III. FAST ANALYSIS OF ACTIVE SWITCHED-CAPACITOR FILTERS

T. PÖRNECZI

Research and Development Institute,
Telefongyár H-1143 Budapest Hungária krt. 126–132.

Received December 15, 1985
Presented by Prof. S. Csibi

Summary

A technique is presented for efficient computer analysis of active switched capacitor (SC) filters. A special block partitioning method based on partitioning the circuit into elementary voltage-controlled charge sources and charge to voltage converters is used. The effects of the finite op amp gain and the top plate to substrate parasitics are also considered. The presented work was made during the postgraduate studies of the author at the Institute of Communication Electronics, Technical University of Budapest.

Introduction

Many of the recently published SC filter computer analysis techniques strive for complete generality with respect to topology and some with respect to the number of switching phases and switching instances, too [1]–[3]. In many practical SC filter implementations topological constraints are established in order to eliminate the degrading effects of parasitics and common mode gain. The noninverting inputs of the op amps are connected to ground.

Both plates of all switched-capacitors are switched between a voltage source and ground or between ground and virtual ground. Only a two phase clock is used. In the case of active SC filters it is enough to compute the output voltages of the op amps (the nodal voltages*).

In analysis it will pay to make use of these restrictions. The analysis will require less computational time and this allows us to use statistical tolerance analysis or design centering technique.

* The definition of the nodal voltage in this paper slightly differs from the usual definition.

Analysis technique

In a general two phase SC filter all the nodal voltages are permitted to change in both of the switching phases. Sampling the i-th nodal voltage in phase φ^1 $V_i^{\varphi 1}(z)$ and sampling in phase φ^2 $V_i^{\varphi 2}(z)$ can be defined. This way every nodal voltage is interpreted as a vector

$$V_i(z)=[V_i^{\varphi 1}(z), \quad V_i^{\varphi 2}(z)]. \tag{1}$$

A two phase SC filter is characterized by four transfer functions depending on input and output sampling:

$$\mathbf{H(z)}=\begin{bmatrix} H^{\varphi 1,\varphi 1}(z), & H^{\varphi 2,\varphi 1}(z) \\ H^{\varphi 1,\varphi 2}(z), & H^{\varphi 2,\varphi 2}(z) \end{bmatrix}. \tag{2}$$

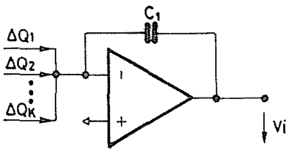
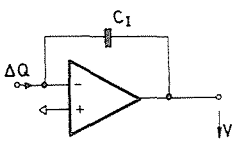
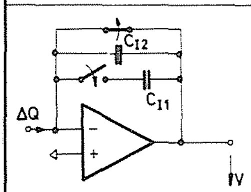
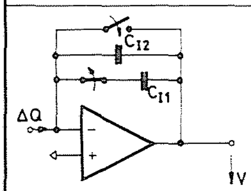


Fig. 1. Charge variation controlled voltage source

Table 1

CVCVS In phase ∅1	∅1, ∅1 $H_i(Z)$	∅1, ∅2 $H_i(Z)$	∅2, ∅1 $H_i(Z)$	∅2 ∅2 $H_i(Z)$
	$-\frac{1}{C_I(1-Z^{-1})}$	$-\frac{Z^{-1/2}}{C_I(1-Z^{-1})}$	$-\frac{Z^{-1/2}}{C_I(1-Z^{-1})}$	$-\frac{1}{C_I(1-Z^{-1})}$
	0	0	0	$-\frac{1}{C_{I1}+C_{I2}-C_{I1}Z^{-1}}$
	$-\frac{1}{C_{I1}+C_{I2}-C_{I1}Z^{-1}}$	0	0	0

In $\mathbf{H}^{\varphi i, \varphi j}(z)$ transfer functions φi indicates the phase of the input sampling and φj indicates the phase of the output sampling.

The op amps are focal points in the active SC filters. With their switched or unswitched capacitive feedback they independently sum the variation of the charge coming to their inputs from different parts of the filter, and convert into voltage.

