

# ANALYTIC AND EXPERIMENTAL METHOD OF MECHANICAL CHARACTERISTICS DETERMINATION OF CYLINDRICAL LINEAR INDUCTION MOTOR

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## Summary

On the basis of the accepted design of cylindrical motor the method of mechanical characteristics determination is presented. Measuring system and the data of velocity  $v=v(t)$  are presented. This was the basis for determination of mechanical characteristics of electromechanical energy transducer of linear motion exemplified by cylindrical motor and linear induction flat motor of comparable exploitation parameters.

## Introduction

One of the numerous directions of development of contemporary electric drives is the branch connected with linear electromechanical transducers.

Considering the development of drives with linear induction motors it should be said that majority of theoretical works, designs and experiments concern flat linear induction motors of different kinds and versions [4, 5, 6].

Linear induction motors due to their special features and mainly due to directness of electrical energy transformation on kinetic energy of translatory motion are widely applied in different fields of industrial technology [1, 2, 5, 8]. In the field of drives with linear induction motors different experiments are carried out generally concerning the problems of exploitation and designing.

To one of those groups belong the works dealing with the subgroup of linear induction motors consisting of machines with a closed primary or secondary part. In literature [2, 3], the machine with a closed primary part (exciter) is called the tubular motor or Bivkeland's gun. Linear induction machine with a closed secondary part (track) completely covering the primary part is called cylindrical linear machine [9, 10, 11]. To the most popular designs of tubular and cylindrical motors belong the devices with immobile exciter and immobile track, respectively.

Examples of such designs are as the following:

- tubular linear induction motor, LMPK-type, Demag-Conz Elektrizität-Ges., versions: 19/6, 19/9, 19/12, 19/18, 19/24 of synchronous velocity 3 m/s
- tubular linear induction motor, LMKK-type, "Polysolenoid", Demag Conz Elektrizität-Ges.
- tubular linear induction motor, STL-54-4-type with the track in 3 variants [3]
- tubular linear induction motor, LCIA- and LCIB-types [2].

Besides of the advantages concerning the general properties of drive systems with linear induction motors the designs of cylindrical motors are characterized by additional advantages, such as:

- reduction of magnetic pull forces between the exciter and track
- possibilities for convenient choice of characteristics and exploitaltional parameters.

### **Description of Design and Data of Cylindrical Motor**

The motor consists of two main parts: exciter and track. The track is in the form of thick-walled ferromagnetic tube, subjected to honing, of dimensions: length 2000 mm, internal diameter 204 mm, external diameter 223 mm.

The tracks of two-layers were made on the ferromagnetic basis with electrolytic copper 0.1 and 0.2 mm thick.

The exciter consists of core assembly of profiled sheets, winding, wedges and supporting tube. The assembly of profiled sheets were placed along the axis of the supporting tube in such arrangement that there are six of them on the tube circumference. Between the assembly of sheets there are wedges fixed to the supporting tube. Such a magnetic core is closed with covers from both sides. The covers are also the supports for the assembly of guiding rollers. In the magnetic core there are 12 ring coils (20 winds, Cu-wire 2 mm diam., cotton covered). The arm of the motor is in the form of a steel tube of 50 mm diam, 4000 mm long. The guides are in the form of rollers that can be adjusted separately for precise adjustment of air gap (0.5 mm). In order to avoid vibrations of the arm the rollers are fixed on ends of the track in the covers that are the limiters during inverse motion.

The design described above is schematically shown in Fig. 1 (without the assembly of guides of the exciter and arm). The wedges in the cross-section A-A are not shaded.

For the described design the core assembly is made of 60 sheets of BS-0.5/2.6-type with semi-closed rectangular groove. The model of the motor given in Fig. 1 is in the department of Electric Machines and Drives of the Institute of Electroenergetics of Technical University of Częstochowa. The general view of the cylindrical motor exciter is shown in Fig. 2.

