DESIGN OF ELECTRONIC CIRCUITS FOR OPTIMAL PRODUCTION YIELD*

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Summary

The new methods of electronic circuit design make possible not only to determine the production yield, but also to calculate new nominal element values and optimal tolerances. The design methodology is called design centering and tolerance assignment.

The elaboration of methods demands further development of the sensitivity and tolerance theory of electronic circuits. To solve the task, Monte Carlo simulation and optimization methods are applied.

The paper deals with the development of the design concepts for optimal production yield and presents algorithms to determine new nominal element values, tolerances and production specification. Reference is made to the computer programs developed in the Institute of Communication Electronics of the Budapest Technical University and successfully used in industry. The efficiency of the new design methodology is illustrated by examples of LC filters.

Introduction

The scientific session at the Faculty of Electrical Engineering held on the occasion of the 200 years anniversary of the foundation of the Budapest Technical University offers a solemn opportunity to review the results achieved in the economical design of electronic circuits. The paper relies on the work realized at the Institute of Communication Electronics. These special circumstances are the main reason why only publications of the institute staff figure in the references.

Among the steps of electronic circuit design of paramount importance is the synthesis of the starting circuit, its analysis and its optimization. During the last decade the interaction of demand and possibility gave rise to the computer aided design of electronic circuits. The design methods which take mass

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production and economic consequences into consideration are worthy of attention. This design methodology is called design centering and tolerance assignment and achieves production improvement by increasing the yield and/or increasing the tolerances.

The investigation of deviations between nominal and measured parameters and reduction of their adverse effects is treated by the theory of tolerances. The derivative of the network function with respect to circuit parameters has a decisive role in tolerance calculation. This derivative is called sensitivity. The following methods were applied by the designers to receive electronic circuits insensitive to parameter deviations: (i) suitable fulfilment of the specification (e.g. overdimensioning), (ii) selecting a proper structure (e.g. feedback, ladder network), (iii) prescribing a strict technology (e.g. low tolerances). In critical situations worst case design was done and the selected structure was qualified by comparing different sensitivity measures.

With the advent of fast computers the Monte Carlo simulation of mass production became possible as well as the simulation of environmental effects (e.g. temperature). There was no change in the assignment of tolerances, where the designer's experience remained the basis.

The methodology of design centering and tolerance assignment gives new nominal element values for the original circuit indicates new tolerances for these network elements, in order to achieve a maximal yield. Reducing the cost of suitable electronic circuits it improves the economical production.

An additional difficulty in the course of design is to take into consideration the environmental effects (e.g. temperature). This, as will be seen later, can be dealt with the concept of production specification. If the circuit fulfils the production specification at the time and temperature of production, it will meet the original one under environmental influence as well.

In the following — without going into details — a review of the mathematical formulation of design centering and tolerance assignment will be given including the algorithms of solution. And finally, reference will be made to the computer programs and their application.

Region of acceptability

In Fig. 1 two network functions (attenuation response) are shown as functions of frequency ω. Besides frequency the network function is also dependent on the vector of the network parameters x. The specification is represented by α_l (lower) and α_u (upper) values. The fulfilment of the specification is investigated at s discreet frequencies. In one respect the network

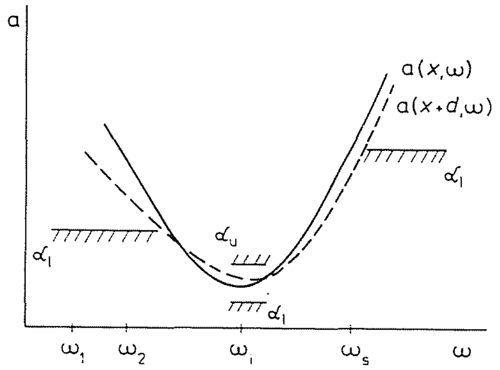

Fig. 1. Specification in the frequency domain. The dotted line presents the network function in the case of drift *d*

function fulfils the following inequalities:

$$\begin{gathered}\alpha_{li} \leq a_i(x) \leq \alpha_{ui} \\ i = 1, 2, \ldots s\end{gathered} \tag{1}$$

