JENŐ EGERVÁRY
A GREAT PERSONALITY
OF THE HUNGARIAN MATHEMATICAL SCHOOL

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Jenő Egerváry 1891—1958

Summary

The paper deals with the life and work of Jenő Egerváry, professor of mathematics at the Technical University Budapest from 1941 to 1958. A comprehensive bibliography of his papers is attached.

Dénes König's paper Graphs and Matrices [I] and Jenő Egerváry's paper On Combinatorial Properties of Matrices [11], containing results which are classical by now, appeared more than 50 years ago in Vol. 38 of the Hungarian periodical "Matematikai és Fizikai Lapok." König also published his results in German [II] and some years later included that paper in his book entitled Theory of Finite and Infinite Graphs (in German, [III]). The importance of his work could not be better demonstrated than by the fact that the book was republished in New York in 1950, by the Chelsea Publishing Corp. The victorius career of König's and Egerváry's theorems made its real start when American mathematicians recognized their applicability in several problems of operation research. H. W. Kuhn translated and published Egerváry's work in 1955 [IV]. Soon after, in his papers [V] [VI] he showed how König's and Egerváry's theorem could be applied for solving the so-called assignment problem. From that time on, authors quote the method of proving the above theorems as Hungarian Method applicable in various fields and different forms (cf. e.g. [VII], [VIII]).

It is very sad that Dénes König did not live to see the international success of the Hungarian Method in the classical literature of operation research: in 1944 he committed suicide to escape from Fascist persecution. Egerváry obtained knowledge of the American mathematicians' work in 1957 only: under their influence, his interest turned towards economical applications. This is how — more than a quarter of a century after his fundamental paper had appeared — he applied this method to solve the so-called transport problem [70], [71].

The path accomplished by the mentioned theorems of König and Egerváry, from their graph-theoretical formulation to the solution of the transport problem, convincingly represents the unity of theory and practice and the interaction which necessarily promotes progress of both.

Let us now take a closer look at the man whose brilliant intellect, after decades, still has a productive effect on generations of scientists active in the broad spectrum of applied mathematics. We shall briefly recite the main stations of his life and attempt to survey his major achievements in mathematics.

Jenő Egerváry was born in 1891 in Debrecen. He studied at the Pázmány Péter University in Budapest, and received his doctor's degree in 1914. Subsequently he became assistant at the Seismological Institute in Budapest (till 1917) and from 1918 on, professor at the Superior Industrial School in Budapest. In 1921 he was appointed to privat-docent at the University of Kolozsvár (today University of Szeged) which at the time functioned in the capital. In 1932 he was rewarded with the Gyula König Prize for his scientific activity. In 1938 he became privat-docent at the Pázmány Péter University in Budapest, and in 1941 he was appointed to full professor at the Technical University Budapest. The Hungarian Academy of Sciences elected him to corresponding member in 1943 and to ordinary member in 1946. In 1947, he founded the Institute of Applied Mathematics of the Hungarian Academy of Sciences. He was honoured with the Kossuth Prize in 1949 and in 1953 and with the Order of Labour in 1956. On November 30, 1958 he died by his own hand under tragic circumstances.

The first results of Egerváry's scientific activity are related to those of Lipót Fejér, who also inspired his doctor's thesis. He wrote several papers in the field of analysis and theory of functions, meanwhile, his interest turned soon towards algebraic equations. It is characteristic already for his first papers that the theory of determinants frequently is an important and useful tool. Dénes König characterized Egerváry in 1932 among others with the following words [IX]: "Egerváry's calculative ability — above all regarding determinants — may be traced right through all his work; in this sense, his activity appears as a renascence of Hungarian mathematical tradition, having in mind Jenő Hunyadi and Ágoston Scholtz." Egerváry's later work — and particularly the papers published in the last years of his life — fully support Dénes König's statement. In the meanwhile, however, Egerváry's interest turned almost parallelly towards geometry and towards theoretical physics. From 1938 on, for about 15 years, he continued to publish his results in the fields of geometry and differential equations, above all those related to the orthocentric system of coordinates and its applications, as well as to those dealing with the three-body problem. In the last six years of his life Egerváry devoted his activity almost exclusively to matrix theory with particular regard to applications.

The major part of his papers were published in the last 14 years of his life. His creative powers were largely stimulated by the material and moral appreciation of scientific work. He contributed valuable achievements to international mathematical literature and won recognition for Hungarian mathematicians all over the world.

All of Egerváry's papers — as well as his lectures — are characterized by lucidity, accuracy and elegance. His style is concise, one can never find a single superfluous word, but this will never affect intelligibility. One of the main features attracting the reader is his suggestiveness, and another feature — found in almost all his works — is the endeavour to find and demonstrate the applicability of the results.

