SIMPLIFIED DESIGN OF POWER RECTIFIERS

A. AMBRÓZY

Department of Electronics Technology,
Technical University, H-1521 Budapest

Received 2 Juni, 1985

Summary

In contrary to the simple topology of the power rectifier with a buffer capacitor its calculation is rather cumbersome. The classical equations are manageable with difficulties therefore designers often rely on guesses. The design procedure introduced in the paper is based on a clear physical view and composed of well approximating, simple equations.

There are very few such simple topology circuits as the half-wave power rectifier with buffer capacitor (Fig. 1). Its design equations have been well known for a long time; their use, however, is cumbersome partly for the implicite appearance of an important variable, partly for the many trig-

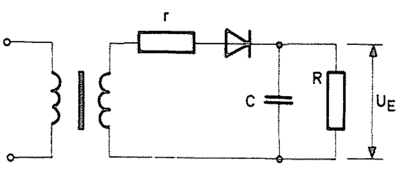

Fig. 1. Schematic diagram of half-wave rectifier

onometric functions. Probably for these reasons most of the textbooks treat the subject in a rough-and-ready way. Working conditions characterizing semiconductor diodes make possible the introduction of approximations which simplify design equations considerably. This design method has a straightforward physical background therefore its application in the education is justified.

Exact equations [1] [2]

The voltage and current waveforms of the half-wave rectifier shown in Fig. 1 can be seen in Fig. 2. For the present the threshold voltage of the diode is considered to be zero.

With a finite value buffer capacitor the dashed charging and discharging lines are valid. In this case the midpoint of the charging line and the maximum

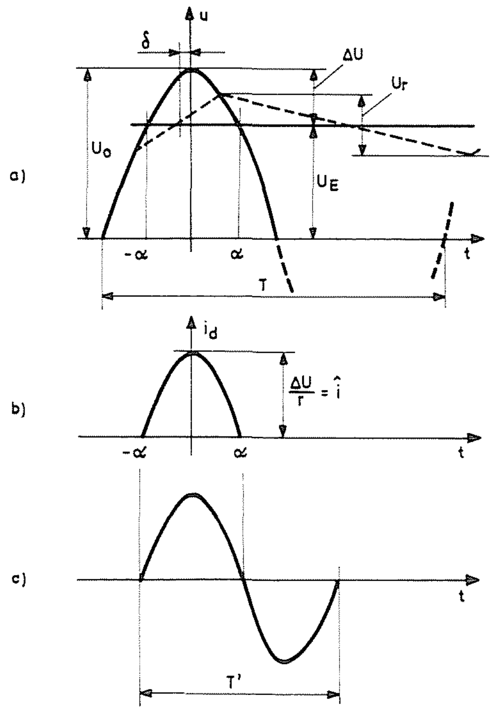

Fig. 2. Waveforms: a) *AC*-voltage, b) diode-(charging-) current, c) approximation of curve *b* by halfcosine

of the cosine curve are at different times; let their difference be δ. At first, our calculations will be simplified suggesting $C=\infty$; then the full line of Fig. 2 shows the voltage across C and the load resistor R, furthermore, $\delta=0$. It is easy to see that

$$U_E=U_0 \cos \alpha \tag{1}$$

where α is the angle of flow. The connection between the r series and R load resistance and α is

$$\frac{\operatorname{tg} \alpha-\alpha}{\pi}=\frac{r}{R} \tag{2}$$

which is the aforementioned implicite expression for α. It is a difficulty that the independent variable of all the following equations is α. For example, the ratio of the periodic peak current $\hat{i}$ of Fig. 2b and the load current I is

$$\frac{\hat{i}}{I}=\frac{\pi}{\operatorname{tg} \alpha-\alpha} \frac{1-\cos \alpha}{\cos \alpha} \tag{3}$$

The ratio of the root mean square value of this periodic current and the load current is

$$\frac{I_{rms}}{I} = \frac{1}{\cos\alpha(\operatorname{tg}\alpha-\alpha)}\left[\frac{\pi}{2}(\alpha+2\alpha\cos^2\alpha-3\sin\alpha\cos\alpha)\right]^{1/2} \tag{4}$$

Finally, the efficiency of the rectifier

$$\eta = \frac{P_{DC}}{P_{AC}} = (\operatorname{tg}\alpha-\alpha)\frac{2(1+\cos 2\alpha)}{2\alpha-2\sin\alpha} \tag{5}$$

Eqs. (1)...(5) were worked out during the electron tube era. Then $r/R = 0.05\ldots0.1$ and correspondingly $\alpha = 40°\ldots50°$ were usual. Semiconductor diodes make possible much smaller r/R ratios so α lies in the range of $15°\ldots30°$. This permits some approximations.