In addition also the environmental effects (temperature, ageing) are taken into consideration with the aid of drift vector d. In this manner further inequalities are prescribed for the circuit:

$$\begin{gathered}\alpha_{li} \leq a_i(x+d) \leq \alpha_{ui} \\ i = 1, 2, \ldots s\end{gathered} \tag{2}$$

Thus the circuit which meets the specification satisfies the following inequalities:

$$\begin{gathered}\alpha_{li} \leq a_i(x) \leq \alpha_{ui} \\ \alpha_{li} \leq a_i(x+d) \leq \alpha_{ui} \\ i = 1, 2, \ldots s\end{gathered} \tag{3}$$

In the following we generally assume to have a preliminary network which satisfies (3). Our aim is to improve this starting circuit. For this reason the notion of the region of acceptability has to be introduced.

The region of acceptability is the set of circuit parameters for which the specification is fulfilled:

$$R=\left\{ \;\middle|\; x \begin{array}{c} \alpha_{li}\leq a_i(x)\leq\alpha_{ui} \\ \alpha_{li}\leq a_i(x+d)\leq\alpha_{ui} \\ i=1,2,\ldots s \end{array} \right\} . \qquad (4)$$

It is extremely complicated to make a general statement about the form of the region of acceptability. The graphical illustration is simple only in the case of two circuit parameters; an example is shown in Fig. 2. For parameter values inside the region of acceptability the specification is fulfilled, outside this region it is not. It can be seen that the nominal value of the circuit parameter has to be located in the "center" of the region of acceptability, that is the basis for the expression "design centering". In the special case of Fig. 2 the region of acceptability is single connected and convex.

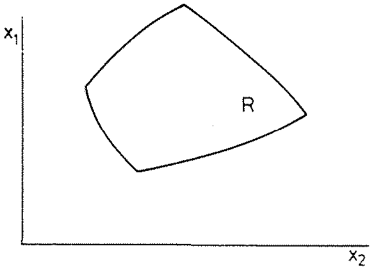

Fig. 2. The region of acceptability

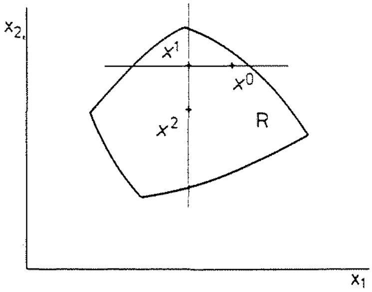

Fig. 3. Design centering by bisection of line segments (one-dimensional search)

A method to determine the new nominal circuit parameters is as follows. In Fig. 3 x^0 denotes the original value of the circuit parameter. Changing the x_1 circuit element only the boundary points of R are determined. Bisection of the line segment gives the new x^1 point. Changing then the x_2 circuit element new boundary points are received only and the bisection of the line segment yields the new x^2 point. Repeating the iteration the center of the region of acceptability is reached. Let us denote the vector of these new nominal values by:

$$x^{\mathrm{opt}} \tag{5}$$

Tolerances

The nominal value of the i-th circuit parameter is x_i^{opt}. Let us suppose that the actual value of x_i varies between $x_i^{\mathrm{opt}} \pm \varepsilon_i$, where ε_i is the tolerance of the circuit parameter. In the case of two circuit parameters the situation is shown in Fig. 4. From the tolerances the vector

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_i \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{6}$$

is constructed and to optimize it various cost functions may be used. Typical cost functions are:

$$\min \varepsilon_i \to \max! \tag{7.a}$$

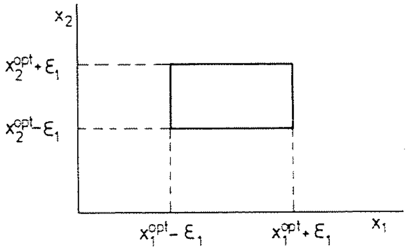

Fig. 4. Tolerance body in two dimensions

$$\prod_{i=1}^{n} \varepsilon_i \rightarrow \max! \tag{7.b}$$

$$\sum_{i=1}^{n} \frac{1}{\varepsilon_i} \rightarrow \min! \tag{7.c}$$

where n is the number of circuit parameters. Naturally, the circuits have to satisfy the constraint (4) during the optimization procedure.

