CLASSICAL FIELD THEORY IN THE EDUCATION OF MICROELECTRONICS*

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Summary

The viewpoints of semiconductor-physics are overemphasized, the electrodynamic concepts are practically neglected in the education of microelectronics. The education of electromagnetic concepts may be fertilized by the microelectronic problems either introducing the electromagnetic background or applying the real structures as examples for electrodynamic concepts.

Introduction

The fundamental phenomena taking place in microelectronic structures, semiconductor devices, monolitic and hybrid IC-s are basically electromagnetic ones. In spite of this fact the viewpoint of semiconductor physics are overemphasized, the electrodynamic concepts are practically neglected in the education of these fields.

In the authors opinion the education of the electromagnetic concepts may be fertilized by the microelectronic problems either introducing the electromagnetic background or applying the real structures as examples for electrodynamic concepts.

In the following the authors try to touch both lines: the first in connection with the semiconductor governing equations and the second showing various examples.

Deduction of the semiconductor equations from Maxwell's equations [1] [2] [3]

In this part of the paper it is shown that the governing equations can be deduced from Maxwell's equations in a consequent manner. The results obtained in this way are called *classical*, the others will be *quasi-classical* assumptions. Other approximations are not necessary.

* Dedicated to Professor Károly Simonyi on the occasion of his Seventieth Birthday

Maxwell's equations

The usual form of Maxwell's equations in SI system is used.

$$\mathrm{rot}\,\bar{H}=\bar{J}+\frac{\partial\bar{D}}{\partial t} \tag{1}$$

$$\mathrm{rot}\,\bar{E}=-\frac{\partial\bar{B}}{\partial t} \tag{2}$$

$$\mathrm{div}\,\bar{D}=\rho \tag{3}$$

$$\mathrm{div}\,\bar{B}=0 \tag{4}$$

The constitutional equations may be of different form. However up to medium frequencies the scalar momentarily interaction between the field quantities is expectable i.e.

$$\bar{D}=\varepsilon\bar{E}\,,\qquad \bar{B}=\mu\bar{H}\,. \tag{5a, b}$$

As for the electric current density the expressions will be introduced later. In the microelectronic structures ε and μ are assumed piecewise constant.

Poisson's equation

To describe the internal behaviour of a semiconductor structure Maxwell's equations are replaced by Poisson's equation. A rather common misunderstanding connected to it, that Poisson's equation can be used only for small frequencies.

One can prove in an easy way the opposite, following the consequent derivation of this equation.

From Eq. (4) the

$$\bar{B}=\mathrm{rot}\,\bar{A} \tag{6}$$

equality follows, where $\bar{A}$ is the vector potential. Applying Eq. (2) and introducing the scalar potential φ one obtains

$$\bar{E}=-\mathrm{grad}\,\varphi-\frac{\partial\bar{A}}{\partial t} \tag{7}$$

while

$$-\varepsilon\Delta\varphi-\varepsilon\frac{\partial}{\partial t}\mathrm{div}\,\bar{A}=\rho\,. \tag{8}$$

Eq. (8) returns to Poisson's equation if

$$\mathrm{div}\,\bar{A}=0 \tag{9}$$

is valid. It means the choice of *Coulomb gauge* and the result is the following

$$\Delta\varphi(t) = -\frac{\rho(t)}{\varepsilon} \tag{10}$$

where the time dependence is explicitly emphasized.

In this way the validity of (10) can be accepted, and as a side-result the concept of gauge can be introduced.

Of course, the approximation must appear elsewhere. One can try to formulate the magnetic equations

$$\bar{H} = \frac{1}{\mu}\operatorname{rot}\bar{A} \tag{11}$$

and substituting to Eq. (1)

$$\frac{1}{\mu}\operatorname{rot}\operatorname{rot}\bar{A} = \bar{J} - \varepsilon\frac{\partial}{\partial t}\operatorname{grad}\varphi - \varepsilon\frac{\partial\bar{A}}{\partial t} \tag{12}$$

where applying the Coulomb gauge (9) again

$$\Delta\bar{A} - \varepsilon\mu\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J} + \varepsilon\mu\frac{\partial}{\partial t}\operatorname{grad}\varphi\,. \tag{13}$$

The current density can be split to a transversal (divergenceless) and a longitudinal (curlless) part, i.e.

$$\bar{J} = \bar{J}_t + \bar{J}_l = \operatorname{rot}\bar{\Gamma} - \operatorname{grad}\psi\,. \tag{14}$$

And obviously—corresponding to Eq. (9)—

$$\Delta\bar{A} - \varepsilon\mu\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J}_t\,, \tag{15}$$

while

$$\mu\operatorname{grad}\psi + \varepsilon\mu\frac{\partial}{\partial t}\operatorname{grad}\varphi = 0\,, \tag{16}$$

i.e.

$$\psi = -\varepsilon\frac{\partial\varphi}{\partial t} + f(t)\,. \tag{17}$$

Now one can follow the neglections. Generally $\bar{J}_t \cong 0$ is assumed, which corresponds to $|\bar{J}_t| \ll |\bar{J}_1|$. In this case $\bar{A} = 0$ is a particular solution of Eq. (15) and the vector potential is neglected in the calculation of the electric field strength in Eq. (7). It is obvious that this kind of neglection may be proved case by case only. But a simple estimation can be done easily on the basis of Eq. (7). The second term contains time derivation, which gives a smaller order if the variation of the potentials is not very quick.

