THE WEIGHTED RESIDUAL METHOD FOR WAVE PROPAGATION IN A ONE-DIMENSIONAL INHOMOGENEOUS MEDIUM

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Abstract

In this article the propagation of plane-waves in a one-dimensional inhomogeneous medium with arbitrarily varying material constant is examined by the weighted residual method. After formulating the boundary value problem the weighted residual method is applied and the resulting equation is solved numerically using the finite element method. At the end of the article a practical example, namely the reflectivity of an inhomogeneous wave absorber, is worked out and its results are presented in forms of graphs.

Introduction

Many practical media of physical interest can be described by a model containing a one-dimensional inhomogeneous slab. Such practical problems are the propagation of an electromagnetic wave through the ionosphere [1], the transmission and reflection properties of the plasma sheat [2, 3], the analysis and synthesis of nonuniform transmission lines [4], or the examination of properties of inhomogeneous wave absorbers.

Because, only a few profiles can have closed-form analytic solution, several approximate methods have been developed for analyzing the wave propagation problem for one-dimensional inhomogeneous media. Such approximate methods are, among others, Wentzel–Kramers–Brillouin (WKB), phase integral, stepped profile [5], generalized WKB [2, 5], integral equation [6], finite difference [7], variational theory [10–13] and the weighted residual method [7, 8, 15].

The advantages of the weighted residual method are that it may be used for problems which do not have suitable variational formulation and this method keeps the error small. Thus, the weighted residual method can handle the more general problems, i.e. polarizations of both types (perpendicular and parallel) and inhomogeneities both in permittivity and permeability, simultaneously.

It is rather easy to extend the weighted residual method for two or three dimensional field problems as well.

Statement of the problem

The geometry of the wave propagation problem is shown in Fig. 1. Two homogeneous media occupy the semi-infinite regions $z<0$ (region 0. $\mu_0\tilde{\mu}_0$, $\varepsilon_0\tilde{\varepsilon}_0$) and $z>a$ (region 2., $\mu_0\tilde{\mu}_2$, $\varepsilon_0\tilde{\varepsilon}_2$) respectively. The inhomogeneous slab (region 1., $\mu_0\tilde{\mu}_1(z)$, $\varepsilon_0\tilde{\varepsilon}_1(z)$) occupies the region from $z=0$ to $z=a$. As usual, μ_0 and ε_0 represent permeability and permittivity of free space, while $\tilde{\mu}_i$ and $\tilde{\varepsilon}_i$ represent the relative complex permeability and permittivity of the i-th region ($i=0, 1, 2$). The e.m. parameters of the inhomogeneous slab vary only in the z direction.

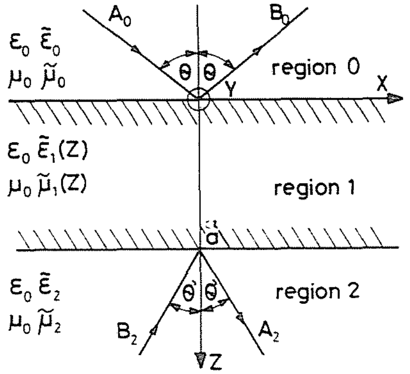

Fig. 1. Geometry of wave propagation problem

The time harmonic variation of $e^{j\omega t}$ is assumed and suppressed throughout the following.

In the case of perpendicular polarization the electric vector has only y component:

$$\bar{E}_y^{\perp}(x, z)=E_y^{\perp}(z)e^{-\gamma_i \sin \Theta}\bar{e}_y \tag{1}$$

where

$\bar{e}_y$ — the unit vector in the y direction

$\gamma_i=j\dfrac{2\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_i\tilde{\mu}_i}$ — the propagation constant in the region i. $i=0, 2$

λ_0 — the free space wavelength.