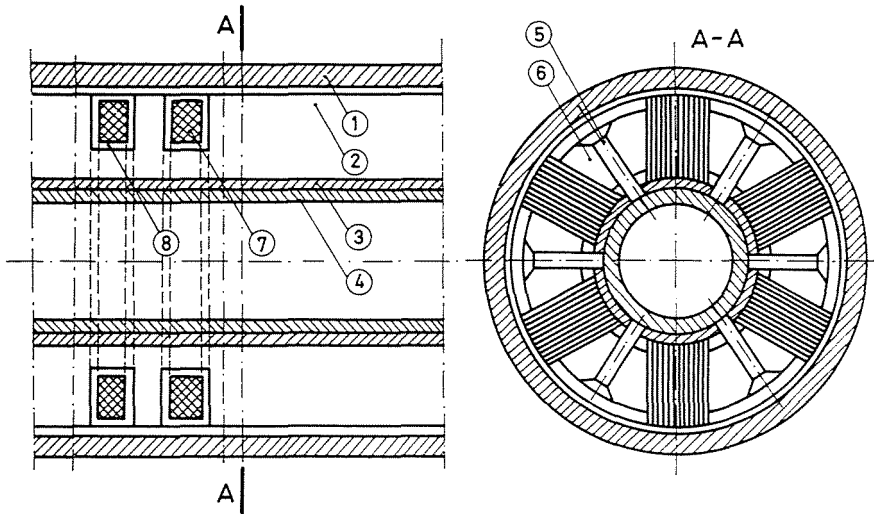


Fig. 1. Cross-section of cylindrical motor

- |                             |                            |
|-----------------------------|----------------------------|
| 1 — track                   | 5 — screw                  |
| 2 — arrangement of sheets   | 6 — wedge                  |
| 3 — exciter supporting tube | 7 — winding of single coil |
| 4 — arm                     | 8 — groove insulation      |

Other technical data of the motor are as the following:

- |   |                              |
|---|------------------------------|
| — input voltage   | $U = 380 \text{ V}$          |
| — mean induction in the slot for concurrent and countercurrent connection in phase band | $B_{\delta} = 0.4 \text{ T}$ |
| — groove pitch  | $t_z = 0.0164 \text{ m}$     |
| — number of grooves   | $z = 12$                     |

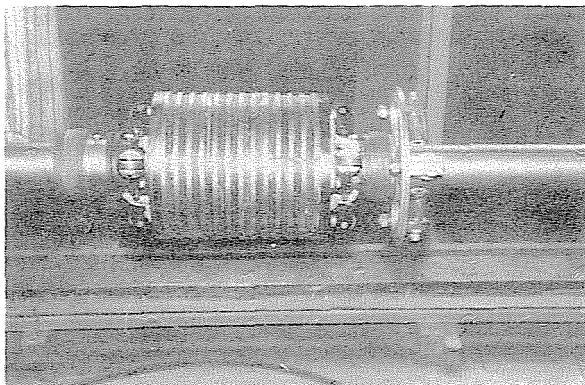


Fig. 2. General view of Cylindrical motor-exciter

|  |                         |
|--|-------------------------|
| — number of grooves for pole and phase | $q = 1$                 |
| — number of pole pairs                 | $z_p = 4$               |
| — main pitch                           | $\tau = 0.05 \text{ m}$ |
| — synchronous velocity                 | $V_s = 5 \text{ m/sec}$ |

### Tractive Force of Cylindrical Motor versus Electric Parameters of Secondary Part

For analysis the following assumptions have been made:

- exciter moves along Ox direction
- effect of exciter magnetic core grooves is neglected
- constant magnetic permeability
- effect of harmonics of higher order is neglected
- there is electric and magnetic symmetry of the machine.

The analysis was performed on the basis of classical methods applied for electric machines. The linear variables  $x_1$  and  $x_2$  connected with exciter and track, respectively, were introduced. Induction of magnetic field of the first harmonic is a wave function defined by the following relationship (1):

$$B(x_1, t) = B_\delta \sin\left(\frac{2\pi}{\lambda} x_1 - \omega t\right) \quad (1)$$

where:  $B_\delta$  — induction amplitude  
 $\lambda$  — wave length  
 $\omega$  — pulsation  
 $t$  — time.

