AN EXTENDED RESONATOR CONCEPT AND ITS APPLICATION IN SWITCHED-CAPACITOR DFT PROCESSORS

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Summary

An extension of the method based on the Goertzel-algorithm is given applying general two-output second-order lossless discrete resonators. In the proposed structure of resonators simple SC integrators are used avoiding the problems of circuit complexity of SC delay elements. The different resonators of the bank are of the same topology and differ only by one capacitor value. The solution is, therefore, very suitable for integration.

Introduction

An n-point discrete Fourier transform of a real sequence of samples can be evaluated either by a transversal structure containing $(n-1)$ delay elements (for an SC implementation see [1]) or by a bank of $M=0.5(n+1)$ for n odd or $M=0.5(n+2)$ for n even second-order lossless resonators each of them containing two delay elements (Goertzel-algorithm, [2]). Both types need further stages for weighted summation. The first version may have the advantage of supplying running DFT values, while the other one results in saving the number of delay elements if a complete set of transformed values is not required.

For a switched-capacitor solution, however, both methods require a lot of unit delay elements as basic building blocks. Most of the known SC delay circuits suffer from being too complicated, e.g. using two opamps per stage, [3] or multi-phase timing, [1]. An integrating capacitor and another capacitor of the same value (the bottom plate of the latter should be grounded while the top-plate is toggle-switched between the output and the inverting input of the same operational amplifier) as a two-phase clocked delay element is sensitive to the top-plate parasitic capacitances.

The present contribution tries to generalize the two-output recursive resonator concept in order to be able to make use of hardware simplicity of the SC integrators, thus avoiding the complexity problems of the *SC* delay element realizations.

The extended second-order resonator concept

Let us start from a second-order z-domain lossless resonator transfer function (poles on the unit circle) and its partial fractioned form

$$F(z)=\frac{D+Ez^{-1}}{1-2\cos\alpha_k z^{-1}+z^{-2}}=\frac{A+jB}{1-\exp(-j\alpha_k)z^{-1}}+\frac{A-jB}{1-\exp(j\alpha_k)z^{-1}} \quad (1)$$

where $\alpha_k=2\pi f_0/f_s$, f_0 is the resonant frequency, f_s is the sampling frequency and $n\alpha_k/2\pi=k$ integer. Let the right-hand terms in (1) be multiplied by $(1-q^n)$ where $q=\exp(-j\alpha_k)z^{-1}$ or $q=\exp(j\alpha_k)z^{-1}$, respectively, yielding expressions of the form

$$\left[\frac{a_0}{1-q}\right](1-q^n) \quad (2)$$

(2) results in a sum of a finite geometric series having the form (even for $q=1$ as a limit value)

$$s=a_0\sum_{i=0}^{n-1}q^i. \quad (3)$$

In our cases $q^n=z^{-n}$ and therefore $(1-q^n)=(1-z^{-n})$ corresponds to a function in which the z^{-n} term becomes effective first after n unit delays. Applying a sequence of n samples to the input of a filter starting from zero energy initial state (Fig. 1) the output at the end of the processing cycle is to be determined by calculating first a properly limited finite part of the impulse response for (1). As the total number of delays involved is $(n-1)$, (2) and (3) can be applied, as well.

An input sequence sampled from a harmonic signal of frequency f_0 and of arbitrary initial phase may be written in the z-domain as

$$v_{\text{in}}=\sum_{i=0}^{n-1}[-S_k\sin(i\alpha_k)+C_k\cos(i\alpha_k)]z^{-i}. \quad (4)$$

Combining (1), (3) and (4) yields the output value at the end of the processing

$$v_{\text{out}}=n(AC_k+BS_k). \quad (5)$$

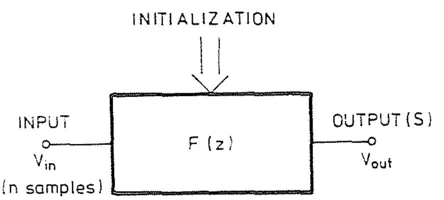

Fig. 1. The resonator structure

As (5) has two unknown values C_k and S_k, two filters are required with the same denominator and linearly independent pairs of numerator coefficients D and E. As in the case of the Goertzel-algorithm, [2], or a corresponding analogous continuous structure, [4], the best solution is to use only one initialized filter with two outputs. From a theoretical point of view there are infinite possibilities choosing the two pairs of coefficients D and E, even second-order numerators can be used (after a division). Some cases for D and E values and the corresponding $A=D/2$ and $B=\dfrac{D\cos\alpha_k-E}{2\sin\alpha_k}$ values of practical interest are given here

1. $D_1=0$ and $E_1=-1$ yields $A_1=0$ and $B_1=\dfrac{0.5}{\sin\alpha_k}$.
2. $D_2=1$ and $E_2=0$ yields $A_2=0.5$ and $B_2=-\dfrac{1}{2\,\mathrm{tg}\,\alpha_k}$.
3. $D_3=1$ and $E_3=-K$ yields $A_3=0.5$ and $B_3=\dfrac{K-\cos\alpha_k}{2\sin\alpha_k}$.