Character of approximations

For example, instead of (1)

$$U_E = U_0\cos\alpha \approx U_0\left[1-\frac{\alpha^2}{2}\right] \tag{6}$$

and instead of (2)

$$\pi\frac{r}{R} = \operatorname{tg}\alpha-\alpha = \alpha+\frac{\alpha^3}{3}+\frac{2\alpha^5}{15}+\ldots-\alpha \approx \frac{\alpha^3}{3} \tag{7}$$

can be written. Combining (6) and (7) we get

$$U_E \approx U_0\left[1-\frac{1}{2}\left(3\pi\frac{r}{R}\right)^{2/3}\right] \tag{8}$$

Similarly, for (3), (4) and (5)

(3)
$$\frac{\pi}{\operatorname{tg}\alpha-\alpha}\,\frac{1-\cos\alpha}{\cos\alpha} \approx \frac{\pi}{\alpha^3/3}\,\frac{1-(1-\alpha^2/2)}{1} = \frac{3\pi}{2\alpha} \tag{9}$$

(4)
$$\frac{1}{\cos\alpha(\operatorname{tg}\alpha-\alpha)}\left[\frac{\pi}{2}(\alpha+2\alpha\cos^2\alpha)-3\sin\alpha\cos\alpha\right]^{1/2} \approx$$

$$\approx \frac{1}{\alpha^3/3}\sqrt{\frac{\pi}{2}\,\frac{4}{15}\alpha^5} = \frac{1.94}{\sqrt{\alpha}} \tag{10}$$

$$(5)\quad (\mathrm{tg}\,\alpha-\alpha)\frac{2(1+\cos 2\alpha)}{2\alpha-\sin 2\alpha}\approx\frac{\alpha^3}{3}\left(1+\frac{2}{5}\alpha^2\right)\frac{2(1+1-2\alpha^2)}{2\alpha-2\alpha+\frac{8}{6}\alpha^3-\frac{32}{120}\alpha^5}=$$

$$=\frac{1}{1+0.4\alpha^2} \tag{11}$$

Although the approximate Eqs. (6)...(11) could be used for design, it is better to approach the problem of the rectifier by a physical way. Its results will be comparable to (6)...(11).

Physical approximations

It is well known that the charge flowing into and out of the buffer capacitor should be equal during a full period:

$$\int_0^T i_d(t)\,\mathrm{d}t=I\cdot T \tag{12}$$

where $i_d(t)$ is the diode current and T is the time of one full period. According to Fig. 2b

$$i_d(t)=\frac{U_0\cos\omega t-U_E}{r};\qquad -\alpha\leqq\omega t\leqq\alpha \tag{13}$$

and zero elsewhere. Introducing the quantities ΔU and b

$$\Delta U=U_0-U_E;\quad b=\frac{\Delta U}{U_E} \tag{14}$$

from which

$$U_E=\frac{U_0}{1+b}\approx U_0(1-b) \tag{15}$$

Comparing (6) and (15) it can be seen that

$$\alpha\approx\sqrt{2b} \tag{16}$$

The charge pumped into the capacitor during one period is proportional to the area under the curve of Fig. 2b and this is the left side of (12). To evaluate this area more easily the curve of Fig. 2b is replaced by the one of Fig. 2c which is a half cosine [3]. Then

$$\frac{T'/2}{T}=\frac{2\alpha}{2\pi};\quad T'=T\frac{2\alpha}{\pi};\quad \omega'=\frac{2\pi}{T'}=\frac{\pi^2}{\alpha T} \tag{17}$$

Now Eq. 12 is solved as

$$\int_0^T i_d(t)\,\mathrm{d}t = \int_{-T'/4}^{T'/4} \frac{\Delta U}{r}\cos\omega' t\,\mathrm{d}t = \frac{\Delta U}{r}\frac{2}{\omega'} = \frac{\Delta U}{r}\frac{2\alpha T}{\pi^2} =$$

$$= \frac{U_E}{R}T \tag{18}$$

Let us express b using (18) and (16)

$$b = \frac{\Delta U}{U_E} = \frac{r}{R}\frac{\pi^2}{2\sqrt{2b}}; \quad b = \left(\frac{\pi^4}{8}\right)^{1/3}\left(\frac{r}{R}\right)^{2/3} = 2.3\left(\frac{r}{R}\right)^{2/3} \tag{19}$$

This is an important result: the relative voltage loss b is given as the function of r/R. Putting (19) into (15) we get an expression similar to (8) — only the multiplier of the independent variable differs a little.