Considering relationships (2)

$$\begin{aligned} \lambda &= 2\tau & v_s &= 2 \cdot f \cdot \tau \\ x_1 &= x_2 + vt & v &= v_s(1 - s) \end{aligned} \quad (2)$$

and identity (3)

$$\frac{\pi}{\tau} x_1 - \omega t = \frac{\pi}{\tau} (x_2 - s \cdot t \cdot v_s) \quad (3)$$

where:  $s$  — slip  
 $f$  — frequency.

Induction in the exciter and track will be defined by relationship (4):

$$\begin{aligned} B(x_1, t) &= B_\delta \sin\left(\frac{\pi}{\tau} x_1 - \omega t\right) \\ B(x_2, t) &= B_\delta \sin\frac{\pi}{\tau} (x_2 - s \cdot t \cdot v_s). \end{aligned} \quad (4)$$

Wave function of current density in the track is defined by equation (5)

$$J(x_2, t) = Jm_2 \sin \frac{\pi}{\tau} (x_2 - s \cdot t \cdot v_s - x') \quad (5)$$

where:  $Jm_2$  — current density amplitude

$x'$  — phase shift of current density wave in the track with respect to induction  $B(x_2, t)$ .

General equation for magnetic energy stored in cylindrical motor is a function of currents and exciter position:

$$Wm = W(i_1, i_2, i_3, \dots, x, y, z). \quad (6)$$

In expression (6) currents  $i_1, i_2, i_3 \dots$  are actual values of independent closed loops generated from the motor winding and local specific electric loadings in the conductive layer of the track. Variables  $x, y, z$  do not exist directly but are arguments of expressions for self-inductances and mutual inductances of the separated circuits of the motor. On the basis of the above assumptions and motor design Fig. 1 the equation for forces acting on the direction of motor motion will take the form (7):

$$m \frac{d^2x}{dt^2} + F_t \frac{dx}{dt} = \frac{\partial}{\partial x} W(i_1, i_2, i_3, \dots, x) - F_0 \quad (7)$$

where:  $m$  — mass of exciter

$F_t$  — friction force

$F_0$  — load.

Elementary magnetic energy is expressed by the partial derivative  $\partial Wm / \partial v$  of energy with respect to volume element, where  $\partial v = \pi \cdot D_w \cdot \delta x \cdot \delta$

$$\frac{\partial Wm}{\partial v} = \int_0^{2\tau_p} J(x_2, t) \cdot B(x_2, t) dx_2 \quad (8)$$

where:  $D_w$  — exciter diameter

$\delta$  — air slot.

Tractive force  $F_e$  defined on the basis of (8) is given by the equation (9):

$$F_e = \pi \cdot D_w \cdot \delta \cdot \tau \cdot p \cdot B_\delta Jm_2 \cos \frac{\pi}{\tau} x'. \quad (9)$$

On the basis of the equivalent scheme of cylindrical motor (similar to the equivalent scheme of induction machine) the following relationships (10) can be given:

$$\begin{aligned}
 E_1 &= \sqrt{2} \cdot \pi \cdot f \cdot z_1 \cdot k_{u1} \cdot \bar{\Phi}_m \\
 \bar{\Phi}_m &= 2 \cdot \tau \cdot p \cdot B_\delta D_w \\
 I'_2 &= \frac{p \cdot \tau \cdot \delta}{m_1 k_{u1} z_1} J m_2
 \end{aligned}
 \tag{10}$$

where:  $E_1$  — electromotive force of transverse branch of equivalent scheme  
 $z_1$  — number of turns of exciter winding  
 $\bar{\Phi}_m$  — flux amplitude  
 $k_{u1}$  — winding factor  
 $I'_2$  — transformed value of secondary current  
 $m_1$  — number of phases of exciter.

On the basis of equation (9) and relationship (10) tractive force of the motor is defined by the relationship (11)

$$\begin{aligned}
 F_e &= \frac{m_1 \cdot |I'_2| \cdot E_1}{2f} \cos \frac{\pi}{\tau} x' \\
 v_s &= 2 \cdot \tau \cdot f \\
 F_e &= m_1 \cdot |I'_2| \cdot E_1 \cdot v_s^{-1} \cdot \cos \frac{\pi}{\tau} x'.
 \end{aligned}
 \tag{11}$$

Using the analogy with the rotary motors the expression  $m_1 |I'_2| E_1 \cos \frac{\pi}{\tau} x' = P_e$  is the internal power of cylindrical machine.

