MONTE CARLO ANALYSIS OF SC FILTERS

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Summary

The Monte Carlo method is used to perform the statistical analysis of MOS integrated SC filters. The basic ideas and services of the computer program SCAMON are presented. The detailed topics are: (i) statistical modelling and simulation of the circuit elements in SC filters at the electric parameter and the geometrical-technological parameter level; (ii) the problem of quickly repeated SC circuit analysis; (iii) the statistical attributes of the circuit function and their consistent estimators.

Introduction

Some technological effects should be taken into account at the electric parameter design stage of switched-capacitor (SC) filters [1]. To calculate the effects of typically random variations of the circuit component values arising in a real technological process a circuit designer needs a computer program performing statistical circuit analysis.

Basic assumption of the statistical analysis is the following: the electric parameters of the network components and the performance function of the network (for example the frequency transfer characteristics) are considered as probability variables.

The task of the statistical circuit analysis is to determine the statistical parameters of the performance function (expected value, variance, probability density function, probability of meeting a given specification, regression . . .) when the statistical parameters of the circuit components are given (specified). [8]

The Monte Carlo procedure is a typical choice to perform statistical analysis of an electronic circuit.

The general scheme of the Monte Carlo procedure is illustrated in Fig. 1.

The Monte Carlo program starts by inputting statistical data, specification on the circuit performance and other control data.

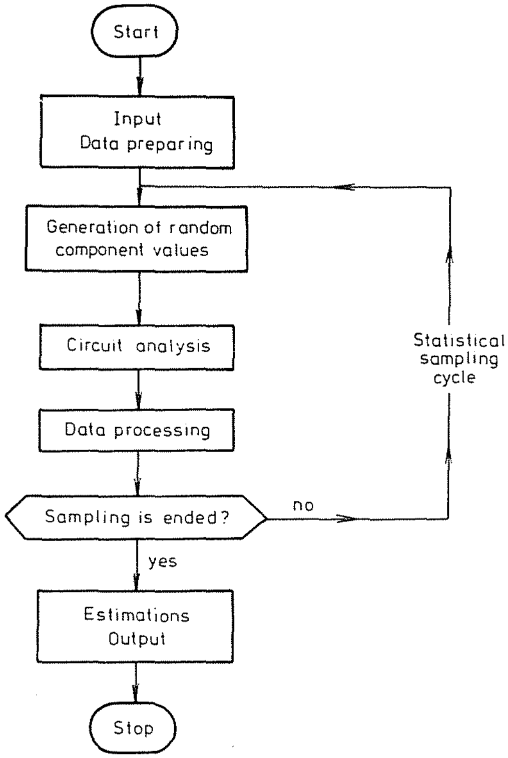

Fig. 1. Scheme of the Monte Carlo circuit analysis

After the input the data structure is converted into a proper form to performe the sampling cycle as quick as possible. In the Monte Carlo cycle first the circuit elements are assigned random values—controlled by their user specified statistical parameters—then using these random values of network elements a circuit analysis is performed. The result—as a random value of the circuit performance function—is processed and stored in a proper form for later computation.

This cycle is repeated n times, so a statistical sample of the performance function with the sample size n is obtained. From this sample the statistical parameters of the network function (expected value, standard deviation, yield . . .) are systematically estimated [8].

The Monte Carlo analysis terminates after the user oriented output of the statistical results.

The computer program SCAMON was developed to perform Monte Carlo analysis of switched-capacitor filters. In the following we shall investigate some details of statistical modelling and the problem of the repeated circuit analysis.

Statistical modelling

Switched capacitor circuits consist of monolithic integrated capacitors, switching transistors and operational amplifiers. The frequency selective transfer characteristics of the SC circuits is essentially determined by the value or the ratio of the capacitances. Therefore at the first approach the capacitances have to be considered as probability variables, have to be handled with tolerances during the Monte Carlo simulation.

There are two different levels of the modelling of the technological statistical effects:

a) "phenomenological" level: statistical modelling of the electric parameters,

b) deeper level: statistical modelling of the geometrical and the technological parameters.

In the SCAMON program there are possibilities of the capacitor modelling at both levels. The op.amp.s can be handled as ideal ones or with normally distributed finite gain. The switches are always considered as ideal switches. In the following the modelling of the capacitors will be detailed.