In the following we calculate $\hat{i}/I$, I_{rms}/I and η, all in function of b. This results in an easily manageable, consistent system of design equations.

The peak value of the current pulse in Fig. 2 can be expressed with the help of Eq. (18)

$$\hat{i} = \frac{\Delta U}{r} = \frac{U_E}{R}\frac{\pi^2}{2\alpha} = \frac{U_E}{R}\frac{\pi^2}{2\sqrt{2b}} = \frac{3.49}{\sqrt{b}}\frac{U_E}{R} \tag{20}$$

Depending on b, the periodic peak current is 6 . . . 12 times of the DC current. If α is written instead of $\sqrt{2b}$ in the denominator we get back Eq. (9) — again with a small difference in the coefficient.

The RMS current flowing through the diode — or the charging circuit — will heat the diode, the secondary winding of the transformer and the protecting resistor if it is applied. For the calculation of I_{rms} we use its definition and (18)

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,\mathrm{d}t} = \sqrt{\frac{2\alpha}{2\pi}\frac{1}{2}\left(\frac{\Delta U}{r}\right)^2} = \frac{\Delta U}{r}\sqrt{\frac{\alpha}{2\pi}} =$$

$$= \frac{U_E}{R}\frac{\pi^2}{2\alpha}\sqrt{\frac{\alpha}{2\pi}} = \frac{U_E}{R}\left(\frac{\pi}{2}\right)^{3/2}\frac{1}{2^{1/4}}\frac{1}{b^{1/4}} =$$

$$= \frac{1.655}{b^{1/4}}\frac{U_E}{R} \tag{21}$$

In the second step the factor 1/2 comes from the fact that the curve of Fig. 2b was replaced by a half cosine. I_{rms}/I is much lower than $\hat{i}/I$ since the nominator of (21) is less than that of (20) and $b^{1/4} > b^{1/2}$. Eq. (21) corresponds also to our expectations since it can directly be compared to (10).

To calculate the efficiency of the rectifier, the *DC* and *AC* work for a period should be evaluated. The former is $U_E I T$, the latter — according to the Fig. 2b-2c transformation —

$$\int_{-T'/4}^{T'/4} (U_E + \Delta U \cos \omega' t) \frac{\Delta U}{r} \cos \omega' t \, \mathrm{d}t = U_E \frac{\Delta U}{r} \frac{2}{\omega'} + \frac{\Delta U^2}{r} \left[\frac{1}{2} t + \frac{1}{4\omega'} \sin 2\omega' t \right]_{-T'/4}^{T'/4} \tag{22}$$

From (18), (19) and (22)

$$\eta = \frac{U_E I T}{\frac{\Delta U}{r} \frac{\alpha T}{\pi} \left(U_E \frac{2}{\pi} + \frac{\Delta U}{2} \right)} = \frac{1}{1 + \frac{\pi}{4} b} \tag{23}$$

This result is comparable to Eq. (11).

The errors of approximations

Fig. 3 shows the relative errors between the approximate and precise values, in function of angle of flow. Fig. 3a refers to U_E/U_0 [results calculated from (19) and (15) compared to ones from (1) and (2)]. The difference is not very high even at $\alpha = 45°$ although it increases rapidly at this point.

In Fig. 3b one can see the error of $\hat{\imath}/I$ calculated from (20) instead of (3). The difference is more marked here. The error *vs* α function can be obtained if the power series in Eq. (9) is extended. It turns out that (9) should be multiplied by $(1 + k_1 \alpha^2)$ for a better approximation. This multiplier corresponds to $(1 + 2k_1 b)$ in Eq. (20). Finding k_1 and modifying the coefficient of (20) we get

$$\frac{\hat{\imath}}{I} = 3.384 \frac{1 + 0.29b}{\sqrt{b}} \tag{24}$$

whose error lies in the horizontal axis of Fig. 3b, within a linewidth.

Fig. 3c shows the relative difference between (21) and (4). Applying the above reasoning to Eq. (10) we get again a multiplier of type $(1 + k_2 \alpha^2)$, by which (21) can be modified:

$$\frac{I_{rms}}{I} = 1.646 \frac{1 + 0.135b}{b^{1/4}} \tag{25}$$

The error curve lies again in the horizontal axis.