The value  $|I'_2|$  is a quotient of electromotive force of transverse branch of equivalent scheme and impedance  $Z'_b$ .

Tractive force expressed by the parameters of secondary part is defined by the relationship (12):

$$F_e = \frac{m_1 R'_b}{v_s s} \frac{E_1}{|Z'_b|^2}
 \tag{12}$$

where:  $Z_b$  — track impedance  
 $R_b$  — track resistance

### Tractive Force of Cylindrical Motor versus Induction in the Exciter Slot

On the basis of (12) elementary impedance of metallic semi-space subjected to a polarized plane wave generating the current flow is defined by the equation (13):

$$Z_j = r + jx = (1 + j) \frac{1}{\gamma \cdot \Delta} \quad (13)$$

$$\Delta = (\pi \cdot s \cdot f \cdot \mu \cdot \gamma)^{-\frac{1}{2}}$$

where:  $\gamma$  — conductivity

$\Delta$  — depth of wave penetration

$\mu$  — magnetic permeability.

For ferromagnetic semi-space equation (13) should be completed by constant coefficients  $a_r = (1.3 \sim 1.5)$  and  $a_x = (0.8 \sim 0.9)$ , according to [7]:

$$Z_{jFe} = (a_r + ja_x) \frac{1}{\gamma \cdot \Delta p} \quad (14)$$

$$\Delta_{Fe} = \frac{\Delta p}{\sqrt{2}}; \quad \Delta_{Fe} = (2\pi \cdot f_i \cdot \mu_p \cdot \gamma_{Fe})^{-\frac{1}{2}}$$

where:  $\Delta_p$  — surfacial depth of wave penetration

$\Delta_{Fe}$  — depth of wave penetration in ferromagnetic materials

$\mu_p$  — surfacial magnetic permeability

$\gamma_{Fe}$  — conductivity of ferromagnetic material.

On the basis of (12) for cylindrical motor of single-layer track the equation for secondary part impedance is in the form:

$$\begin{aligned} Z_b &= Z_{jFe} \cdot 2p \cdot \pi \cdot D_w \cdot \tau^{-1} \\ Z_b &= (a_r + ja_x) \cdot 2p \cdot \pi \cdot D_w (\Delta p \cdot \gamma_{Fe} \cdot \tau)^{-1}. \end{aligned} \quad (15)$$

Resistance and reactance of the track are, respectively:

$$R_b = \operatorname{Re} \{Z_b\}$$

$$X_b = \operatorname{Im} \{Z_b\}$$

$$R_b = a_r \frac{2p \cdot \pi \cdot D_w}{\Delta p \cdot \gamma_{Fe} \cdot \tau} \quad (16)$$

$$X_b = a_x \cdot \frac{2p \cdot \pi \cdot D_w}{\Delta p \cdot \gamma_{Fe} \cdot \tau}.$$

On the basis of eqs. (12), (13), (14) the tractive force of cylindrical motor will be:

$$F_e = \frac{a_r \cdot p \cdot \tau \cdot D_w \cdot v_s \cdot B_\delta^2}{a_r^2 + a_x^2 \cdot s^2} \sqrt{\frac{\pi \cdot \gamma_{Fe} \cdot s}{\mu_p \cdot f}}. \quad (17)$$

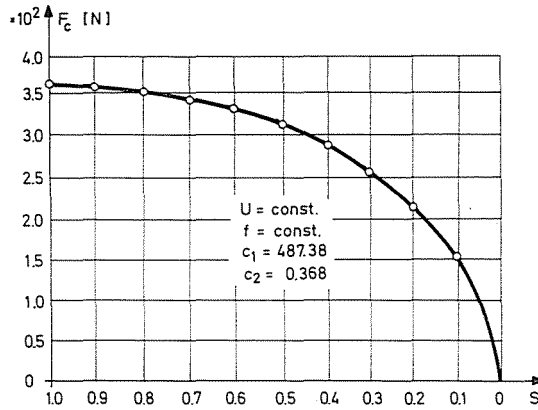


Fig. 3. Theoretical mechanical characteristics

The static starting force  $F_e$  for  $s=1$  will then be:

$$F_e(s=1) = \frac{a_r \cdot p \cdot \tau \cdot D_w \cdot v_s \cdot B_\delta^2}{a_r^2 + a_x^2} \sqrt{\frac{\pi \cdot \gamma_{Fe}}{\mu_p \cdot f}} \quad (18)$$