Modelling at the electric parameter level

The capacitances are always simulated as normally distributed random values. The normally distributed capacitor value C_i is described by its expected value C_{i0} as its nominal value and by its standard deviation σ_i, as its tolerance data. Using ϑ_i with standard distribution (zero mean, unit variance) the random value (with the specified statistical character) of C_i is:

$$C_i = C_{i0} + \sigma_i \cdot \vartheta_i. \tag{1}$$

The standard deviation of C_i is often given in percentage related to the nominal value:

$$\sigma_i' = 100 \cdot \frac{\sigma_i}{C_{i0}} \quad (\%).$$

In special cases the absolute or the relative standard deviations are constant in the circuit:

$$\sigma_i = \sigma_o, \qquad \sigma_i' = \frac{100 \cdot \sigma_0}{C_i}$$

or

$$\sigma_i = \frac{\sigma_0'}{100} \cdot C_i, \qquad \sigma_i' = \sigma_0'$$

respectively.

To avoid process-caused systematic capacitance-ratio errors MOS capacitors are often realized as the parallel combinations of several smaller unit capacitors. These provide constant area-to-perimeter ratio. In that case

$$C_i = n_i \cdot C_0$$

where C_0 is the unit capacitance. Assuming that the random errors are uncorrelated and normally distributed the following can be derived: the absolute standard deviations are proportional and the relative standard deviations are inversely proportional to the square root of the nominal value:

$$\sigma_i = \sqrt{n_i} \cdot \sigma_0, \qquad \sigma_i' = \frac{\sigma_0'}{\sqrt{n_i}}$$

where σ_0 is the standard deviation of the unit capacitor.

In the circuit there are m capacitors. Let us introduce the vectors C, C_0 and the matrix $\Sigma = \text{diag}(\sigma_i)$ representing the actual C_i and the nominal values C_{i0} and the standard deviations of the capacitances σ_i, $i=1, 2, \ldots, m$ respectively. Then (1) can be written in the more general form:

$$C = C_0 + \Sigma \cdot \vartheta \tag{2}$$

where the vector ϑ has m different uncorrelated, normal, standard distributed random values.

When we use (2) we get uncorrelated random capacitance values. But in many cases the random errors of the monolithic integrated capacitors can't be considered to be independent variables. The statistical interdependence between the capacitances C_i and C_j can be represented by their correlation coefficients $r_{i,j}$, $i, j=1, 2, \ldots, m$. The whole correlation feature of all capacitances in the circuit is described by the correlation matrix R consisting of the correlation coefficients. In the diagonal of the matrix there are units ($r_{i,i}=1$, $i=1, 2, \ldots, m$). The correlation matrix is always a positive definite matrix.

To generate the random vector C with a given expected value vector C_0 and with given standard deviations and correlation matrix R we use the linear transformation

$$C = C_0 + \Sigma \cdot A \cdot \vartheta \tag{3}$$

where ϑ is a standard distributed random vector (with uncorrelated elements) and A is the steering matrix related to the correlation matrix as

$$A \cdot A^t = R \tag{4}$$

(A^t is the transpose of A).

The simplest way to generate decomposition of a symmetric, positive matrix according to (4) is to determine the LU factorization of the correlation matrix R. (LU factorization means the lower and upper triangular matrix decomposition.)

The elements $a_{i,j}$ of the lower tringular form steering matrix A can be computed from the correlation coefficients $r_{i,j}$ as follows:

$$a_{1,1}=1, \qquad a_{i,1}=\sqrt{r_{1,i}}, \qquad i=1, 2, \ldots, m$$

$$a_{i,i}= \quad 1-\sum_{k=1}^{i-1} a_{i,k}^2 \qquad i=2, 3, \ldots, m$$

$$a_{j,i}=\frac{r_{i,j}-\sum_{k=1}^{i-1} a_{i,k}\cdot a_{j,k}}{a_{i,i}} \qquad j=i+1, \ldots, m.$$

In the SCAMON program's input there are two possibilities to specify the correlations of the capacitances that is, to specify the correlation matrix. In the case of a global specification only one r correlation coefficient has to be given and the correlation matrix will be filled in by that value: $r_{i,j}=r$, $i\neq j$. It results in the same correlation coefficient r between all C_i, C_j pairs.

