 + v(z_0)(t - t_0) \\ z &= z_0 \end{aligned} \tag{3}$$

where x_0, y_0, z_0 are the coordinates of a particle at time t_0. The components of a vector joining two particles near each other in time t_0 are obtained in the form:

$$\begin{aligned} \mathrm{d}x &= \mathrm{d}x_0 + \frac{\mathrm{d}u}{\mathrm{d}z_0}(t - t_0)\mathrm{d}z_0 \\ \mathrm{d}y &= \mathrm{d}y_0 + \frac{\mathrm{d}v}{\mathrm{d}z_0}(t - t_0)\,\mathrm{d}z_0 \\ \mathrm{d}z &= \mathrm{d}z_0 \end{aligned} \tag{4}$$

From (4), we get the particular form of the deformation gradient tensor:

$$\boldsymbol{x} = \boldsymbol{\delta} + (t - t_0)\left(\frac{\mathrm{d}u}{\mathrm{d}z_0}\mathbf{i} + \frac{\mathrm{d}v}{\mathrm{d}z_0}\mathbf{j}\right)\mathrm{o}\mathbf{k} \tag{5}$$

where 'o' stands for dyadic product.

Introducing the quantities:

$$\varkappa^2 = \left(\frac{\mathrm{d}u}{\mathrm{d}z_0}\right)^2 + \left(\frac{\mathrm{d}v}{\mathrm{d}z_0}\right)^2 \qquad \mathrm{e} = \frac{1}{\varkappa}\left(\frac{\mathrm{d}u}{\mathrm{d}z_0}\mathbf{i} + \frac{\mathrm{d}v}{\mathrm{d}z_0}\mathbf{j}\right) \tag{6}$$

the deformation gradient tensor is casted into the form:

$$\boldsymbol{x} = \boldsymbol{\delta} + \varkappa(t - t_0)\ \mathbf{e}\ \mathrm{o}\ \mathbf{k} \tag{7}$$

Because e and $\mathbf{k}$ are perpendicular to each other, we can see that the velocity field (1) describes a viscometric flow. Hence the stress tensor is given in the form [1, 4]:

$$\boldsymbol{t} = -\left(p + \frac{\sigma_1(\varkappa) + \sigma_2(\varkappa)}{2}\right)\delta + \tau(\varkappa)(\mathbf{e}\,\mathrm{o}\,\mathbf{k} + \mathbf{k}\,\mathrm{o}\,\mathbf{e}) + \sigma_1(\varkappa)\mathbf{e}\,\mathrm{o}\,\mathbf{e} + \sigma_2(\varkappa)\mathbf{k}\,\mathrm{o}\,\mathbf{k} \tag{8}$$

Here $\tau(\varkappa)$ is the shear stress, $\sigma_1(\varkappa)$ and $\sigma_2(\varkappa)$ the normal stress differences and p the pressure.

The form of the stress tensor becomes considerably simple if Onsager's linear laws hold. This case

$$\sigma_1(\varkappa) + \sigma_2(\varkappa) = 0 \tag{9}$$

so

$$\boldsymbol{t} = -p\boldsymbol{\delta} + \tau(\varkappa)(\mathbf{e}\,\mathrm{o}\,\mathbf{k} + \mathbf{k}\,\mathrm{o}\,\mathbf{e}) + \sigma_1(\varkappa)(\mathbf{e}\,\mathrm{o}\,\mathbf{e} - \mathbf{k}\,\mathrm{o}\,\mathbf{k}) \tag{10}$$

It is worth expressing (10) in a matrix form:

$$t=\begin{bmatrix} \dfrac{u'^2}{\varkappa^2}\sigma_1-p & \dfrac{u'v'}{\varkappa^2}\sigma_1 & \dfrac{u'}{\varkappa}\tau \\ \dfrac{u'v'}{\varkappa^2}\sigma_1 & \dfrac{v'^2}{\varkappa^2}\sigma_1-p & \dfrac{y'}{\varkappa}\tau \\ \dfrac{u'}{\varkappa}\tau & \dfrac{v'}{\varkappa}\tau & -\sigma_1-p \end{bmatrix} \tag{10a}$$

here we used the symbols:

$$u'=\frac{du}{dz_0}, \qquad v'=\frac{dv}{dz_0} \tag{11}$$

The dynamic equations

For motions with velocity field (1) and without body forces, Cauchy's equation of motion reads:

$$\text{Div } t=0 \tag{12}$$

Notice that all the quantities depend on z only, except p. Applying it and substituting the expression (10) for the stress into Cauchy's equation, we get:

$$\begin{gathered} -\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left[\frac{u'}{\varkappa}\tau(\varkappa)\right]=0 \\ -\frac{\partial p}{\partial y}+\frac{\partial}{\partial z}\left[\frac{v'}{\varkappa}\tau(\varkappa)\right]=0 \\ -\frac{\partial p}{\partial z}-\frac{\partial \sigma_1}{\partial z}=0 \end{gathered} \tag{13}$$