In the case of parallel polarization the magnetic vector has only y component

$$\bar{H}_y^{||}(x, z)=H_y^{||}(z)e^{-\gamma_i \sin \Theta}\bar{e}_y \tag{2}$$

The solution of the Maxwell's equations in the two homogeneous region may be written as

$$\Phi(z)=H_y^{||}(z)=E_y^{\perp}(z)=A_0e^{-\gamma_0 \cos(\Theta)z}+B_0e^{\gamma_0 \cos(\Theta)z} \qquad z<0 \tag{3.a}$$

$$\Phi(z)=H_y^{||}(z)=E_y^{\perp}(z)=A_2 e^{-\gamma_2 \cos(\Theta')(z-a)}+B_2 e^{\gamma_2 \cos(\Theta')(z-a)} \qquad z>a \quad (3.b)$$

where A_0, B_0, A_2, B_2 are normalized amplitudes and the meanings of them together with that of Θ' can be seen in Fig. 1.

Utilizing that

$$\gamma_0 \sin\Theta=\gamma_2 \sin\Theta' \tag{4}$$

it can be written that

$$\gamma_2 \cos\Theta'=j\frac{2\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_2\tilde{\mu}_2-\tilde{\varepsilon}_0\tilde{\mu}_0 \sin^2\Theta} \tag{5}$$

The problem with $A_0=1$ and $B_2=0$ corresponds to the case of a uniform plane wave incident obliquely upon the slab from region 0. In that case $B_0=\Gamma(\Theta)$ and $A_2=T(\Theta)$ may be interpreted as the reflection and transmission coefficients with Θ and Θ' as the angles of incidence and transmission, respectively. A backward propagation problem may be interpreted similarly, where $A_0=0$ and $B_2=1$. This corresponds to a uniform plane wave incident obliquely upon the inhomogeneous slab (region 1.) from region 2. In this case $A_2=\Gamma(\Theta)$ and $B_0=T(\Theta)$ will be the reflection and transmission coefficients.

From Maxwell's equations and boundary conditions — the tangential component of $\bar{E}$ and $\bar{H}$ should be continuous at the interface $z=0$, $z=a$ — it can be shown that $\Phi(z)$ must satisfy the following boundary value problem [13]

$$\frac{d}{dz}\left[p(z)\frac{d\Phi(z)}{dz}\right]+q(z)\Phi(z)=0 \qquad 0<z<a \quad (6.a)$$

$$\left.\frac{d\Phi}{dz}\right|_{z=0}+\alpha_1\Phi(0)+\alpha_2=0 \quad (6.b)$$

$$\left.\frac{d\Phi}{dz}\right|_{z=a}+\beta_1\Phi(a)+\beta_2=0 \quad (6.c)$$

The corresponding definitions for $p(z)$, $q(z)$, α_1, α_2, β_1 and β_2 are given in Table I. Once the (6) boundary value problem is solved, the coefficients B_0 and A_2 can be determined from

$$B_0=\Phi(0)-A_0 \tag{7}$$
$$A_2=\Phi(a)-B_2$$

where A_0 and B_2 have already been specified.

There are several methods to solve these boundary value problems, here the weighted residual method will be used for numerical evaluation.

