EFFECT OF TUNING THE SECOND HARMONIC FREQUENCY ON THE BEHAVIOUR OF GUNN DIODE OSCILLATORS

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Summary

Effect of the second harmonic sinusoide on the Gunn-diode oscillator performance is studied using the Two Input Sinusiode Describing Function.

The effects of tuning the second harmonic termination on the generated power at the fundamental frequency on the oscillator stability and on the oscillator tuning characteristic are presented.

Introduction

Negative resistance nonsinusoidal oscillators had been discussed by several authors. L. Gustafsson [1] introduced the use of the describing function in analyzing negative resistance oscillators and amplifiers. K. W. H. Foulds [2] discussed the general characteristic of negative resistance oscillators for which the voltage waveform across the active device consists of fundamental and second harmonic components.

H. Pollmann [3] and E. M. Bastida [4], [8] found that the variation of the second harmonic frequency terminations cause the fundamental output power to change by a factor of up to 5. Quine [5] derived the restrictions which a negative resistance oscillator must satisfy if the characteristic shows no hysteresis. In this paper we use the Two Sinusoide Input Describing Function "T. I. S. D. F." [6] to study the effect of the second harmonic component on the behaviour of the Gunn-diode oscillators. Section II is concerned with the block diagram of the oscillator circuit. The effect of the second harmonic sinusoide, and the T. I. S. D. F. is investigated in section III.

In section IV, the effect of the second harmonic sinusoide on the generated power at the fundamental frequency is studied. Section V deals with the oscillator stability in the presence of a fundamental and of a second harmonic component. Section VI is concerned with the oscillator tuning characteristic.

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Block Diagram of the Oscillator circuit

The general form of the oscillator circuit shown in Fig. 1 consists of the active device in parallel with a noise current source which represents the intrinsic noise source of the active device. The four terminal network shown in Fig. 1 represents the equivalent circuit of the diode parasitic elements, diode mounting structure, and the stabilizing circuit cavity resonator, with or without tuning elements.

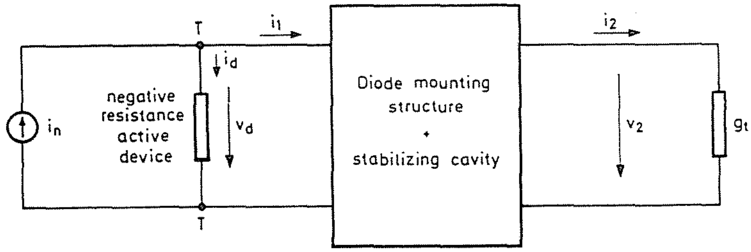

Fig. 1. Equivalent circuit of the oscillator

Let the above coupling circuit be described by its voltage current transmission matrix [T], where

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{1}$$

The voltages and currents in (1) are shown in Fig. 1, then

$$V_1 = V_d = T_{11}(\omega)V_2 + T_{12}(\omega)I_2$$

$$I_1 = I_n - I_d = T_{21}(\omega)V_2 + T_{22}(\omega)I_2$$

$$I_2 = g_t V_2$$

Eliminating V_2 and I_2

$$I_n - I_d = \frac{T_{21}(\omega) + T_{22}(\omega)g_t}{T_{11}(\omega) + T_{12}(\omega)g_t} V_d = y_L(\omega)V_d \tag{2}$$

Where $y_L(\omega)$ represents the input admittance seen at the active device reference plane $T-T$ looking toward the load side. From (2) and replacing the nonlinear active element by its describing function, the system can be represented by the block diagram shown in Fig. 2. The difference between the block diagram shown in Fig. 2 and that obtained by [1] is that in our case the noise source represents the active device noise source while in [1] it represents the load noise source or an injected signal.

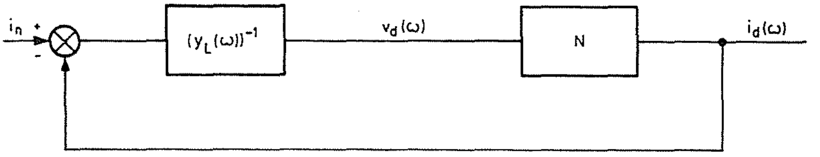

Fig. 2. Block diagram representation of the oscillator circuit

Effect of the second harmonic sinusoide

Let the i—V characteristic of the diode be represented by the Van-der Pol-type (cubic) nonlinearity

$$i_d = -a_0 v_d + a_1 v_d^2 + a_2 v_d^3 \tag{3}$$

Where i_d and v_d are the instantaneous current and voltage deviations from the d—c bias point [1], and the coefficients a_0, a_1 and a_2 are positive numbers [7].