Production specification and optimized specification

The qualification of the electrical circuit in the factory is carried out for a more stringent specification than the original one, with the aim to come up to the original specification even after environmental influences. In Fig. 5 the situation is illustrated by an example. The specifications if Fig. 1 are repeated in Fig. 5 but are extended with the production specifications β_l and β_u. If during the production the

$$\beta_l = \begin{bmatrix} \beta_{l1} \\ \beta_{l2} \\ \vdots \\ \beta_{ls} \end{bmatrix}, \qquad \beta_u = \begin{bmatrix} \beta_{u1} \\ \beta_{u2} \\ \vdots \\ \beta_{us} \end{bmatrix} \tag{8}$$

so-called production specifications are met, then in the case of a circuit parameter, changed by drift vector d, the original specification will be fulfilled.

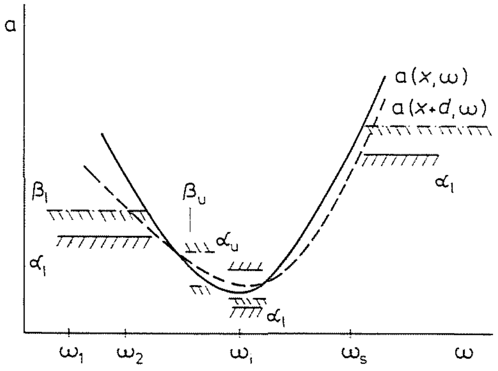

Fig. 5. Points and lines trace the production specification

We succeeded to elaborate algorithms for the determination of the production specification.

The original specification that takes in account systems engineering considerations only and neglects the circuit designer's view, may result in very stringent requirements. Production yield may be considerably improved if the original specification α is changed slightly at a specific frequency. With this final correction the optimized specification

$$\gamma_{\mathrm{l}}, \gamma_{\mathrm{u}} \tag{9}$$

is produced. In this manner the circuit designer and the systems engineer receive numerical information about the consequence of changing the original specification. For example, diminishing the specification at frequency ω_j by 1 dB results in a 5% improvement in production yield.

Summing up the above, our task is to calculate new nominal values x^{opt}, optimal tolerances ε, production specification β and in certain critical situations the optimized specification γ.

Design methodology

The starting point in discussing the design methodology is Fig. 6. The block diagram shows that a preliminary circuit is chosen, circuit analysis is performed and a decision is made whether the requirements are met by the circuit. In a YES case this particular phase of design work is finished. In a NO

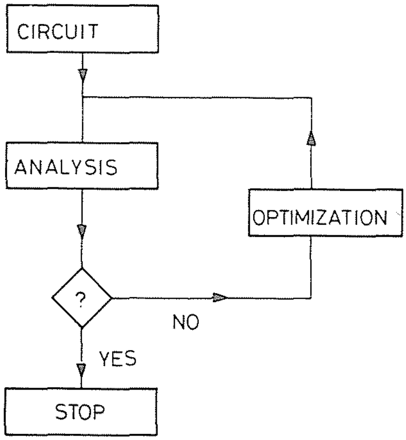

Fig. 6. Block diagram of the electronic design

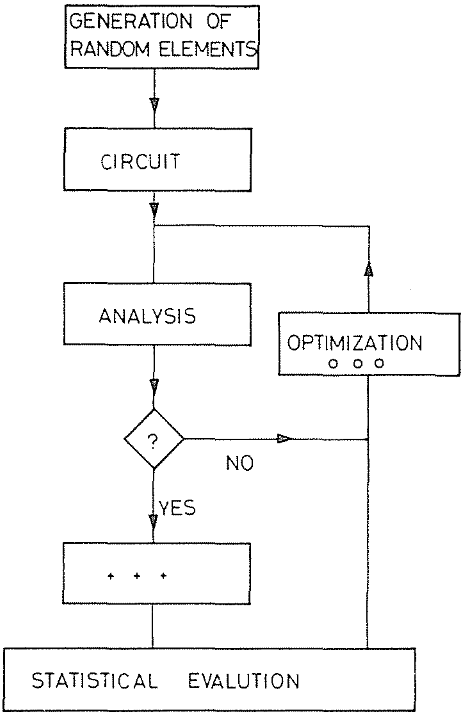

Fig. 7. Block diagram of the design for optimal yield. The suitable circuits are marked by +, the fail circuits are symbolized by o