Table I

Definitions of parameters

Symbol	Perpendicular pol.	parallel pol.
$\Phi(z)$	$E_y^{\perp}(z)$	$H_y^{\parallel}(z)$
$p(z)$	$1/\tilde{\mu}_1(z)$	$1/\tilde{\varepsilon}_1(z)$
$q(z)$	$\left(\frac{2\pi}{\lambda_0}\right)^2\left[\tilde{\varepsilon}_1(z)-\frac{\tilde{\mu}_0\tilde{\varepsilon}_0}{\tilde{\mu}_1(z)}\sin^2\Theta\right]$	$\left(\frac{2\pi}{\lambda_0}\right)^2\left[\tilde{\mu}_1(z)-\frac{\tilde{\mu}_0\tilde{\varepsilon}_0}{\tilde{\varepsilon}_1(z)}\sin^2\Theta\right]$
α_1	$-j\frac{2\pi}{\lambda_0}\sqrt{\frac{\tilde{\varepsilon}_0}{\tilde{\mu}_0}}\cos\Theta\tilde{\mu}_1(0)$	$-j\frac{2\pi}{\lambda_0}\sqrt{\frac{\tilde{\mu}_0}{\tilde{\varepsilon}_0}}\cos\Theta\tilde{\varepsilon}_1(0)$
α_2	$j\frac{4\pi}{\lambda_0}\sqrt{\frac{\varepsilon_0}{\tilde{\mu}_0}}\cos\Theta\tilde{\mu}_1(0)A_0$	$j\frac{4\pi}{\lambda_0}\sqrt{\frac{\tilde{\mu}_0}{\tilde{\varepsilon}_0}}\cos\Theta\tilde{\varepsilon}_1(0)A_0$
β_1	$j\frac{2\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_2\tilde{\mu}_2-\tilde{\varepsilon}_0\tilde{\mu}_0\sin^2\Theta}\,\frac{\tilde{\mu}_1(a)}{\tilde{\mu}_2}$	$j\frac{2\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_2\tilde{\mu}_2-\tilde{\varepsilon}_0\tilde{\mu}_0\sin^2\Theta}\,\frac{\tilde{\varepsilon}_1(a)}{\tilde{\varepsilon}_2}$
β_2	$-j\frac{4\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_2\tilde{\mu}_2-\tilde{\varepsilon}_0\tilde{\mu}_0\sin^2\Theta}\,\frac{\tilde{\mu}_1(a)}{\tilde{\mu}_2}B_2$	$-j\frac{4\pi}{\lambda_0}\sqrt{\tilde{\varepsilon}_2\tilde{\mu}_2-\tilde{\varepsilon}_0\tilde{\mu}_0\sin^2\Theta}\,\frac{\tilde{\varepsilon}_1(a)}{\tilde{\varepsilon}_2}B_2$

The Weighted Residual Method (WRM) and the Finite Element Solution [7, 8]

To summarize the fundamentals of the WRM let us consider a differential equation, which can be written quite generally as

$$D(\Phi)=L(\Phi)+p=0 \qquad \text{in} \quad \Omega \tag{8}$$

and the associated boundary condition in a general form is

$$P(\Phi)=M(\Phi)+r=0 \qquad \text{on} \quad \Gamma \tag{9}$$

where Ω is the domain of the problem bounded by a closed curve Γ. Here L, M are appropriate linear differential operators, p and r are known functions independent of Φ, where it is the unknown function to be found. The WRM starts by expanding Φ in terms of a set of known trial functions $\{N_m\}$ with unknown coefficients a_m, i.e.

$$\Phi \cong \hat{\Phi} = \sum_{m=1}^{M} a_m N_m \tag{10}$$

Substituting (10) into (8) and (9) the residual can be written as

$$\begin{aligned} R_\Omega &= D(\hat{\Phi}) = L(\hat{\Phi}) + p \qquad \text{in} \quad \Omega \\ R_\Gamma &= P(\hat{\Phi}) = M(\hat{\Phi}) + r \qquad \text{on} \quad \Gamma \end{aligned} \tag{11}$$

The weighted sum of the residual on the boundary and on the domain can be reduced by the following integral

$$\int_{\Omega} W_l R_{\Omega} \mathrm{d}\Omega + \int_{\Gamma} \bar{W}_l R_{\Gamma} \mathrm{d}\Gamma = 0 \tag{12}$$

where W_l and $\bar{W}_l$ are weighted functions which in general can be chosen independently.

If Equation (12) is satisfied for a very large number of arbitrary functions W_l and $\bar{W}_l$, then the approximation $\hat{\Phi}$ must approach the exact solution Φ.