There is also a possibility to specify locally different correlations. In that case to all capacitors C_i maximum six $C_{i_1}, \ldots, C_{i_6}$ capacitor pairs can be designated with different correlations r_{i,i_j}. This local specification can be used when the lay-out dependent statistical interdependence must be taken into account. (For example sometimes the neighbouring components have more significant correlations.)

Modelling at the geometrical-technological parameters level

Consider a rectangle shaped MOS capacitor (Fig. 2). Its capacitance is

$$C_i=\varepsilon_i\cdot\frac{x_i\cdot y_i}{t_i}$$

where ε_i and t_i are the permittivity and thickness of the SiO_2 layer, x_i, y_i are the

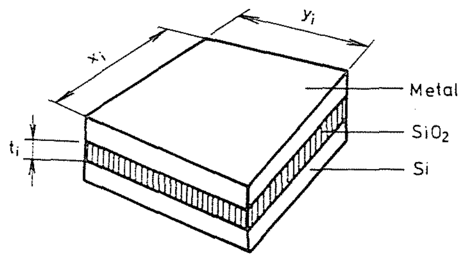

Fig. 2. Rectangle shaped MOS capacitor

edge length in the x- and y-directions, respectively. All these parameters are supposed to be random variables with normal distribution [2].

To characterize the statistical feature of the oxid layer parameters we use the expected values ε_0, t_0 the standard deviations σ_ε, σ_t and the correlations r_ε, r_t. These user specified global parameters are constant for all capacitors of the circuit. Using the standard distributed global random numbers $\vartheta_{0\varepsilon}$, ϑ_{0t} and local random numbers $\vartheta_{i\varepsilon}$, ϑ_{it}, $i=1, 2, \ldots, m$ we get the random value for the i-th permittivity and thickness:

$$\varepsilon_i=\varepsilon_0+\sigma_\varepsilon\cdot\sqrt{r_\varepsilon}\cdot\vartheta_{0\varepsilon}+\sigma_\varepsilon\cdot\sqrt{1-r_\varepsilon}\cdot\vartheta_{i\varepsilon}$$

$$t_i=t_0+\sigma_t\cdot\sqrt{r_t}\cdot\vartheta_{0t}+\sigma_t\cdot\sqrt{1-r_t}\cdot\vartheta_{it}.$$

The random values of the rectangle shaped capacitor edge lengths are generated by the superposition of global and local random errors. The source of the global error could be the error of the mask position. These errors are equal for all capacitances in the circuit and are specified by their expected values Δx_0, Δy_0 and standard deviations σ_{0x}, σ_{0y} in the x- and y-directions, respectively.

The local errors are different for the capacitances and could be used to describe local inaccuracies of the mask geometry and of the lithographic process. These errors are also specified by their expected values Δx_i, Δy_i and standard deviations σ_{ix}, σ_{iy}, $i=1, 2, \ldots, m$.

The resultant random values are:

$$x_i=x_{0i}+\Delta x_0+\sigma_{0x}\cdot\vartheta_{0x}+\Delta x_i+\sigma_{ix}\cdot\vartheta_{ix}$$

$$y_i=y_{0i}+\Delta y_0+\sigma_{0y}\cdot\vartheta_{0y}+\Delta y_i+\sigma_{iy}\cdot\vartheta_{iy}$$

$$i=1, 2, \ldots, m$$

where ϑ_{0x} and ϑ_{0y} are global (constant for all capacitances) and ϑ_{ix}, ϑ_{iy}, $i=1, 2, \ldots, m$ are local, independent random numbers with standard Gaussian distribution.

Repeated circuit analysis

When using the Monte Carlo method the most significant factor affecting the total CPU time of the computer program is the time required for one circuit analysis. Therefore it is very important to use the fastest analysis method. At the choice of the suitable method we have to find the trade-off between the wide-ranging applicability and the CPU time needed for one circuit analysis.

We get the quickest method when the repeated circuit analysis means repeated evaluation of a given and not too complex explicit formula. Of course the given formula is applicable in the case of a given circuit with fixed structure (fixed circuit topology). In that case to analyse an other circuit we have to use a new program with a new formula. This extreme type of the repeated circuit analysis provides the fastest, but the most special application of the Monte Carlo program.