The voltage v_d will be assumed to contain only a fundamental and a second harmonic component

$$v_d = A\cos\omega t + B\cos(2\omega t + \Theta) = \mathrm{Re}(Ae^{j\omega t} + Be^{j\Theta}e^{j2\omega t}) \tag{4}$$

where A and B are the amplitudes of the fundamental and of the second harmonic sinusoides, respectively, and Θ is the phase angle of the second harmonic sinusoide with respect to the fundamental sinusoide.

In our analysis we neglect the variation of the diode capacitance with the r—f voltage amplitude.

The resultant device current from (3) and (4) will be

$$i_d(t) = -a_0 A\cos\omega t - a_0 B\cos(2\omega t + \Theta) + a_1 A\, B\cos(\omega t + \Theta) +$$
$$+\frac{a_1 A^2}{2}\cos 2\omega t + a_2\left(\frac{3}{4}A^2 + \frac{3}{2}B^2\right)A\cos\omega t +$$
$$+a_2\left(\frac{3}{4}B^2 + \frac{3}{2}A^2\right)B\cos(2\omega t + \Theta) + \text{higher frequency terms} =$$
$$= \mathrm{Re}\left[\left(-a_0 + a_1 Be^{j\Theta} + a_2\left(\frac{3}{4}A^2 + \frac{3}{2}B^2\right)\right)Ae^{j\omega t} +\right.$$
$$\left.+\left(-a_0 + \frac{a_1 A^2}{2B}e^{-j\Theta} + a_2\left(\frac{3}{4}B^2 + \frac{3}{2}A^2\right)\right)Be^{j(2\omega t + \Theta)}\right] +$$
$$+\text{higher frequency terms} \tag{5a}$$

$$i_d(t) = |I_{d1}|\cos(\omega t + \Phi_1) + |I_{d2}|\cos(2\omega t + \Phi_2)$$
$$= \mathrm{Re}[\,|I_{d1}|e^{j\omega t}e^{j\Theta_1} + |I_{d2}|e^{j2\omega t}e^{j\Theta_2}] \tag{5b}$$

The describing function "N" at frequency f is defined as the ratio of the phasor representation of the current component of frequency f, to the phasor representation of the voltage component at the same frequency.

Using the notations of N_A and N_B for the fundamental and for the second harmonic describing functions, respectively, we get

$$N_A = -a_0 + a_1 B e^{j\Theta} + a_2\left(\frac{3}{4}A^2 + \frac{3}{2}B^2\right) \tag{6}$$

$$N_B = -a_0 + \frac{a_1 A^2}{2B} e^{-j\Theta} + a_2\left(\frac{3}{4}B^2 + \frac{3}{2}A^2\right) \tag{7}$$

Now the system can be resolved to two systems. Neglecting the effect of noise, the condition of oscillation is obtained from Fig. 2.

$$1 + \text{open loop gain} = 0$$

$$1 + (y_L(\omega))^{-1} N = 0$$

which can be written separately for the fundamental and for the second harmonic sinusoides as follows:

$$N_A + y_L(\omega) = 0 \tag{8}$$

$$N_B + y_L(2\omega) = 0 \tag{9}$$

where

$$y_L(\omega) = g_L(\omega) + jb_L(\omega)$$

$$y_L(2\omega) = g_L(2\omega) + jb_L(2\omega)$$

Equating to zero the real and the imaginary parts of (8) and (9) using (6) and (7), we get

$$-a_0 + a_1 B\cos\Theta + a_2\left(\frac{3}{4}A^2 + \frac{3}{2}B^2\right) + g_L(\omega) = 0 \tag{10}$$

$$a_1 B\sin\Theta + b_L(\omega) = 0 \tag{11}$$

$$-a_0 + \frac{a_1 A^2}{2B}\cos\Theta + a_1\left(\frac{3}{2}A^2 + \frac{3}{4}B^2\right) + g_L(2\omega) = 0 \tag{12}$$