Quite generally the system of equations to be solved can be written in the form

$$\mathbf{K}\boldsymbol{a} = \boldsymbol{f} \tag{13.a}$$

where

$$K_{lm} = \int_{\Omega} W_l L(N_m) \mathrm{d}\Omega + \int_{\Gamma} \bar{W}_l M(N_m) \mathrm{d}\Gamma \tag{13.b}$$

$$f_l = -\int_{\Omega} W_l p \mathrm{d}\Omega - \int_{\Gamma} \bar{W}_l r \mathrm{d}\Gamma \tag{13.c}$$

After solving this set of Equations the unknowns $\boldsymbol{a} = [a_1, a_2, \ldots, a_M]^T$ can be determined.

Depending on the choice of the weighted function set different types (point collocation, overdetermined collocation, least square, Galerkin [7]) of the weighted residual approximation are used.

Applying the general weighted residual statement to the (6) boundary value problem, it can be written that

$$\int_0^a W_l \left[\frac{\mathrm{d}}{\mathrm{d}z}\left(p \frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} \right) + q\hat{\Phi} \right] \mathrm{d}z + $$

$$+ \left[\bar{W}_l \left(\frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} + \alpha_1 \hat{\Phi} + \alpha_2 \right) \right]_{z=0} + \left[\bar{W}_l \left(\frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} + \beta_1 \hat{\Phi} + \beta_2 \right) \right]_{z=a} = 0 \tag{14}$$

where $\{W_l\}$ and $\{\bar{W}_l\}$ is a set of independent weighting functions $(l = 1, 2, \ldots, L)$.

The first term of (14) can be rewritten by using Green's lemma

$$\int_0^a W_l \left[\frac{\mathrm{d}}{\mathrm{d}z}\left(p \frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} \right) + \mathrm{d}\hat{\Phi} \right] \mathrm{d}z = -\int_0^a \left[p \frac{\mathrm{d}W_l}{\mathrm{d}z} \frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} - q\hat{\Phi} \right] \mathrm{d}z + \left[W_l p \frac{\mathrm{d}\hat{\Phi}}{\mathrm{d}z} \right]_{z=0}^{z=a} \tag{15}$$

Limiting now the choice of the weighting functions so that

$$
\begin{aligned}
[\bar{W}_l]_{z=0} &= [W_l \cdot P]_{z=0} \\
[\bar{W}_l]_{z=a} &= -[W_l \cdot P]_{z=a}
\end{aligned}
\tag{16}
$$

it can be seen that the term involving the weighted integral of the gradient of $\hat{\Phi}$ on the boundary ($z=0$, $z=a$) disappears and the approximating equation becomes

$$
\int_0^a \left[p \frac{dW_l}{dz} \frac{d\hat{\Phi}}{dz} - q\hat{\Phi}W_l \right] dz - \left[W_l p(\alpha_1 \hat{\Phi} + \alpha_2) \right]_{z=0} +
$$

$$
+ [W_l p(\beta_1 \hat{\Phi} + \beta_2]_{z=a} = 0 \tag{17}
$$

The resulting expression is often termed as the weak form of the weighted residual statement [7]. If it is possible to eliminate the integral involving $\hat{\Phi}$ derivatives along the boundary, the boundary conditions are often termed as natural boundary conditions. So eqs (6.b) and (6.c) are the natural boundary conditions of the (6) boundary value problem.

Note that, the weighted residual formulation (17) applies only to the case where $\tilde{\mu}_1(z)$ and $\tilde{\varepsilon}_1(z)$ are continuous throughout the slab. If $\tilde{\mu}_1(z)$ or $\tilde{\varepsilon}_1(z)$ is discontinuous across some point $z_i \in (0, a)$ in the inhomogeneous slab, the integral should be separated into two parts to take the profile discontinuity into account. In this way (17) may be rewritten as

$$
\left[\int_0^{z_i^-} + \int_{z_i^+}^a \right] \left(p \frac{dW_l}{dz} \frac{d\hat{\Phi}}{dz} - q\hat{\Phi}W_l \right) dz + \text{bound. terms} = 0 \tag{18}
$$