Egerváry's first paper [1] deals with a class of integral equations of second kind in which the kernel is a periodic function of the difference $(x-\xi)$ and therefore the matrix of the approximating system of algebraic equations is cyclic. Egerváry calculated the limit of the determinant for $n\to\infty$ and represented the so-called Fredholm transcendent thus obtained in the prime factor form. This product form evidences the fact already recognized earlier that the eigenvalues are the reciprocals of the coefficients in the Fourier series expansion of the kernel.

The first paper by Egerváry on polynomials and algebraic equations [4] appeared in 1918. The starting point of this work consists of the following four apprehensible properties of the sine curve: (i) The area between any chosen zero and the neighbouring minimum is equal to the area between the zero and the neighbouring maximum. (ii) All areas between two neighbouring zeros are equal. (iii) All zeros are inflexion points. (iv) The absolute values of all maxima and minima are identical. Egerváry raised and answered the following question: which, among the polynomials of degree n whose zeros are real and single, will possess properties (i), (ii), (iii) and (iv) resp. He proved that the functions which have these properties are the only ones that satisfy the above-mentioned conditions.

In [7], [8] and [12] Egerváry gave the application of characteristic equations in the theory of power series and polynomials. It is an important result of [8] (with Ottó Szász as co-author) that an upper bound is provided for the amplitudes of certain terms of non-negative trigonometric polynomials and for the absolute value of the derivative of the polynomial. In [12], extremal value problems regarding general harmonic polynomials being non-negative in the unit circle are discussed. Egerváry here — among others — solves the following problem as a generalization of a problem solved earlier by S. Bernstein: the value of n-th order harmonic polynomials being non-negative in the unit circle is given in two points of the perimeter of the unit circle; the minimum of the polynomials is to be determined in an arbitrary point inside the unit circle. (Bernstein was looking for the minimum in the centre of the circle.)

In [15], Egerváry generalized the following result of Lipót Fejér's: not a single partial sum of the geometrical series $\sum_{n-1}^{\infty} z^k$ is univalent. He pointed out that the third order arithmetical means of the series are univalent within the closed unit circle. In [37] he generalized further results of Lipót Fejér and Ottó Szász. In [72] and [73], with Pál Turán as co-author, he deals with the determination of the "most economical stable interpolation".

Papers [5], [6] and [9] deal with the localization of the roots of algebraic equations and with the determination of the domains in which — under certain restrictions to the coefficients or the roots — each root can vary. Papers [5] and [6] contain studies on extremal value problems connected with symmetric multilinear forms. [9] discusses the localization of the roots of trinomial equations. The basic idea of the paper is that the general trinomial is the derivative of the product of two binomials; hence he determines $n+m$ annular sectors, each containing exactly one root of the trinomial equation $Az^{n+m}+Bz^m+C=0$. [10] contains the generalization of the Kakeya's theorem.

Egerváry's work on geometry starts with [17]. In this paper he deals, introducing symmetric parameters, with the main properties of tetrahedrons whose altitudes meet in one point. Such orthocentric tetrahedrons show many analogies with the triangle which do not exist in the general tetrahedron.

[36] deals with a property of spatial curves. In [24] (co-author: György Alexits) he established the general theory of linear curvatures in semi-metric

spaces and proved that it includes, as a special case, the theory of curvatures of the rectificable curves in the Euclidian space.

Egerváry published his first paper on differential equations [19] in 1938, dealing with the differential equations of electron motion.

[30], [31] and [32] discuss a new form of the differential equation of the three-body problem and its solution in special cases. Egerváry observed an interesting analogy between the differential equations of the three-body problem and those of the force-free gyroscope. The discovery of this analogy was made possible by the recognition that in the cases of both three-body problem and the force-free gyroscope, kinetic and potential energies depend merely on the speed components of the principal inertia axes of the system and on the coordinates and speed components of the bodies relative to these axes, but they are independent of the absolute position in space. It is known from the theory of gyroscope that if the principal inertia axes of the gyroscope are being considered a moving system of coordinates, the 12-th order system of differential equations of gyroscope motion will split into three second order systems and two third order systems. The second order systems define the motion of the centre of gravity; one of the third order systems provides the kinematic equations, while the other is identical with the Euler equations of the gyroscope. Egerváry showed that if the principal inertia axes are considered as moving system of coordinates in the three-body problem (two principal radii of inertia and one angular coordinate are chosen as general coordinates), the 18th order system of differential equations of the three-body problem will split into three second order systems, two third order systems and one 9-th order system. The meaning of the second order and third order systems will be the same as in the case of the gyroscope, while the 9-th order system can be regarded as a novel form of the equations of the three-body problem. Obviously, first integrals of this system are the energy integral and the integral expressing the relationship between the angular velocities. Making use of them and eliminating time, the system can be reduced to a sixth order system.