The other extreme is the use of a general purpose circuit analysis method based on some kind of topological description of the circuit without any restrictions of the possible circuit topology (nodal-, modified nodal-, state space analysis, etc.). In that case the repeated circuit analysis needs repeated building up of a set of equations and then their solution. Using this method almost all kind of circuits with different structures can be analysed, with the same program on the other hand this type of obtaining the random value of the circuit performance function as a function of the random network parameters is very time consuming comparing to the fixed formula evaluation and the repeated use of it in a Monte Carlo program leads to unacceptable increase of computer time.

A good trade-off between the above-mentioned two extremes is obtained when we accept a practically not too strict restriction of the possible circuit topology and so only a reduced set of equations has to be repeatedly built up and solved.

In the SCAMON program there are two types of circuit analysis:

— analysis of the Fleischer-Laker circuit by the known explicite formula,

— analysis of stray-insensitive SC circuits.

In the following these two possibilities will be detailed.

Analysis of the Fleischer-Laker circuit

As the cascade approach of active filter design is very much in use and as the general purpose biquad is a building block in this design, it is a circuit of extreme importance. Its z-domain transfer function is

$$H(z)=\frac{a+b\cdot z^{-1}+c\cdot z^{-2}}{d+e\cdot z^{-1}+f\cdot z^{-2}}.$$

In the competition of numerous variants the FL general purpose active SC biquad seemed to be the most frequently used and referenced. It was developed by Fleischer, Laker and other in the Bell Laboratories [4]. It is realized in an NMOS mask programmable building block chip capable of up to 22 poles of switched-capacitor filtering.

The schematic circuit diagram of the FL building block is shown in Fig. 3. It is always assumed that the clocks which drive the switches are biphase nonoverlapping pulse trains with 50 per cent duty cycles and that the input is sampled-and-held for the full clock period. The transfer function as an explicit

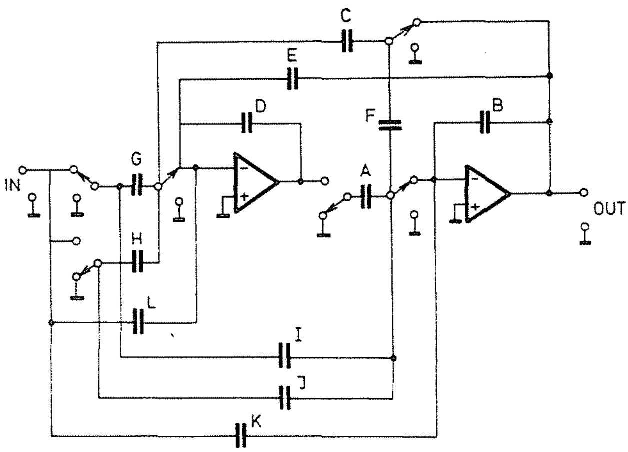

Fig. 3. Circuit diagram of the FL biquad

formula of the capacitances is the following:

$$H(z)=$$

$$=-\frac{D(I+K)+(AG+AL-DI-DJ-2DK)z^{-1}+(DJ+DK-AH-AL)z^{-2}}{D(F+B)+(AC+AE-DF-2BD)z^{-1}+(DB-AE)z^{-2}}$$

By evaluating this formula the SCAMON program is able to perform a repeated analysis of cascade structured SC circuits built up from FL biquads very quickly.

Analysis of stray-insensitive SC circuits

To avoid the use of the time consuming SC analysis methods that can handle all kinds of topologies we have to accept some restrictions on the topology of the circuit. When we want to perform Monte Carlo analysis of an SC circuit by the SCAMON program we have to accept the following topological constraints:

— the noninverting inputs of the operational amplifiers must be connected to ground the inverting inputs are always virtual grounds,
— both plates of all capacitors must be connected to a voltage source or to a virtual ground or ground,
— the switches are always switched between a voltage source and ground or between ground and virtual ground.

In the practical SC filter implementations these restrictions assure elimination of the degrading effects of parasitics and common mode gain. This stray-insensitive topology gives the opportunity of partitioning the circuit into elementary subnetworks [5]. These so-called functional blocks are individually characterized by their transfer function. The whole set of functional blocks and the transfer functions are presented in [7].

A computer algorithm was developed to generate the set of linear equations from the user specified circuit topology based on the library of the functional blocks [6]. The size of the set of equations is 2N × 2N where N is the number of the op. amp.s in the circuit. You can find more detailed informations on this type of SC circuit analysis in [7].