The $\hat{\Phi}$ function in (18) should satisfy the (6.a) differential equation in the region $(0, z_i^-)$ and (z_i^+, a) as well as the boundary conditions of (6.b), (6.c) at $z=0$ and $z=a$, and continuity condition for tangential $\bar{E}$ and $\bar{H}$ at $z=z_i$. The continuity conditions must be written as follows

$$
\begin{aligned}
\hat{\Phi}(z_i^-) &= \hat{\Phi}(_i^+) \\
p(z_i^-) \frac{d\hat{\Phi}}{dz}(z_i^-) &= p(z_i^+) \frac{d\hat{\Phi}}{dz}(z_i^+)
\end{aligned}
\tag{19}
$$

The extension to the cases of several profile discontinuities is straightforward.

The most popular weighted residual method is often termed the Galerkin Method. In this case the set of weighting functions equal to the set of trial functions,

$$
W_l = N_l \qquad l = 1, 2, \ldots, M. \tag{20}
$$

The advantage of this choice is that the resulting coefficient matrix is a symmetric matrix.

Substituting (20) and (10) into (17) yields the following matrix equation:

$$\mathbf{K} \cdot \boldsymbol{a} = \boldsymbol{f} \tag{21.a}$$

where

$$K_{l,m} = \int_0^a \left(p \frac{dN_l}{dz} \frac{dN_m}{dz} - qN_l N_m \right) dz - [p\alpha_1 N_l N_m]_{z=0} + [p\beta_1 N_l N_m]_{z=a} \tag{21.b}$$

$$f_l = [p\alpha_2 N_l]_{z=0} - [p\beta_2 N_l]_{z=a} \qquad 1 \le l, m \le M. \tag{21.c}$$

and $\boldsymbol{a}$ is the vector of parameters that can be calculated from (21.a). The finite element method will be used for evaluating eq. (21). For numerical computation, the interval $z \in [0, a]$ have to be divided into $(M-1)$ subintervals

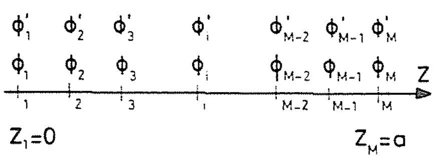

Fig. 2. Subdivision of interval $[0, a]$ into (M-1) subinterval of equal spacing h, $h = a/(M-1)$, $z_i = (i-1)h$

(elements) of equal spacing as shown in Fig. 2. The idea of the finite element method is that polynomial functions are the trial functions in each element.

The four most popular types of the set of trial functions are the linear, quadratic, cubic and Hermite trial functions [7, 8, 13].

Note that, the number (M) of points selected depends upon the set of trial functions which is used. In the case of linear and Hermite trial functions M may be chosen arbitrarily and the element n occupies the region from $z = z_i$ to $z = z_{i+1}$, where $n = i$. In the case of quadratic trial functions the number of points (M) should be an odd number and the element n occupies the region from $z = z_i$ to $z = z_{i+2}$, where $i = 2n - 1$, $n = 1, 2, \ldots, (M-1)/2$ and for cubic trial functions M should be a number for which $(M-1)/3$ is integer and the element n. occupies the region from $z = z_i$ to $z = z_{i+3}$, where $i = 3n - 2$, $n = 1, 2, \ldots, (M-1)/3$. After dividing the interval $z \in [0, a]$ into E element (21) may be written as

$$\mathbf{K} = \sum_{n=1}^{E} \mathbf{K}^{(n)} \qquad \boldsymbol{f} = \sum_{n=1}^{E} \boldsymbol{f}^{(n)} \tag{22}$$

where

$$K_{l,m}^{(n)} = \int_{\Omega_n} \left(p \frac{dN_l^{(n)}}{dz} \frac{dN_m^{(n)}}{dz} - qN_l^{(n)}N_m^{(n)} \right) dz -$$

$$-p[\alpha_1 N_l^{(n)} N_m^{(n)}]_{z=0} + [p\beta_1 N_l^{(n)} N_m^{(n)}]_{z=a} \qquad (23.a)$$

$$f_l^{(n)} = [p\alpha_2 N_l^{(n)}]_{z=0} - [p\beta_2 N_l^{(n)}]_{z=a} \qquad (23.b)$$

Ω_n — the region of the element n. and subscript (n) denotes the n-th element.