The output voltage of the charge variation controlled voltage source (CVCVS) is:

$$V_i(z) = \mathbf{H}_i(\mathbf{z}) \sum_{m=1}^{k} Q_m^*(z). \tag{3}$$

The Q_m charges are generated by switched or unswitched capacitor branches working like voltage controlled charge sources (VCCS). The inputs of these VCCSs are driven by voltage sources and the outputs are virtually grounded. Only limited types of VCCSs and CVCVSs are existing in practical SC filters. The commonly used set of the CVCVSs and their transfer functions are presented in Table 1., Table 2. showes the VCCSs with the transfer functions.

The limited type of CVCVSs and VCCSs allows us to store their transfer functions in a library of a computer program.

Using this partitioning method the topology of a SC filter is given by the system of interconnections between different VCCSs and CVCVSs.

To every node belongs a CVCVS with the transfer function $\mathbf{H}_i(\mathbf{z})$. Between the nodes and the inputs of the CVCVSs there are VCCSs with the transfer function $\mathbf{G}_{ijl}(\mathbf{z})$ where i indicates the CVCVS to which the output of the VCCS is connected; j indicates the nodal voltage controlling the input of the VCCS; l distinguishes the parallel connected VCCS from each other. The input node is denoted by 0. Fig. 2 shows the partitioning for one node.

Then a nodal voltage can be written in the following form:

$$V_i(z) = \mathbf{H}_i(\mathbf{z}) \left[\sum_{j=1}^{N} \left(\sum_{l=1}^{k_{i,j}} \mathbf{G}_{i,j,l}(\mathbf{z}) \right) V_j(z) + \left(\sum_{l=1}^{k_{i,0}} \mathbf{G}_{i,0,l}(\mathbf{z}) \right) V_{IN}(z) \right]. \tag{4}$$

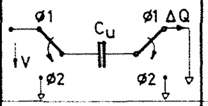
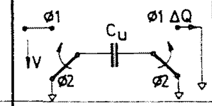
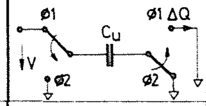
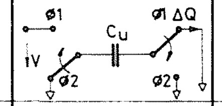
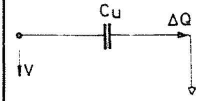
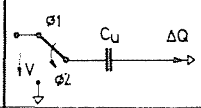
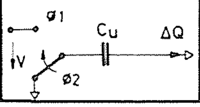
In equation (4) N indicates the number of the op amps, e.g. the number of the nodal voltages in the filter; $k_{i,j}$ indicates the number of the parallel connected VCCSs between the nodes i and j. If there is no direct connection between the nodes i and j via a VCCS the corresponding $\mathbf{G}$ matrices are 0 matrices.

The application of the procedure for every node leads to a system of function equations completely characterizing the SC filter.

$V_{IN}(z)$ is defined as a sampled and held signal. It is held during the whole clock period.

$$V_{IN}(z) = V_{IN}^{\varphi 1}(z)[1, z^{-1/2}]. \tag{5}$$

Table 2

	VCCS	$G^{\phi1,\phi1}(Z)$	$G^{\phi1,\phi2}(Z)$	$G^{\phi2,\phi1}(Z)$	$G^{\phi2,\phi2}(Z)$
1		C_u	0	0	0
2		0	0	0	C_u
3		0	$-C_u Z^{-1/2}$	0	0
4		0	0	$-C\ Z^{-1/2}$	0
5		$C_u(1-Z^{-1})$	0	0	$C_u(1-Z^{-1})$
6		C	$-C_u Z^{-1/2}$	0	0
7		0	0	$-C_u Z^{-1/2}$	C

Using the substitution $z_i = \exp(j2\pi f_i/f_c)$ the (4) system of function equation can be solved point by point in f_i frequencies.

In a general case, the number of the variables is $2N$. If any of the nodal voltages is held during the whole clock period the number of the variables in (4) can be reduced with the following substitution

$$V_i(z) = V_i^{\varphi 1}(z)[1 \quad , \quad z^{-1/2}] \tag{6}$$
$$V_i(z) = V_i^{\varphi 2}(z)[z^{-1/2}, \quad 1 \quad] \tag{7}$$

depending on the phase the nodal voltage changes in. If all of the nodal voltages are held during the whole clock period, the number of the variables is N.