The SCAMON program analysis routine computes the frequency response $y(f)$ of the circuit. When we consider the sampled output as a discrete time sequence, the amplitude-frequency transfer characteristics of the circuit is

$$y(f)=|H(z=e^{2\pi\cdot f/f_c})|$$

where f_c is the clock (=sampling) frequency.

If the input sampling frequency f_s is smaller that the clock frequency f_c of the circuit (sample repeating sampling) then frequency domain characteristics of the output sequence is:

$$y(f)=\frac{\sin(\pi\cdot f/f_s)}{\pi\cdot f/f_s}\cdot\frac{\pi\cdot f/f_c}{\sin(\pi\cdot f/f_c)}\cdot H(z=e^{2\pi\cdot f/f_c}).$$

When the output is sampled-and-held for the full clock period continuous time output signal then the frequency domain transfer characteristics of the SC circuit is:

$$y_h(f)=y(f)\cdot\frac{\sin(\pi\cdot f/f_c)}{\pi\cdot f/f_c}.$$

The statistical attributes of the network function and their estimations

By repeating the random value generation, circuit analysis and data assembling cycle n-times we get a random sample of size n for the random variable $y(f)$. From that sample—using the standard, elementary techniques of the mathematical statistics—we can estimate the most important statistical attributes of $y(f)$ [8]. In the following these quantities and their estimators are listed ($-$ is the symbol of sample averaging).

Expected values:

$$\hat{E}\{y(f)\}=\overline{y(f)}.$$

Standard deviation:

$$\sqrt{\hat{\text{var}}\{y(f)\}}=\hat{\sigma}_y=\sqrt{\frac{n}{n-1}\left(\overline{y^2(f)}-\overline{y-(f)}^2\right)}.$$

Experienced sample range:

$$\hat{b}_l=\min_i\{y_i(f), \quad i=1,2,\ldots,n\}$$

$$\hat{b}_u=\max_i\{y_i(f), \quad i=1,2,\ldots,n\}.$$

Linear regression coefficients:

$$\hat{\text{reg}}\{y(f),C_j\}=\frac{\overline{y(f)\cdot C_j}-\overline{y(f)}\cdot\overline{C_j}}{\overline{C_j^2}-\bar{C}_j^2}.$$

The regression coefficient of the network function $y(f)$ versus the capacitance value C_j can be considered as a statistical sensitivity measure.

Histograms: This estimator of the probability density function of $y(f)$ is an equidistant histogram consisting of 12 intervals. The position of the histogram can be directly specified by input data or can be automatically determined by the program. In this second case the experienced range of the first 50 sample elements determine the lengths and the positions of the intervals.

All of these above-mentioned attributes are functions of frequency so they are computed in all frequency points.

The most important statistical result of a Monte Carlo analysis is the yield that is the probability of satisfying a given specification. In the case of the SCAMON program there is possibility to give more specifications (max. 3). The i-th specification means:

$$S_i\colon a_{\text{lower},i}(f_j)\le y(f_j)\le a_{\text{upper},i}(f_j), \qquad f_j\in\Omega_i$$

and the i-th partial yield is:

$$Y_i=\Pr\{S_i \text{ is satisfied}\}.$$

The total yield:

$$Y=\Pr\{\text{all the } S_i \text{ specifications are satisfied}\}.$$

The yields are estimated by the relative frequencies.

All the estimators used in the SCAMON program are consistent estimators.

Conclusion

One of the most important aids a circuit designer can use is a computer program performing statistical analysis of the circuit. This computer supported service allows the designer to take into account the statistical effects of the random variations of the circuit components on the network performance function.

The problem of the statistical circuit analysis is a complex, multidimensional, nonlinear problem. The typical method used to solve this problem is the Monte Carlo method, which is a computer oriented simulation method based on the sampling– and the estimation theory of mathematical statistics.

In this paper the basic principles of the computer program SCAMON were presented.

To obtain realistic results one needs to use realistic modelling and simulation. This program allows the user to specify the statistical parameters of the monolithic integrated capacitors both at the level of electric parameters and at the level of geometrical-technological parameters.

One of the most important questions of an effective Monte Carlo simulation is how quickly can one circuit analysis be carried out. The SCAMON program can quickly analyse the filters built up of Fleischer-Laker biquad blocks (by evaluating an explicit formula) and the stray-insensitive circuits (by building up and solving a reduced set of linear equations).

The statistical results of the Monte Carlo simulation are produced in the form of consistent estimators.

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