For the discontinuity problem described by (18) the point z_i of profile discontinuity should be chosen as the node point of the subdivision.

Using one of the trial function sets which was mentioned in this paper the resulting linear system of equations obtained this way can be expressed as

$$\mathbb{K}\boldsymbol{\Phi} = \boldsymbol{f} \qquad (24)$$

where $\mathbb{K}$ is a known symmetric band matrix determined by (23.a), (22), $\boldsymbol{f}$ is a known column vector determined by (23.b), (22) and $\boldsymbol{\Phi}$ is an unknown column vector to be determined.

For linear, quadratic or cubic trial functions, $\boldsymbol{\Phi} = [\Phi_1, \Phi_2, \ldots, \Phi_M]^T$ where $\Phi_i = \Phi(z = z_i)$, while for Hermite trial functions,

$$\boldsymbol{\Phi} = [\Phi_1, \Phi_1', \Phi_2, \Phi_2', \ldots, \Phi_M, \Phi_M']^T \text{ where } \Phi_i' = \frac{d\Phi}{dz}(z = z_i).$$

The matrix $\mathbb{K}$ is of bandwidth 1, 2 or 3 depending on whether the trial function is linear, quadratic or Hermite and cubic.

Note that the Galerkin method gives the same matrix equation that was obtained from the variational method, when the differential equation (6.a) is linear and symmetric [7, 8]. These conditions are satisfied at the Sturm–Liouville problem.

Numerical results

Numerical results for an absorber consisting of three homogeneous layers and an inhomogeneous wave absorber were obtained and examined with special attention to the forward propagation problem, where $A_0 = 1$, $B_2 = 0$, $B_0 = \Gamma(\Theta)$ (the reflection coefficient) and $A_2 = T(\Theta)$ (the transmission coefficient). First the stepped profile absorber consisting of three homogeneous layers is investigated for which measured data are also available.

The model of the layered absorber can be seen in Fig. 3. The measured data of individual homogeneous slabs are given in Table II. The measurements were made in a microwave anechoic chamber. A detailed description of the measurements is to be published in a separate paper [14].

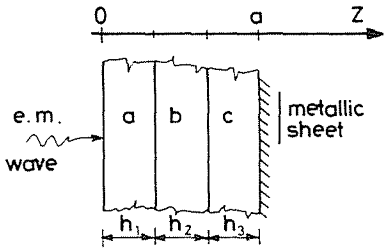

Fig. 3. Model of the stepped profil containing three homogeneous region of equal spacing, $h_1 = h_2 = h_3 = a/3$

The function of the relative permittivity $\tilde{\varepsilon}_1(z)$ and relative permeability $\tilde{\mu}_1(z)$ of the inhomogeneous absorber is constructed in such a way that at three points ($z=0$, a/3, 2a/3) the values of these functions have to be equal to the values of the electromagnetic parameters of the stepped profile absorber. This means that these functions can be approximated by a polinomial function of degree of two.