$$-\frac{a_1 A^2}{2B}\sin\Theta + b_L(2\omega) = 0 \tag{13}$$

Equations (10), (11), (12) and (13) are nonlinear coupled equations in A, B, ω and Θ. For a given load condition $g_L(\omega)$, $b_L(\omega)$, $g_L(2\omega)$, and $b_L(2\omega)$ are given, they can be used to determine A, B, ω and Θ. If, however, the values of A, B, ω and Θ are selected according to special conditions, then the proper load admittances can be found from (10)—(13).

We get an interesting by-product if we eliminate sin Θ from (11) and (13)

$$\frac{A^2}{2B^2} = -\frac{b_L(2\omega)}{b_L(\omega)} \tag{14}$$

Equation (16) is the same as that obtained by Quine [5] except that in our case we consider only the fundamental and the second harmonic components.

Maximum Generated Power at the Fundamental Frequency

The generated power at the fundamental frequency is given by

$$P_g = \frac{1}{2} g_L(\omega) A^2 \tag{15}$$

using (10) in (15) P_g can be written

$$P_g = \frac{1}{2}\left(a_0 - a_1 B \cos\Theta - a_2\left(\frac{3}{4} A^2 + \frac{3}{2} B^2\right)\right) A^2 \tag{16}$$

For maximum generated power

$$\frac{\partial P_g}{\partial A} = 0 \quad \text{and} \quad \frac{\partial P_g}{\partial B} = 0, \quad \text{which give}$$

$$0 = a_0 - a_1 B_m \cos\Theta - \frac{3}{2} a_2 (A_m^2 + B_m^2) \tag{17}$$

$$0 = -a_1 \cos\Theta - 3a_2 B_m \tag{18}$$

From (18), the amplitude of the second harmonics for maximum generated power at the fundamental frequency is given by

$$B_m = -\frac{a_1}{3a_2} \cos\Theta \tag{19}$$

The value of cos Θ must be negative as seen from (19) since a_1, a_2 and a_3 are positive i.e. $\pi/2 < \Theta < \frac{3\pi}{2}$

Substituting (19) in (17) the amplitude of the fundamental for maximum generated power is given by

$$A_m^2 = \frac{2a_0}{3a_2}\left(1 + \frac{a_1^2}{6a_0 a_2} \cos^2\Theta\right) \tag{20}$$

At maximum generated power, the conductances to be seen by the active device for the fundamental and for the second harmonic frequencies can now be calculated from (10), (12) taking into account (19) and (22)

$$g_L(\omega)_{max} = \frac{a_0}{2}\left(1 + \frac{a_1^2}{6a_2a_0}\cos^2\Theta\right) \tag{21}$$

$$g_L(2\omega)_{max} = a_0\left(1 - \frac{a_1^2}{12a_0a_2}\cos^2\Theta\right) \tag{22}$$

The value of the maximum generated power is obtained by substituting (20) and (21) into (15)

$$P_g = \frac{a_0^2}{6a_2}\left(1 + \frac{a_1^2}{6a_0a_2}\cos^2\Theta\right)^2$$

$$P_g = P_{go}\left(1 + \frac{a_1^2}{6a_0a_2}\cos^2\Theta\right)^2 \tag{23}$$

Here P_{go} is the maximum generated power assuming pure sinusoidal voltage waveform [8], [9].

From (23) it is seen that the generated power increased by a factor $\left(1 + \frac{a^2}{6a_0a_2}\cos^2\Theta\right)^2$ which is always greater than unity.

The value of this factor can be obtained from (21) and (22) in terms of $g_L(\omega)_{max}$ and $g_L(2\omega)_{max}$

$$\left(1 + \frac{a_1^2}{6a_0a_2}\cos^2\Theta\right)^2 = \left(\frac{3g_L(\omega)_{max}}{g_L(\omega)_{max} + g_L(2\omega)_{max}}\right)^2 \tag{24}$$

From the measured values of the load conductances at the fundamental and at the second harmonic frequencies corresponding to the maximum output power obtained by [10], the value of (24) is equal to 5, which is in agreement with the observed increase in the output power in [3], [4], [10] and [11]. The value of the conductance seen by the active device when the generated power is maximum can be calculated from (21).

$$g_L(\omega)_{max} = \frac{\sqrt{5}}{2}a_0 = 1{,}118\, a_0 > a_0 \tag{25}$$

Equation (25) can be interpreted by the existence of a trapped domain mode [12], [13].