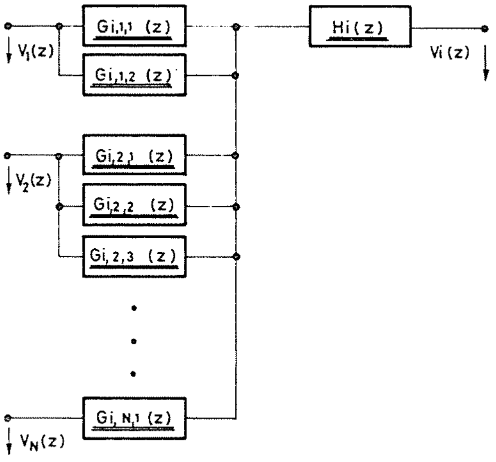


Fig. 2. Partitioning for one node

Nonidealities

The finite op amp's gain causes two effects:

(i) the SC filter loses the complete insensitiveness to parasitics,

(ii) the CVCVSs loses the independent summation properties of their inputs,

(i) means that the top plate to substrate parasitics must be taken into consideration,

(ii) means that we have to establish a new model for analysing the VCCSs and CVCVSs. It is easy to admit that only a common model is good for computing the transfer functions.

Figure 3. showes the model for one type of CVCVS.

On the Fig. 3 C_1–C_5 branches represent the effects of the other VCCSs connected to the same CVCVS. Computing the transfer functions for all kinds of active inputs a new set can be established. These transfer functions are voltage transfer functions. To use the formalism given in (4) we are allowed to split these transfer functions into two parts. The common part will be the transfer function of the nonideal CVCVS.

The whole set of the transfer functions of the nonideal VCCSs is presented in Table 3 for the model given in Fig. 3.

The library of the transfer functions was enlarged with the nonideal transfer functions. The computation of the effect of the nonidealities was checked by example given in Fig. 4.

The measured and the computed results are given in Fig. 5.

Table 3

VCCS	$G^{\varphi1,\varphi1}(z)$	$G^{\varphi1,\varphi2}(z)$
1	$\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z^{-1})}{1-z^{-1}\dfrac{(1+P^2)}{(1+P+Q)(1+P+R)}}$	$\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z)\dfrac{1+P}{1+P+R}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$
2	0	0
3	$-\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z^{-1})z^{-1/2}\dfrac{1+P}{1+P+R}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$	$-\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z^{-1})z^{-1/2}\dfrac{1+P+Q}{1+P+R}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$
4	0	0
5	$\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z^{-1})\left(1-z^{-1}\dfrac{1+P}{1+P+R}\right)}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$	$-\dfrac{Cu}{A(1+P+Q)}\,\dfrac{(1+A)(1-z^{-1})\dfrac{Q}{1+P+R}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$

VCCS	$G^{\varphi2,\varphi1}(z)$	$G^{\varphi2,\varphi2}(z)$
1	0	0
2	$\dfrac{Cu}{A(1+P+R)}\,\dfrac{(1+A)(1-z^{-1})\dfrac{1+P}{1+P+Q}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$	$\dfrac{Cu}{A(1+R+R)}\,\dfrac{(1+A)(1-z^{-1})}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$
3	0	0
4	$-\dfrac{Cu}{A(A+P+R)}\,\dfrac{(1+A)(1-z^{-1})z^{-1/2}\dfrac{1+P+R}{1+P+Q}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$	$-\dfrac{Cu}{A(1+P+R)}\,\dfrac{(1+A)(1-z^{-1})z^{-1/2}\dfrac{1+P}{1+P+Q}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$
5	$-\dfrac{Cu}{A(1+P+R)}\,\dfrac{(1+A)(1-z^{-1})\dfrac{R}{1+P+R}}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$	$\dfrac{Cu}{A(1+P+R)}\,\dfrac{(1+A)(1-z^{-1})\left(1-z^{-1}\dfrac{1+P}{1+P+Q}\right)}{1-z^{-1}\dfrac{(1+P)^2}{(1+P+Q)(1+P+R)}}$