So the one-dimensional inhomogeneous media is characterized by the following two functions (see Fig. 1).

$$\tilde{\varepsilon}_1(z) = 3.0645z'^2 - 0.7815z' + 1.3 - j[1.5975z'^2 - 0.49z' + 0.023] \quad (25.a)$$

$$\tilde{\mu}_1(z') = -1.3635z'^2 + 0.4335z' + 0.988 - $$
$$- j[2.0475z'^2 - 0.3975z' + 0.161] \quad (25.b)$$

and

$$\tilde{\varepsilon}_0 = \tilde{\mu}_0 = \tilde{\mu}_2 = 1 \qquad \tilde{\varepsilon}_2 = 1 - j \cdot 10^{12} \qquad z' = z/a. \quad (25.c)$$

Table II

The measured values of the e.m. parameters for the stepped profile (see Fig. 3)

Layer	Re ($\tilde{\varepsilon}$)	Im ($\tilde{\varepsilon}$)	Re ($\tilde{\mu}$)	Im ($\tilde{\mu}$)
a	1.3	−0.023	0.988	−0.161
b	1.38	−0.037	0.981	−0.256
c	2.141	−0.406	0.671	−0.806

Region 0. ($z<0$) corresponds to free space ($\tilde{\varepsilon}_0 = \tilde{\mu}_0 = 1$),
Region 2. ($z>$a) corresponds to metal termination ($\tilde{\varepsilon}_2 = 1 - j \cdot 10^{12}$, $\tilde{\mu}_2 = 1$) and
Region 1. ($0<z<a$) corresponds to the inhomogeneous slab.

In Figure 4 the amplitude and the phase of the reflection coefficient for stepped and inhomogeneous profile are plotted as the function of a/λ_0 with $\Theta = 0^0$.

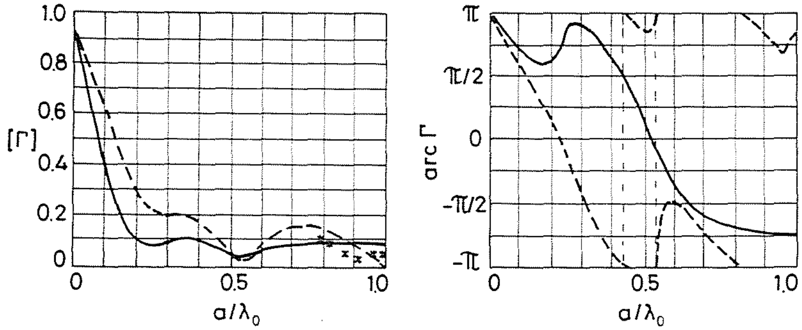

Fig. 4. Amplitude and phase of reflection coefficient as function of a/λ_0, inhomogeneous slab (continuous line), stepped profile (dashed line), $*$ — measured value for stepped profile, $\Theta = 0°$

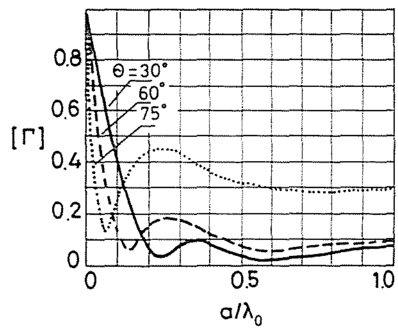

Fig. 5. Amplitude of reflection coefficient (parallel pol.) as function of a/λ_0 with $\Theta = 30°, 60°, 75°$

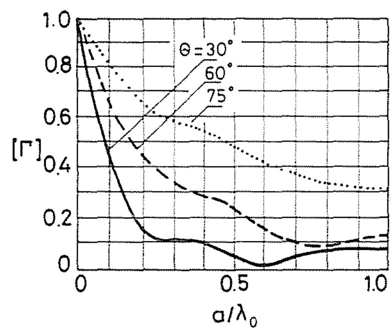

Fig. 6. Amplitude of reflection coefficient (perpendicular pol.) as function of a/λ_0 with $\Theta = 30°$, $60°, 75°$

In this figure the measured values of the reflection coefficient for stepped profile are plotted, too. It can be concluded that the inhomogeneous profile is more advantageous than the stepped profile containing three homogeneous slabs. The measured and calculated values of the amplitude of the reflection coefficient for the stepped profile are in close agreement. Figure 5–9 refer to the inhomogeneous profile.