According to this loading condition, the oscillation cannot be initiated since $g_L(\omega)$ is greated than the small signal negative conductance of the device. The oscillation at maximum generated power can be initiated either by large signal injection of the oscillator [12] or by tuning the oscillator circuit in such a way that when the oscillator is switched on the conductance seen by the active device is smaller in magnitude than the small signal negative conductance of the device, and then the conductance is increased during tuning.

Oscillator stability

From the foregoing analysis of the oscillator circuit we have two resonance conditions at the fundamental and the second harmonic frequencies.

The two conditions can be obtained from the block diagram shown in Fig. 2, equations (8) and (9).

The system stability can be analyzed by using the Incremental-Input-Describing-Function [6]. Figure 3 shows the block diagrams of the perturbed systems, where ΔA, ΔB and $\Delta\omega$ are the perturbations in A, B and ω respectively. N_{Ai} and N_{Bi} are the incremental input describing functions for the fundamental and for the second harmonic amplitude perturbation, respectively, and are given by [6].

$$N_{Ai} = N_A + \frac{A}{2}\frac{\partial N_A}{\partial A}$$

$$N_{Bi} = N_B + \frac{B}{2}\frac{\partial N_B}{\partial B} \tag{26}$$

To check the system stability ω is to be replaced by $\omega + j\sigma$ where σ is the damping coefficient, given by

$$\sigma = -\frac{1}{\Delta A}\frac{d\Delta A}{dt} = -\frac{1}{\Delta B}\frac{d\Delta B}{dt}$$

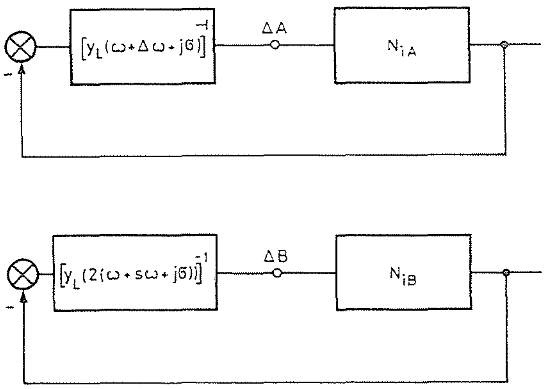

Fig. 3. Block diagram representations of the perturbed oscillator

Solving the characteristic equations for each system shown in Fig. 3, if σ is positive, the perturbation will decay with time, and the system is stable.

If σ is negative, the perturbation will grow with time and the system is unstable.

Before solving for σ, the value of $y_L(\omega+\Delta\omega+j\sigma)$ and $y_L(2(\omega+\Delta\omega+j\sigma))$ are to be expanded around ω by Taylor series.

The characteristic equations as seen from Fig. 3.

$$y_L(\omega+\Delta\omega+j\sigma)+N_{Ai}=0$$

$$y_L(2(\omega+\Delta\omega+j\sigma))+N_{Bi}=0$$

$$y_L(\omega)+\frac{\partial y_L}{\partial\omega}(\Delta\omega+j\sigma)+N_A+\frac{A}{2}\frac{\partial N_A}{\partial A}=0 \tag{27.a}$$

$$y_L(2\omega)+\frac{\partial y_L(2\omega)}{\partial\omega}(\Delta\omega+j\sigma)+N_B+\frac{B}{2}\frac{\partial N_B}{\partial B}=0 \tag{27.b}$$

Using (8) and (9), (27.a) and (27.b) are reduced to

$$\frac{\partial y_L(\omega)}{\partial\omega}(\Delta\omega+j\sigma)+\frac{A}{2}\frac{\partial N_A}{\partial A}=0 \tag{28.a}$$

$$\frac{\partial y_L(2\omega)}{\partial\omega}(\Delta\omega+j\sigma)+\frac{B}{2}\frac{\partial N_B}{\partial B}=0 \tag{28.b}$$