case an optimization procedure is used to improve the circuit parameters. The iterative procedure is continued until the final circuit is found that satisfies the specification or a prescribed number of iteration steps is reached. (In the latter case a new starting circuit or a less severe specification may help.)

In our design centering and tolerance assignment algorithms the Monte Carlo simulation is applied. Pseudo-random circuit element values are determined and circuit analysis and optimization is performed. The statistical properties (including histograms) are evaluated by repeating this procedure many times. The block diagram of the design methodology for optimal production yield is shown in Fig. 7. We found that to realise this block diagram fast circuit analysis program, efficient optimization methods and a broad knowledge of mathematical statistics are necessary. The design for optimal production yield calls for new methods in circuit theory.

The principles of the algorithms

In many cases it is practical to break down the design task into two steps. In the first the new nominal element values are determined, in the second the tolerances are calculated. For the first step we already presented an algorithm at the end of the chapter on the region of acceptability, namely the bisection of line segments.

Further important methods are based upon the application of the Monte Carlo simulation. Besides the region of acceptability, Fig. 8 shows a tolerance body as well, traced by a continuous line. In the two-dimensional case the tolerance body is a parallelepiped, its center is the nominal circuit element value and its side lengths are twice the tolerances. With the Monte Carlo analysis the center of gravity of the suitable circuit elements may be determined. The same Monte Carlo analysis presents the center of gravity of the failed circuit elements. In the knowledge of these centers the nominal circuit element is shifted in the direction of the center of gravity of the suitable circuit elements. The new location of the tolerance body is marked by a dashed line. It can be seen that the percentage of suitable circuits is greater resulting in an improved yield.

The Monte Carlo analysis may be used to approximate the region of acceptability with a regular body. Figure 9 shows a special case of approximation, namely covering by an n dimension ellipsoid which is reduced to an ellipse in two dimensions. The size and location of the ellipsoid are optimized by Monte Carlo cycles, thus the algorithm belongs to the family of recursive Monte Carlo methods. The calculation of tolerances follows direct from the approximating regular body and the new nominal values are identical with the center of the body.

Nominal values and tolerances may be calculated from data collected during optimization. Two histograms are built up for each circuit element as

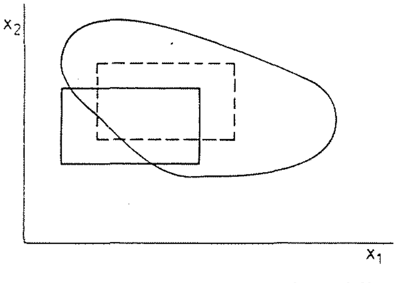

Fig. 8. The new location of the tolerance body, dotted lines offer a better yield

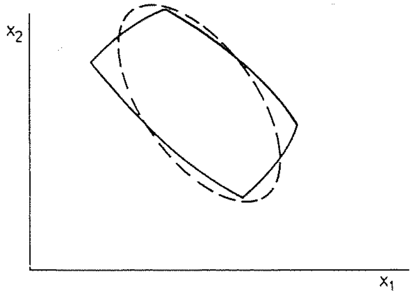

Fig. 9. The approximation of the region of acceptance by a regular body

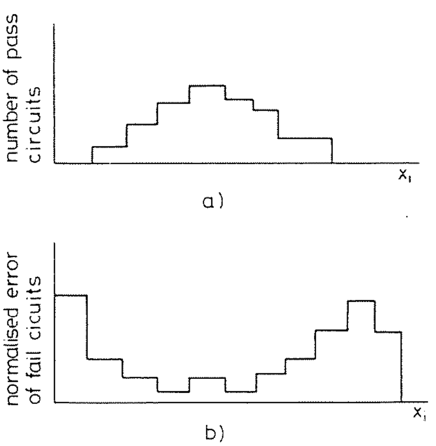

Fig. 10. The new nominal value and the new tolerances are calculated from two histograms