Introducing denotation (19)

$$C_1 = \frac{B_\delta^2 \cdot v_s \cdot p \cdot \tau \cdot D_w}{s_r} \sqrt{\frac{\pi \cdot \gamma_{Fe}}{\mu_p \cdot f}} \quad (19)$$

$$C_2 = \left( \frac{a_x}{a_r} \right)^2$$

equation (17) will be expressed in the convenient form for determining the theoretical mechanical characteristics:

$$F_e = C_1 \frac{\sqrt{s}}{1 + C_2 \cdot s^2} \quad (20)$$

On the basis of design parameters the 2 constants  $C_1$  and  $C_2$  were determined and theoretical mechanical characteristics of cylindrical motor was plotted (Fig. 3) for single-layer track of ferromagnetic material ( $a_x=0.85$ ,  $a_r=1.4$ )

$$C_1 = 487.38$$

$$C_2 = 0.368$$



## Experimental Determination of Mechanical Characteristics of Cylindrical and Flat Linear Induction Motors

The measuring system presented in Fig. 4 consists of the following elements:

|                               |          |
|-------------------------------|----------|
| — rotary—pulse transducer     | PJ       |
| — pulse decade counter        | LD       |
| — analogue-digital transducer | PAC      |
| — recorder with attachment    | $R_{xy}$ |
| — digital voltmeter           | $V_c$    |
| — D. C. feeder                | Z        |

For comparison of the obtained results with other linear energy transducers the measurements of flat linear induction motor SL-5-270-type have been performed along the track of the length 6000 mm with parameters of the secondary part kept according to the requirements of the rated duty. Comparing the behaviour of flat and cylindrical motor we observed the following criteria:

- similar values of phase currents for  $v=0$
- identical values of synchronous velocities  $v_s=5$  m/s
- identical conditions of feeding (three-phase A. C., 380 V, frequency 50 Hz)
- similar possibilities of application in driving systems.

General view of the measuring device of the flat motor is shown in Fig. 5.

The measurements were performed for the following values of the load:  $F_e = 5, 10, 15, 20$  and  $25$  kg. Because both motors were loaded by the use of the same system (stand with weights providing sliding of about 3000 mm) the additional inaccuracies connected with the losses in loading system were excluded. The signal emitted by PJ rotary-pulse transducer was transmitted from the rubber roller of 45 mm diam. (turned by the arm) through the flexible shaft to the terminal of the above mentioned transducer.

The real force acting in the system for the mentioned loads  $F_0$  is the resultant force  $F_0 + F_t$ , where  $F_t$  is the friction force originating from mechanical losses of the exciter motion system.