Equating to zero the real and the imaginary parts of (28.a) and (28.b) and eliminating $\Delta\omega$ from each set, for σ to be positive we get:

$$\frac{\partial N_{AP}}{\partial A}\frac{\partial b_L(\omega)}{\partial\omega}-\frac{\partial N_{Aq}}{\partial A}\frac{\partial g_L(\omega)}{\partial\omega}>0 \tag{29.a}$$

$$\frac{\partial N_{BP}}{\partial B}\frac{\partial b_L(2\omega)}{\partial\omega}-\frac{\partial N_{Bq}}{\partial B}\frac{\partial g_L(2\omega)}{\partial\omega}\geqq 0 \tag{29.b}$$

where

$$N_A=N_{Ap}+jN_{Aq}$$

$$N_B=N_{Bp}+jN_{Bq}$$

The stability conditions given by (29.a) and (29.b) are of the same form as that obtained by Kurokawa [14] when a single sinusoide was only assumed. For nonsinusoidal oscillator containing fundamental and second harmonic sinusoides, the stability conditions must be satisfied at both the fundamental and the second harmonic frequencies.

Tuning characteristic

The oscillator cavity resonators have a set of discrete resonance frequencies corresponding to a set of resonant modes. Coupling the cavity resonator with the active device and with the load, the set of resonance frequencies will be perturbed and the resultant perturbed set of resonance frequencies are the resonance frequencies of the oscillator circuit. (i.e. the cavity resonator, the mounting structure of the active device, the load, and the coupling lines)

At any frequency f, the oscillator circuit can be represented by an equivalent circuit corresponding to the resonant mode which have a resonance frequency close to f.

The susceptance seen at the active device terminals at the fundamental and at the second harmonic frequencies are given by

$$b_L(\omega)=Q_{e1}(f/f_{o1}-f_{o1}/f)$$
$$b_L(2\omega)=Q_{e2}(2f/f_{o2}-f_{o2}/2f) \qquad (30)$$

where $f_{o'}$ and f_{o2} are the resonance frequencies closest to f and 2f, respectively. Q_{e1} and Q_{e2} are the external quality factors of the equivalent circuits at the fundamental and at the second harmonic frequencies, respectively. Substituting (30) in (14) we get:

$$A^2Q_{e1}(f/f_{o1}-f_{o1}/f)+2B^2(Q_{e2}2f/f_{o2}-f_{o2}/2f)=0 \qquad (31)$$

The oscillator frequency is given by

$$f^2=f_{o1}^2\frac{1+\frac{B^2}{A^2}\frac{Q_{e2}}{Q_{e1}}(f_{o2}/f_{o1})}{1+4\frac{B^2}{A^2}\frac{Q_{e2}}{Q_{e1}}(f_{o1}/f_{o2})} \qquad (32)$$

Or simply the relative deviation of the oscillation frequency

$$\Delta f=\frac{f-f_{o1}}{f_{o1}}=\frac{2(B/A)^2\frac{Q_{e2}}{Q_{e1}}(1-2f_{o1}/f_{o2})}{1+2(B/A)^2Q_{e2}/Q_{e1}} \qquad (33)$$

The nonlinearity in the tuning characteristic which can be observed in practice is mainly due to the variation of the relative amplitude B/A with a frequency as seen from (33).

A linearization scheme can be employed to improve the F−M characteristic of the oscillator [15].

In [15], the oscillator resonance circuit was assumed to have only a single resonance frequency. From (32) or (33) the oscillation frequency can be determined in terms of the resonance frequency and the external quality factor of the oscillator circuit at the fundamental and at the second harmonic frequencies.

Conclusion

The effect of the second harmonic sinusoide on the performance of the Gunn-diode oscillator had been discussed using the T.I.S.D.F.

The calculated increase in the output power was found to be in agreement with the observed increase.

The second harmonic tuning causes a peaking in the output power at the fundamental frequency, and also causes an increase in the negative conductance at the fundamental frequency.

The oscillator stability had been also studied, and simple stability conditions were derived.

The effect of the variation in the relative amplitudes in the tuning characteristic were discussed and the pulling effect had been derived in terms of the resonance circuit parameters at the fundamental and at the second harmonic frequencies.

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