It is in good correspondence with the usual practice again, that the magnetic field effect is generally neglected as a second order.

In one-dimension the whole question is irrelevant. The curlless property means $J_z=0$ and its consequence $A_z=0$ is correct, the calculation on the basis of Poisson's equation is without any assumption not speaking about the $1D$ approach itself.

The continuity equations

What is to say about the longitudinal part of the current? Its divergence has the following property (see Eq. (17))

$$\operatorname{div} \bar{J}_l = -\Delta\psi = \varepsilon \frac{\partial}{\partial t} \Delta\varphi = -\frac{\partial \rho}{\partial t}$$

i.e. the continuity equation is satisfied by the longitudinal current only. Taking $\bar{J}=\bar{J}_l$

$$\operatorname{div} \bar{J} + \frac{\partial \rho}{\partial t} = 0 \tag{18}$$

is a *classical result* as well: the overall continuity equation.

If one takes

$$\rho = q(p-n+N) \tag{19}$$

and

$$\frac{\partial N}{\partial t} = 0$$

can be assumed, then Eq. (18) can be split formally to two equations

$$\operatorname{div} \bar{J}_n - q\frac{\partial n}{\partial t} = R \tag{20a}$$

$$\operatorname{div} \bar{J}_p + q\frac{\partial p}{\partial t} = -R\,. \tag{20b}$$

It is obvious that (20) has meaning only in the case if R can be filled by physical content. It means that Eq. (20) is a classical approach as far as Eq. (19) can be taken as classical, but the detailed expression of R can be obtained only on semi-classical assumptions.

The expressions for currents and mobile carrier densities

Far more difficult to explain the expressions of currents. The simple equation

$$\bar{J}=\sigma\bar{E}+\bar{J}_i \tag{21}$$

only paves the way, how to find the correct expressions in classical way.

On the basis of physical electronics one can explain, that σ must be proportional to the mobile charge density and the proportionality factor can be called *mobility*. However one cannot tell too much about either the behaviour of the mobility or its dependence on the different quantities.

After having expressed the currents above

$$\bar{J}_n=qn\mu_n(-\text{grad}\ \varphi) \tag{22a}$$

$$\bar{J}_p=qp\mu_p(-\text{grad}\ \varphi) \tag{22b}$$

which supposes the validity of $\bar{A}=0$. They are the so-called *drift currents*, the currents of electrodynamic origin.

It can be demonstrated that currents can flow even when $\bar{E}\equiv 0$, but it is represented above by a faceless current $\bar{J}_i$. If one knows something about non-equilibrium thermodynamics, he or she can be led to the recognition: every current has to depend on the gradient of an intensive quantity. Neglecting the temperature, pressure etc, gradients, the only remaining possibility is the gradient of a non-uniform carrier distribution.

So we can illustrate the existence of diffusion currents

$$\bar{J}_n=qD_n\ \text{grad}\ n \tag{23a}$$

$$\bar{J}_p=-qD_p\ \text{grad}\ p\,. \tag{23b}$$

But it has to be emphasized: *these equations are not classical.* One can recognize it if we wish to express the carrier densities as the function of potential. The only hopeful situation is the no-current state. Combining (22) with (23) it means

$$q(-\mu_n n\ \text{grad}\ \varphi+D_n\ \text{grad}\ n)=0\,, \tag{24a}$$

$$q(-\mu_p p\ \text{grad}\ \varphi-D_p\ \text{grad}\ p)=0\,. \tag{24b}$$

From these two equations

$$n=Ae^{\frac{\mu_n}{D_n}\varphi} \tag{25a}$$

$$p=Be^{-\frac{\mu_p}{D_p}\varphi}. \tag{25b}$$

With the usual approach $\mu_n/D_n=\mu_p/D_p=q/kT$ (the validity is out of the scope

of EM field theory) one can write

$$n = n_i e^{\frac{q}{kT}(\varphi - \varphi_n)}, \tag{26a}$$

$$p = n_i e^{\frac{q}{kT}(\varphi_p - \varphi)} \tag{26b}$$

where n_i, $\varphi_n \varphi_p$ appear formally as constants.