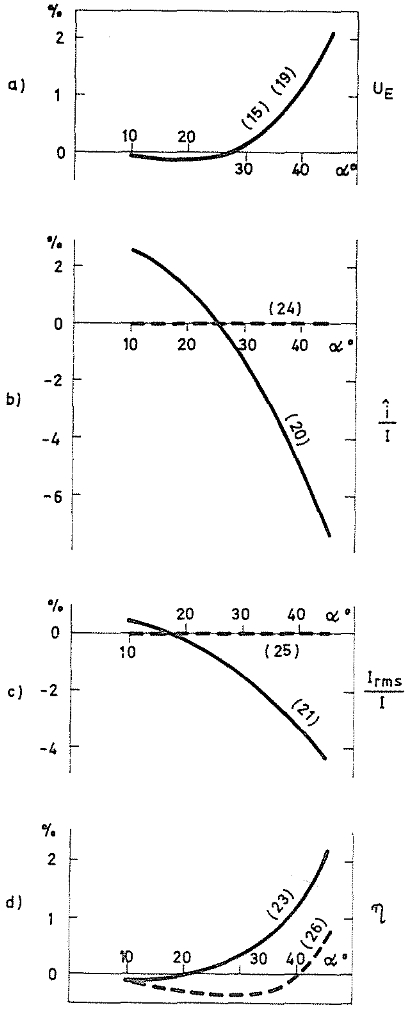

Fig. 3. Differences between the approximate and accurate equations: a) *DC*-voltage, b) periodic peak current, c) *RMS* charging current, d) efficiency of the rectifier

Finally, Fig. 3d shows the error of the approximation of efficiency if Eq. (23) is compared to (5).

The error is rather low but can be decreased if the multiplier of b in the denominator is 0.84 instead of $\pi/4$:

$$\eta = \frac{1}{1+0.84b} \qquad (26)$$

Design equations

Until now we made some simplifying assumptions:
— half-wave rectifier has been studied
— the threshold voltage of the diode has been neglected
— the buffer capacitor considered as infinite.

In practice the power rectifiers are generally full-wave ones and for higher powers 3 or 6 phase systems are usual. Our former results can be extended to higher phase numbers keeping in mind that Eq. (12) should be fulfilled: when the phase number is n then the left side of Eq. (12) should be multiplied by n. During the latter calculations n appears in the denominator of the right side.

The threshold voltage of the diode should be considered with the calculation of U_E/U_0 and η. It is apparent that U_E should be replaced by $U_E + mU_T$ where U_T is the threshold voltage (0.5 . . . 1 V) and generally $m=1$, but in the case of bridge rectifiers — where always 2—2 diodes are in series — $m=2$. To estimate U_T is not easy: the $U_F - I_F$ forward characteristics extended to $\hat{i}$ should be used.

In the case of $C \neq \infty$ two new phenomena appear: there is a ripple superimposed to U_E and $\delta \neq 0$.

From [2]

$$\operatorname{tg} \delta = \frac{\pi/n - \alpha}{\omega CR \operatorname{tg} \alpha} \tag{27}$$

If $\delta \leqq 8^\circ$, the results of this paper deteriorate by less than one per cent. When $\delta > 8^\circ$ the use of Ref. [2] is advised.

The ripple is considered always linear, so

$$U_r = \frac{I}{C} T\gamma; \qquad \gamma = \frac{1}{n} - \frac{\alpha}{\pi} = \frac{1}{n} - \frac{\sqrt{2b}}{\pi} \tag{28}$$

Our design equations, together with (27) and (28)

$$b = 2.3 \left(\frac{r}{nR}\right)^{2/3} \tag{29}$$

$$U_E = \frac{U_0}{1+b} - mU_T \tag{30}$$

$$\frac{\hat{i}}{I} = \frac{3.49}{n\sqrt{b}} \tag{31}$$

$$\frac{I_{rms}}{I} = \frac{1.655}{n^{1/2} b^{1/4}} \tag{32}$$

$$\eta = \frac{1}{1+\frac{\pi}{4}b}\left(1-\frac{U_T}{U_E}\right) \tag{33}$$

Here the better approximations Eq. (24)...(26) were not repeated since there are numerous, hardly considerable factors which deteriorate the accuracy. One of them is that the I—V curve of the diode is not a broken line; another, even more serious that the line voltage differs from pure sinusoidal as the thyristor-controlled loads propagate.

References

1. KAMMERLOHER, J.: Hochfrequenztechnik III. Gleichrichtertechnik. Winter'sche Verlagshandlung, Leipzig, 1942
2. LIEDERS, A.: Single phase rectifier circuits with CR filters. Part 1. Electronic Components and Applications. 153 (1979)
3. BARTA, I.: Rádiókészülékek és erősítők. (Radio receivers and amplifiers). Tankönyvkiadó, Budapest, 1956

Prof. Dr. András AMBRÓZY, H-1521 Budapest