Combining case 1. and 2. corresponds to the Goertzel-algorithm while 1. and 3. proved to be very useful in simple SC realizations.

It is worth-while to note that case 1. corresponds to a matched filter for an input signal with $C_k=0$ and therefore results in an output value directly proportional to S_k. The selectivity properties of the DFT are well known and are not dealt with here.

The resonator circuit

The calculation of a total independent set of n transformed values requires a bank of M filters. The corresponding resonant frequency (f_0) parameter of the k-th filter will be denoted by $\alpha_k=2\pi\dfrac{k}{n}f_s$, the only varying capacitor value in Fig. 2 by β_k while the peak values of the incoming quadrature components of frequency f_0 by S_k and C_k, respectively. SW_1 and SW_2 guarantee the energy-free initial conditions and the processing starts when these switches are opened. The following equations hold

$$v_{\text{out}3}(1-z^{-1})=(v_{\text{in}}+\beta_k v_{\text{out}1})z^{-1} \tag{6}$$

$$v_{\text{out}1}(1-z^{-1})=-v_{\text{out}3}. \tag{7}$$

Combining (6) and (7) results in

$$\frac{v_{\text{out}1}}{v_{\text{in}}}=\frac{-z^{-1}}{1-(2-\beta_k)z^{-1}+z^{-2}} \tag{8}$$

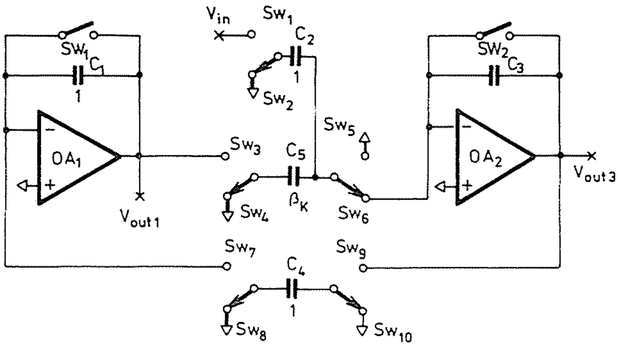

Fig. 2. The resonator circuit
2 operational amplifiers ($OA_1 \ldots OA_2$) 10+2 switches ($Sw_1 \ldots Sw_{10}$, $SW_1 \ldots SW_2$) 4 unit capacitors ($C_1 \ldots C_4$) 1 individual capacitor (C_5)

$$\frac{v_{\text{out}3}}{v_{\text{in}}} = -1 + \frac{1-(1-\beta_k)z^{-1}}{1-(2-\beta_k)z^{-1}+z^{-2}}. \tag{9}$$

(Choosing, however, another timing frame results in a z^{-1} multiplier in the right-hand side of (7) instead of that in (6) leading to different equations i.e. numerators).

β_k should be equal to $2(1-\cos\alpha_k)$ thus $0 \le \beta_k \le 4$. Let the final values at the end of n cycles be denoted in the following way

$$v_{\text{out}1}(n) = V_1 \tag{10}$$

$$v_{\text{out}3}(n) = V_3 \tag{11}$$

$$v_{\text{in}}(n) = V \tag{12}$$

(8) corresponds to case 1. and therefore

$$S_k = \frac{2\sin\alpha_k}{n} V_1 = K_1 V_1 \tag{13}$$

(9) contains a constant term too, and therefore the last input sample should also be involved. The other term corresponds to case 3. Rearranging yields

$$C_k = \frac{2}{n}[V_3 + V + (1-\cos\alpha_k)V_1] = K_2(V_3+V) + K_3 V_1. \tag{14}$$

The weighting circuits can be realized by initialized summing SC-integrators (two operational amplifiers for each frequency). The weighted evaluation should be performed just as the last input sample is about to enter into the system (before Sw_2 of Fig. 2 closes). It seems also possible to make use of the same two operational amplifier integrator stages at the end of the processing for weighting purposes.

As an illustrative example Table 1 summarizes the parameters for a bank of $n=8$.

Table 1
The parameters for $n=8$

k	α_k	β_k	$8K_1$	$8K_2$	$8K_3$
0	0	0.0000	0.0000	2.0000	0.0000
1	$\pi/4$	0.5858	1.4142	2.0000	0.5858
2	$\pi/2$	2.0000	2.0000	2.0000	2.0000
3	$3\pi/4$	3.4142	-1.4142	2.0000	3.4142
4	π	4.0000	0.0000	2.0000	0.0000

For $k=0$ (zero frequency) one of the integrators and one of the weighting circuits are not needed, while for $n=4$ ($f_0=f_s/2$) one weighting circuit is unnecessary.

References

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