If one regards φ_n and φ_p as variables, then the currents can be expressed as

$$\bar{J}_n = -q\mu_n n \operatorname{grad} \varphi_n = q\mu_n n_i \frac{kT}{q} e^{\frac{q}{kT}\varphi} \operatorname{grad} e^{-\frac{q}{kT}\varphi_n} \tag{27}$$

$$\bar{J}_p = -q\mu_p p \operatorname{grad} \varphi_p = -q\mu_p n_i \frac{kT}{q} e^{-\frac{q}{kT}\varphi} \operatorname{grad} e^{\frac{q}{kT}\varphi_p}. \tag{28}$$

At this point it is suitable to introduce the concept of equilibrium. Let us form the product

$$np = n_i^2 e^{\frac{q}{kT}(\varphi_p - \varphi_n)}. \tag{29}$$

If $pn = n_i^2$ the semiconductor is in *equilibrium* state. In this case $\varphi_p = \varphi_n = \varphi_F$, where φ_F is the Fermi-level. After this property φ_p and φ_n are called quasi-Fermi-level in the non-equilibrium case.

The equilibrium is a concept out from the classical frame.

A new question: what is the condition of the equality $n = p$. It turns over a new concept, the concept of (quasi-) *neutrality*.

The neutrality means $\rho = 0$, i.e. by eq. (19)

$$p - n + N = 0. \tag{30}$$

Here is another point where a great deal of misunderstanding has appeared in the literature. The problem is: the condition of Eq. (30) is generally accepted but in Eq. (10) ρ is not counted zero, the potential is not harmonic.

The ambiguous situation is the result of the inconsequent derivation of Eq. (30), which appeared heuristically. In the reality (30) is the result of the scaling of Eq. (10). After having applied the scale factors of Table 1 the

Table 1

Quantity	Symbol	Value
$\bar{r}$	l	$\max(\bar{r}_1 - \bar{r}_2)$
φ	U_t	kT/q
n, p, N	α	$\max \lvert N \rvert$

normalized form of Eq. (10) is the following [5]

$$\lambda^2 \Delta\varphi = n - p + N \tag{31}$$

where

$$\lambda^2 = \frac{\varepsilon kT}{l^2 q^2 \alpha}. \tag{32}$$

If $\lambda^2 \to 0$, a singular perturbation of Eq. (31) occurs. In the case $\alpha \gg 1$ the value of λ^2 can be very small.

A numerical example

$$l = 2{,}5\ \mu\text{m}$$

$$\alpha = 10^{20}\ \text{cm}^{-3}$$

$$\text{for}\quad Si\quad \lambda^2 \cong 4 \cdot 10^{-10}.$$

In any point where the doping profile and therefore the potential changes slowly the solution can be searched in the following form

$$\varphi = \sum_{i=0}^{\infty} \varphi_i \lambda^i \tag{33}$$

where φ_0 belongs to the solution with $\lambda = 0$, i.e.

$$n_0 - p_0 - N = 0 \tag{34}$$

i.e. the neutrality is valid as a very good approximation. On the other hand if the profile and potential changes rapidly i.e. in the $p-n$ junction the potential must be searched as

$$\varphi = \sum_{i=0}^{\infty} \left[\varphi_i + \tilde{\varphi}_i \left(\frac{t}{\lambda} \right) \right] \lambda^i; \qquad \tilde{\varphi}_0 \equiv 0 \tag{35}$$

where t is the distance from the internal layer. This solution forms a rapid change around the layer. Here the assumption of quasi neutrality and therefore the approximation by the zero order solution is not valid. The details require a lot of mathematics and the results can be obtained by computer only.

An important result has been obtained: while the approximation of *equilibrium* is not classical, the approximation of *neutrality* is a classical electrodynamic one.

A survey of elementary electromagnetic examples

It is unnecessary to repeat here the basic tasks in the theory of semiconductor devices which are solved by applying the fundamental ideas of electromagnetic theory. The most important among them are e.g. the MOS

capacitor, the punch-through and saturation in the operation of MOS transistors or the bulk resistance. Several of them are good analytic examples, others can be applied as illustrative ones of the utilization of numerical calculation methods.

In the following we wish to list several less known applications of foundation of electrical engineering in the field of microelectronics.

1. Capacitance calculations for VHSIC-s [6]. The speed of operation is strongly dependent on the electrode and island capacitances.

2. Interconnection modeling in monolit and hybrid IC-s. The interconnections form a multiwire transmission line system. One can model them either as a transmission line (practically a system of distributed RC lines), or as a RC lumped network. In the latter the capacitance calculations, the lumped equivalents of the C and R matrices and the approximations for the largest characteristic time are the goals [7–10].

Similar tools can be applied for the calculation of the effect of the relatively large contacts in microcircuits, [11].

3. Conformal mapping. Application for calculation of layer resistances and contacts [7]. MOS transistors of complicated geometry [12]. Bipolar transistors-base resistance [13] and τ_f recovery time.

4. Capacitance calculations of active nonlinear elements. Symmetrical and non-symmetrical capacitance matrices [2].

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