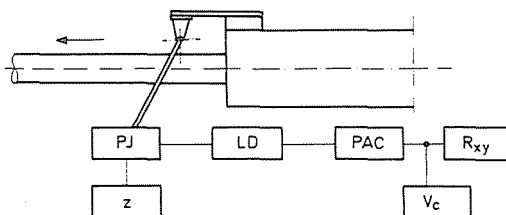


Fig. 4. Scheme of measuring system

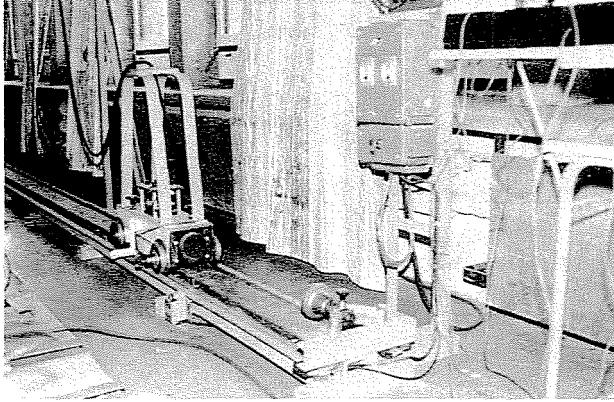


Fig. 5. Linear track of flat motor

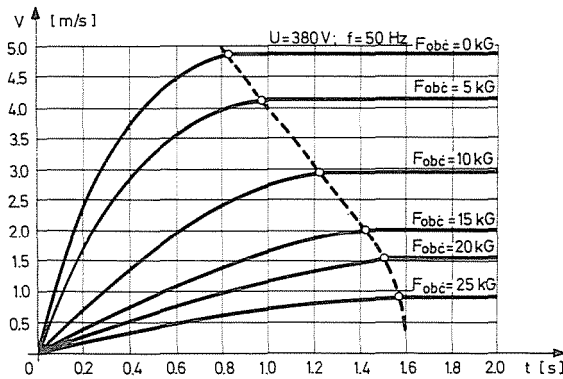


Fig. 6. Measurements of velocity during cylindrical motor starting for different loads

For cylindrical motor the friction force  $F_{t0} = 23$  N and for flat motor  $-F_{tp} = 21$  N. The measurements of  $v = v(t)$  are shown graphically in Figs 6 and 7 for cylindrical and flat motors, respectively.

From  $v = v(t)$  the characteristics of cylindrical and flat motors were determined as  $F_0 + F_{ic} = F(v)$  and  $F_0 + F_{tp} = F(v)$ , respectively. The considerably long time of measurement clearly shows the boundary between starting and work of the motor. The characteristics of both motors are given in Fig. 8 (clear marks denote the data obtained for cylindrical motor and blackened marks—for flat motor). The scatter of the data is due to difficulties connected with loading and average value of the measured friction force.

From  $v = v(t)$  Figs 6 and 7 the areas of steady velocities for the given load were determined broken line. Fig. 9 shows the comparison of the areas of steady works of the tested motors.

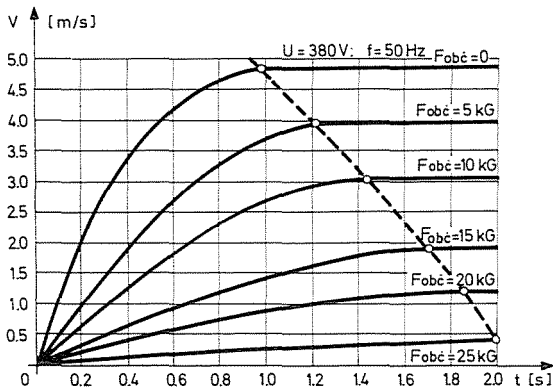


Fig. 7. Measurements of velocity during SL-5-270-type motor starting track Fe + 3 mm Al, slot = 0.001 m

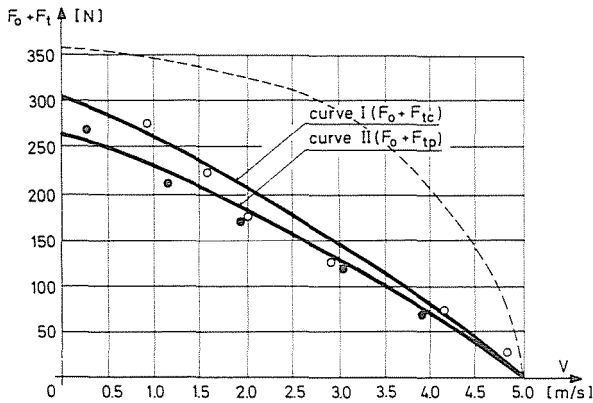


Fig. 8. Experimental mechanical characteristics of cylindrical and linear motors of the data from Fig. 7, broken line denotes the characteristics from Fig. 3

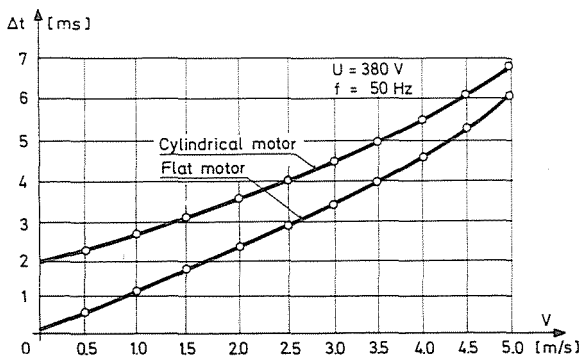


Fig. 9. Comparison of the areas of steady work of cylindrical and flat linear induction motors of the data from Fig. 7