This set of equations can not be solved in general, it is rather difficult even if the viscometric functions are known. The third equation, however, can be integrated readily and we get:

$$p+\sigma_1=p^*(x, y) \tag{14}$$

Next we introduce the quantity p^* into the other equations. As

$$\frac{\partial p}{\partial x}=\frac{\partial p^*}{\partial x} \quad \text{and} \quad \frac{\partial p}{\partial y}=\frac{\partial p^*}{\partial y}$$

the equations (13) are reformulated as

$$\begin{aligned} -\frac{\partial p^*}{\partial x}+\frac{\partial}{\partial z}\left[\frac{u'}{\varkappa}\tau(\varkappa)\right]=0 \\ -\frac{\partial p^*}{\partial y}+\frac{\partial}{\partial z}\left[\frac{v'}{\varkappa}\tau(\varkappa)\right]=0 \end{aligned} \tag{15}$$

From Eq. (10a), we calculate the force acting upon the surface of the clearance. As the normal vector is parallel to the z-axis,

$$\mathbf{dF}=\boldsymbol{t}(\mathrm{d}A\ \mathbf{k}), \tag{16}$$

or in coordinates:

$$\begin{aligned} \mathrm{d}F_x &= \frac{u'}{\varkappa}\tau\mathrm{d}A \\ \mathrm{d}F_y &= \frac{v'}{\varkappa}\tau\mathrm{d}A \\ \mathrm{d}F_z &= -(\sigma_1+p)\mathrm{d}A=-p^*\mathrm{d}A \end{aligned} \tag{16a}$$

Because p^* does not depend on z and the Eq. (15) and (16) contain only p^*, we have the equations to be applied, and they do not contain the normal stress difference functions.

Moderately loaded long bearings

In the case of the moderately loaded long bearings the system of Eq. (15) gets simpler. The condition $v(z)=0$ results in equality $u'=\varkappa$ and p^* depends only on x:

$$\begin{aligned} -\frac{\partial p^*}{\partial x}+\frac{\partial}{\partial z}\left[\tau(\varkappa)\right]=0 \\ \frac{\partial p^*}{\partial y}=\frac{\partial p^*}{\partial z}=0 \end{aligned} \tag{17}$$

For calculation, the correct knowledge of function $\tau(\varkappa)$ is dispensable, because the journal is situated almost co-axially, if the load is small and so the shear rate is near an average.

Here, of course, we suppose that no severe pressure gradient is formed.

The function $\tau(\varkappa)$ is replaced by a linear approximation:

$$\tau(\varkappa)=\tau(\varkappa_0)+\left.\frac{\mathrm{d}\tau}{\mathrm{d}\varkappa}\right|_{\varkappa_0}(\varkappa-\varkappa_0) \tag{18}$$

Introducing the notation:

$$\left.\frac{\mathrm{d}\tau}{\mathrm{d}\varkappa}\right|_{\varkappa_0}=\eta$$

and substituting it into (17), we obtain:

$$-\frac{\partial p^*}{\partial x}+\frac{\partial}{\partial z}(\eta\varkappa)=0 \tag{19}$$

This equation, formally, is the same as that for the Newtonian lubricants so we can apply the results of classical hydrodynamic lubrication theory. The only difference is that the relation

$$\tau=\eta\varkappa$$

valid for Newtonian fluids has to be replaced by:

$$\tau=\tau_0+\eta\varkappa_0 \tag{20}$$

where

$$\tau_0=\tau(\varkappa_0)-\eta(\varkappa_0) \tag{21}$$

It means that in the evaluation of drag, an additional term not depending on the load appears.

Conclusion

In the outlined train of ideas it is shown how to use the results of the classical hydrodynamic lubrication theory for non-Newtonian lubricants. Of course, the approximation given by Eq. (20) is rough and it is useful only within restricted range. Nevertheless, we believe that the approximation is worth, because it is a first step on the way to go along, sooner or later. A better approximation, however, involves a reformulation of the classical formulae and a rather hard and careful work starts with the choice of the shear stress function and goes on with the necessary but proper neglections. We hope that the considerations based on formulate (20) will prove to be helpful.

Lastly, we mention that the parameters τ_0 and η in Eq. (20) are not material constants but functions of $\varkappa_0$ and they change with the speed of the bearing. This gives the designer an opportunity to determine the most convenient shear stress function varying properly with run. We think that our considerations contribute to a better design of lubricants.

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