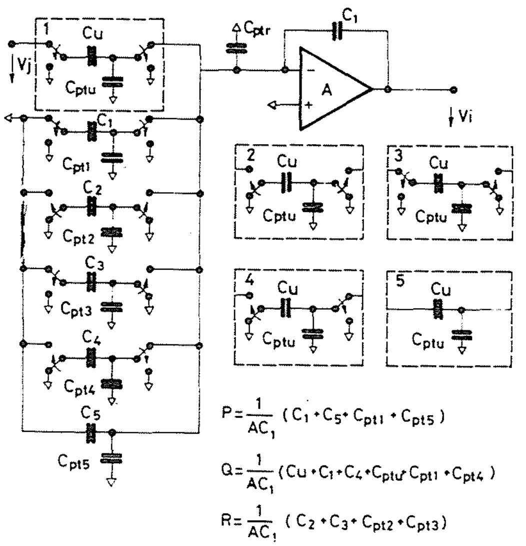


Fig. 3. Nonideal model

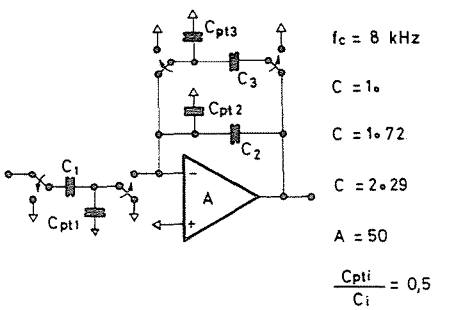


Fig. 4. Nonideal example

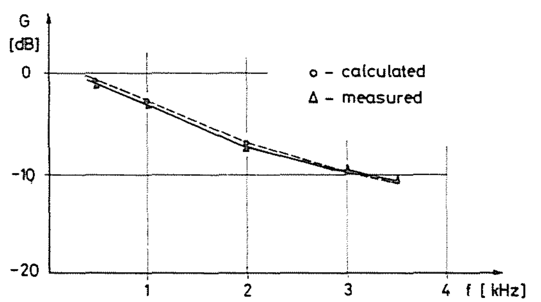


Fig. 5. Measured and computed results

Implementation

The presented technique was implemented on an IBM 4331 computer using double precision arithmetic. The program ANSC (Analysis of Switched-Capacitor filters) has a lot of services. It computes the discrete time frequency response and capable to solve automatically the problem of the dynamic range maximization [4] working as an efficient tool for designing SC filters.

The resulted computational time was low enough to apply this technique for statistical tolerance analysis. The program computes large-change sensitivities, too.

Example

To demonstrate the efficiency of the presented technique a 14th order elliptic bandpass SC filter with coupled biquad structure was chosen. The filter consists of 14 op amps and 68 capacitors. A 300 sample tolerance analysis in 9 frequency points was made. The computational time was 3363 seconds.

Conclusions

A fast and efficient technique is presented for analysing active SC filters. The technique is based on special block partitioning of the circuit and makes use of the topological constraints established in practical SC filters. The computational time was low enough to use for statistical tolerance analysis. The finite op amps gain and top plate to substrate parasitics were considered.

References

1. LIOU, M. L.–KUO, Y. L.: "Exact analysis of switched-capacitor circuits with arbitrary inputs", IEEE Trans. Circuits Syst., vol. CAS–26, pp. 213–223, Apr. 1979.
2. DEMAN, H. J.: "Practical implementation of a general CAD technique for SC circuits", IEEE J. Solid-State Circuits vol. SC–15, pp. 190–200, Apr. 1980.
3. KURTH, C. F.–MOSCHYTZ, G. S.: "Nodal analysis of switched-capacitor networks", IEEE Trans. Circuits Syst., vol. CAS–26, pp. 93–105, Febr. 1979.
4. MARTIN, K. et al.: "Designing leap-frog and SFG filters with optimum dynamic range", Proc. IEEE vol. 65, pp. 1210–1211, Aug. 1979.
5. FÜLÖP, T.–PÖRNECZI, T.: "Effects of the op amp dynamic behaviour in switched-capacitor filters", PROC. SSCT '82. Prague, pp. 244–248, July. 1982.
6. FÜLÖP, T.–PÖRNECZI, T.: "On the design of switched-capacitor bandpass filters. A case study," PROC. ECCTD '83. Stuttgart, Sept. 1983.
7. PÖRNECZI, T.: "Design of active SC ladder filters", Ph. D. thesis, Technical University Budapest, 1983. (in Hungarian).
8. PÖRNECZI, T.: "Fast analysis of active switched-capacitor filters", PROC. ECCTD '85. Prague, pp. 709–712. Sept. 1985.

Tamás PÖRNECZI H-1143 Budapest, Hungária krt. 126–132.