In Figure 5 and 6 the amplitude of the reflection coefficient for perpendicular and parallel polarization is plotted as a function of a/λ_0 with Θ as a parameter ($\Theta = 30°$, $60°$, $75°$). In Figure 7 and 8 the amplitude of the

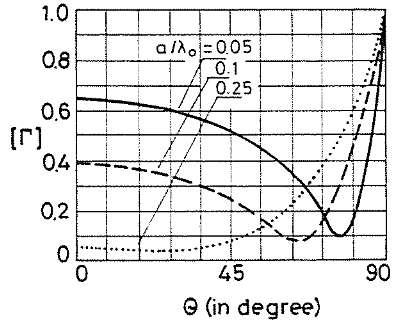

Fig. 7. Amplitude of reflection coefficient (parallel pol.) as function of Θ with $a/\lambda_0 = 0.05, 0.1, 0.25$

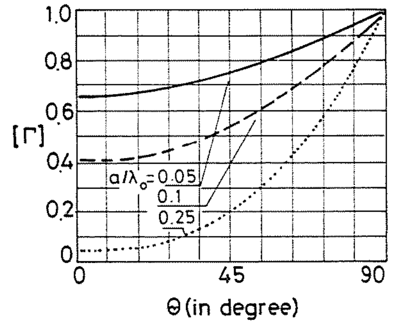

Fig. 8. Amplitude of reflection coefficient (perpendicular pol.) as function of Θ with $a/\lambda_0 = 0.05$, 0.1, 0.25

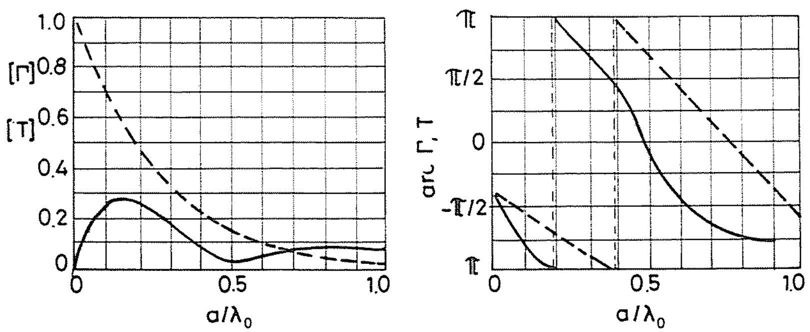

Fig. 9. Amplitude and phase of reflection (continuous line) and transmission (dashed line) coefficient as function of a/λ_0 with $\Theta = 0°$, ($\tilde{\varepsilon}_0 = \tilde{\mu}_0 = \tilde{\varepsilon}_2 = \tilde{\mu}_2 = 1$)

reflection coefficient for polarization of both type is plotted as a function of Θ with a/λ_0 as a parameter ($a/\lambda_0 = 0.05, 0.1, 0.25$). In Figure 9 the amplitude and the phase of the reflection and transmission coefficients are plotted as a function of a/λ_0 for $\Theta = 0°$, but in this case Region 2 corresponds to free space ($\tilde{\varepsilon}_2 = \tilde{\mu}_2 = 1$).

Figures 4–9 show the mechanism of attenuation for inhomogeneous wave absorber.

The error associated with the numerical results can be estimated by the criterian from the conservation of complex power [12, 13]. When the number of elements $E = 20 \cdot a/\lambda_0$ and linear trial function is used, the maximum error is less than three per cent.

Conclusion

The weighted residual method has been applied and the finite element method has been developed to study the wave behaviour in a one-dimensional inhomogeneous medium with variable permittivity and permeability. The advantage of the weighted residual method in contrast with the variational theory is that it can be used for problems which do not have suitable variational formulation. Numerical results have been given for inhomogeneous wave absorbers as well.

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