Table 1

| No | $F_0$  | $F_{ic}$ | $F_{ip}$ | $F_0 + F_{ic}$ | $F_0 + F_{ip}$ | $v_0$ | $v_p$ |
|----|--------|----------|----------|----------------|----------------|-------|-------|
|    | N      | N        | N        | N              | N              | m/s   | m/s   |
| 1  | 0      | 23       | 21       | 23.00          | 21.00          | 4.80  | 4.75  |
| 2  | 49.00  | 23       | 21       | 72.05          | 70.05          | 4.20  | 3.90  |
| 3  | 98.10  | 23       | 21       | 121.11         | 119.10         | 2.90  | 3.05  |
| 4  | 147.15 | 23       | 21       | 170.15         | 168.15         | 2.00  | 1.80  |
| 5  | 196.20 | 23       | 21       | 219.20         | 217.20         | 1.60  | 1.15  |
| 6  | 245.25 | 23       | 21       | 268.25         | 266.25         | 0.95  | 0.35  |
| 7  | —      | 23       | 21       | 313.00         | 268.00         | 0     | 0     |

On the vertical axis  $t$  the time counting was begun from the point of intersection of horizontal axis with the broken line in Fig. 7.

The data obtained during experiments completed with the values of starting forces (at  $v=0$ , item 7) are listed in Table 1.

### Conclusions

On the basis of the analysis and performed experiments the following conclusions can be drawn:

- the discussed measuring system can be applied in all drive systems with transducers of linear motion without additional devices [3] and without design adaptation of the moving element [10].
- the results obtained by the analysis and experiments can be considered as accurate and sufficient for the determination of the characteristics  $F = F(v)$  of different electromechanical linear transducers. Such characteristics are the basis of applicability of the transducers for industrial drive systems.
- the discrepancies between the characteristics obtained theoretically and experimentally (Fig. 8) especially for the velocity range 1.5~4.5 m/s are of secondary importance from the point of view of exploitation (the main parameter is starting force for  $v=0$ ) but this fact requires further analytical considerations. Such an attempt with the application of field theory methods will be presented in the future works.
- for the considered and compared cases of the starts of the analysed motors the cylindrical motor is characterized by larger area of steady work about 25% in comparison with the flat motor (Fig. 9).

## List of symbols

|                |   |
|----------------|---|
| $a_r$          | — constant factor [7]   |
| $a_x$          | — constant factor [7]   |
| $B_\delta$     | — induction in the slot   |
| $B(x_1, t)$    | — wave function of induction in the exciter                     |
| $B(x_2, t)$    | — wave function of induction in the track                       |
| $C_1, C_2$     | — constant factors of mechanical characteristics                |
| $D_w$          | — exciter diameter  |
| $E_1$          | — electromotive force of transverse branch of equivalent scheme |
| $f$            | — frequency of input voltage                                    |
| $F_e$          | — motor tractive force  |
| $F_0$          | — load  |
| $F_{te}$       | — function force of electrical motor                            |
| $F_{tp}$       | — function force of flat motor                                  |
| $I'_2$         | — transformed value of secondary current                        |
| $Jm_2$         | — amplitude of track current density                            |
| $k_{wi}$       | — winding factor of cylindrical motor                           |
| $m$            | — exciter mass  |
| $m_1$          | — number of phases of exciter                                   |
| $p$            | — number of poles' pairs  |
| $R'_2$         | — track resistance  |
| $s$            | — slip  |
| $v$            | — velocity linear   |
| $v_s$          | — synchronous velocity  |
| $X_b$          | — track reactance   |
| $z_1$          | — number of turns   |
| $\delta$       | — air slot  |
| $\bar{\Phi}_m$ | — flux amplitude  |
| $\lambda$      | — wave length   |
| $\tau$         | — pole pitch  |
| $\Delta_{Fe}$  | — depth of wave penetration in ferromagnetic material           |
| $\omega$       | — pulsation   |
| $\gamma$       | — conductivity  |
| $\mu_p$        | — magnetic permeability on the surface                          |

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