shown in Fig. 10. The histogram 10/a is that of suitable circuits. It collects the number of the pass circuits. In the special case seen in Fig. 10/a this histogram would be sufficient to propose a new nominal value and tolerance but in practice a further histogram is necessary. In the course of the Monte Carlo simulation the actual values of the errors are collected. The histogram of Fig. 10/b is received by dividing the values by the number of errors. Based on these two histograms simple heuristical algorithms may be indicated for determining optimal nominal values and tolerances.

Any results received by any one of these algorithms have to be verified by independent standard Monte Carlo analysis from the point of yield.

The determination of the production specification will be illustrated next. Let us denote the network function in the frequency domain by *a*, referring to the attention response. The values of the function at different frequencies ω_1, $\omega_2, \ldots \omega_i \ldots \omega_s$ are given by $a(\omega_1)$, $a(\omega_2)$, $\ldots$ $a(\omega_i) \ldots a(\omega_s)$, respectively. Any one of these function values is dependent on the circuit parameters, as random variables. As a result of the Monte Carlo simulation the related values of $a(\omega_1)$ and $a(\omega_2)$ can be plotted. In Fig. 11/a the coordinate axes are $a(\omega_1)$ and $a(\omega_2)$, + marks the suitable circuits while o the fail. According to Fig. 11/b our task is to find production specifications β_{l1}, β_{u1} and β_{l2}, β_{u2} which separate the pass and fail circuits.

One of the possible methods of calculating the production specification is based upon the specification sensitivity. The definition of the specification sensitivity at the upper boundary of the specification is as follows:

$$S_i^{\mathrm{u}} = \frac{\partial Y}{\partial \alpha_{\mathrm{u}i}} \tag{10}$$

where u stands for the upper boundary, i stands for the i-th frequency, $\alpha_{\mathrm{u}i}$ refers to the upper value of the specification and Y is the yield. The determination of

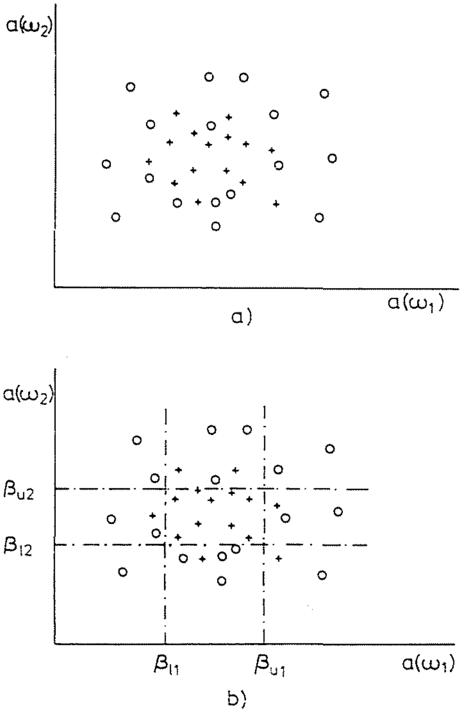

Fig. 11. Interpretation of the production specification in the $a(\omega_1)-a(\omega_2)$ plane

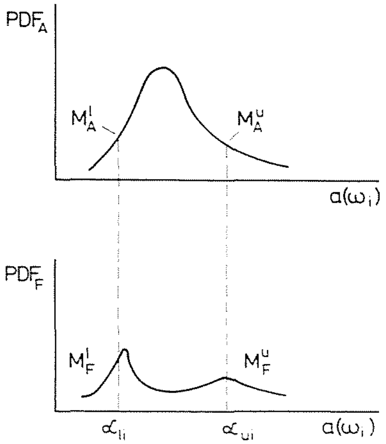

Fig. 12. Explanation of the yield sensitivity with respect to specification. PDF is the abbreviation for a probability density function

(10) is possible by the standard Monte Carlo analysis. Two probability density functions are produced, namely the pdf of all cases and the pdf of fail cases. These are plotted in Fig. 12 as functions of $a(\omega_i)$. Here the index A refers to all circuits and F refers to fail circuits. M_A^u is the value of the pdf of all cases, M_F^u the value of the pdf of fail cases, both at the upper value α_{ui}. Let the upper specification be changed by a differential value. If this change is small enough, it may be proved that

$$S_i^u = \frac{\partial Y}{\partial \alpha_{ui}} = M_A^l - M_F^u \tag{11}$$

A similar result is valid for the lower boundary:

$$S_i^l = \frac{\partial Y}{\partial \alpha_{li}} = M_A^l - M_F^l \tag{12}$$

With the aid of the specification sensitivity the gradient of the yield may be determined. On this basis the specification is changed by an appropriate step. If the circuits — or a prescribed percentage of them — fulfil the new specification, the algorithm will be finished and the production specification has been obtained.

The sensitivity of the yield with respect to the specification offers a possibility to determine a new, optimized specification γ, as well.

Computer programs and their applications

On the basis of algorithms briefly discussed previously, many computer programs were implemented for design centering and tolerance assignment at the Institute of Communication Electronics, Technical University Budapest. The programs have been published elsewhere already, therefore only their list is given in Table 1.

The first research impetus was given and the first programs were supported by the Hungarian Academy of Sciences, mainly for illustrative and educational purposes. On behalf of TELEFONGYÁR Budapest, computer programs for design centering and toleracing of LC filters were developed. At

Table 1

Computer programs and supporting institutes for design centering and tolerancing at the Institute of Communication Electronics, Technical University Budapest

Supporting institution	Name	Function
Hungarian Academy of Sciences	OPAL	Design of active RC circuits by Monte Carlo analysis
TELEFONGYÁR, Budapest	ISOA	Monte Carlo analysis of LC circuits
	INTOPT	Design centering and tolerancing of LC circuits by optimization
	RMC	Design centering and tolerancing of LC circuits by recursive Monte Carlo algorithm
	GHU	Generation of production specification
	SPERZ	Calculation of yield sensitivity with respect to specification
REMIX, Budapest	STARCAN	Monte Carlo analysis of active RC circuits
	TOCENT	Design centering and tolerancing by bisection of line segments

Table 2

Design centering and tolerance assignment for a filter with 4 capacitances and 3 inductances

Preliminary	
Yield	58%
ε_{L1}	+30% −20%
$\varepsilon_{L2}=\varepsilon_{L3}$	0.5%
ε_{C}	1%

Optimized			
Yield	96%	100%	98%
ε_{L1}	+30% −20%	5%	25%
$\varepsilon_{L2}=\varepsilon_{L3}$	0.5%	5%	2%
ε_{C}	2%	5%	2%(∞)

Table 3

Results of design centering and tolerance assignment for a filter with 17 capacitances and 10 inductances

Preliminary	
Yield	
At production	100%
After temperature change	82%
L	tuned
ε_{C}	1%
Production specification	CCITT 1/40

Optimized	
Yield	
At production	100%
After temperature change	94%
L	tuned
ε_{C}	0.5−2%
Production specification	differs in both directions from CCITT 1/40

present programs for optimal design of active RC circuits are elaborated taking into account the possibilities and limitations of hybrid integrated technology. This research is supported by REMIX, Budapest.

The results are demonstrated in Table 2 and Table 3. The mark ∞ in the last row of Table 2 indicates that one of the capacitances had a very large tolerance. This capacitance can be omitted from the circuit.

The permanent application of the programs permitted TELEFONGYÁR the following results. The number of the circuit elements of the filters varied between 5 and 30. The starting yield was about 60—80% and the final one about 90—100%. The starting tolerances of 0.5—1% was increased to 2—5%. If only the yield improvement is taken into consideration, the economical advantage of the new design methodology is well justified.

The development of the algorithms and the program implementation became possible through the support of the Hungarian Academy of Sciences and the industrial cooperation with the TELEFONGYÁR and REMIX.

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