If the general coordinates (that is, the distances of the bodies from each other) are known in function of time, the system can be integrated by quadrature. Hence, the Lagrange's theorem: if the motion of the triangle defined by the three bodies is known in function of time, the problem can be solved by quadrature, — becomes apparent in the novel form of the differential equations of the three-body problem. Egerváry solved the differential equations for special cases and also showed how some results published by other authors (e.g. Pylarinos, Sokolov) follow.

[35] presents an approach for calculating the lowest critical speed of a system consisting of a cylindrical rotor and axially jointed shafts with different diameters. The problem is of great importance in the design of turbogenerators; Egerváry took up an idea of Rayleigh in developing the approach.

Egerváry's comprehensive paper on matrix theory [46] appeared in 1953; it is the introduction to his subsequent activity, the basis of his work in the last six years of his life.

Dyadic reduction of matrices plays an important part in Egerváry's work on matrix theory. In itself, the principle was known earlier; however, it was Egerváry who first recognized its feasibility and applied it to impart a new direction to theory and to simplify numerical computation techniques.

In [47] Egerváry proved the following interesting theorem for projectors (a matrix P is called projector, if it satisfies the equation $P^2 = P$.) If an r-rank projector P is expressed as the sum of linearly independent dyads $P = \sum_{k=1}^{r} u_k v_k^*$, then the vectors u_k and v_k^* are the right and left eigenvectors of the projector matrix, respectively; they are automatically biorthogonalized, that is, they satisfy the relationships $u_k^* v_l = \delta_{kl}$. The theoretical and practical importance of this recognition is that a projector cannot be reduced to the sum of dyads in any other way than the one in which the vectors of the dyads are biorthogonal. The theorem can be applied directly for finding the eigenvectors of matrices, and in general, for the canonical representation of matrix functions.

Egerváry made use of the dyadic reduction of matrices to generalize Stieltjes lemma known in matrix theory [48] [49]. The lemma in question is the following: if all elements of a positive definite matrix off the main diagonal are negative, then all elements of its inverse are positive. Egerváry proved that the above conditions may be replaced by weaker ones, namely: if all principal minors of a matrix are positive, all its elements off the main diagonal are non-positive, and all its columns in the triangle above the principal diagonal as well as all its lines in the triangle below the diagonal contain at least one negative element, then all elements of the reciprocal of the matrix will be positive.

[62], [63] and [68] discuss solution methods of linear systems of equations. It is of great interest and very characteristic for Egerváry that he was able to contribute valuable results to this field of mathematics considered by many as fully closed. In [62] and [69] he described a general rank-diminishing method which may be considered the generalization of the dyadic reduction and allows to solve linear systems of equations in a simple manner by finite iteration.

In [60] Egerváry applies the rank-diminishing method to solve homogeneous linear Diophantian system of equations. In the case of a single Diophantian equation, the result yields the solution formula by Barnett and Mendel.

Egerváry also utilized the rank-diminishing method in [68] to develop another finite iteration technique to solve linear systems of equations; it is the generalization of the Purcell vector method for any arbitrary coefficient matrices. It also allows the simultaneous solution of two systems of equations.

In [63], expanding the concept of the inverse on arbitrary (rectangular) matrices, Egerváry developed a method for the explicit representation of the generalized inverse by means of exclusively rational operations; the method is based on the factorization of the matrix to be inverted to basis factors. The method of factorization is applied most impressively in [75], this paper being the culmination of Egerváry's activity in the field of matrix theory, in the sense that in this paper he discusses and solves a domain of matrix theory closed in itself by a uniform method: Egerváry developed a method to reduce quadratic matrices to the Jordan normal form, which is constructive insofar as it only requires — if the eigenvalues are known — addition and multiplication of matrices, and hence it can readily be programmed for computers. The algorithm used in the reduction of the matrices to basis factors plays an important part in the method; comprehension is greatly enhanced by the circumstance that the right and left eigenvectors and principal vectors of the matrix in question are being used in the whole process of reduction simultaneously and symmetrically.

The papers [53] and [54], dealing with hypermatrices (block matrices) promoted a new direction in the development of matrix theory. In these papers the aim is manifested to extend the concepts and methods of calculation concerning ordinary matrices to hypermatrices consisting of blocks commutable in pairs. Its significance consists in the reduction of operations with matrices of order mn to operations with matrices of order m and n, respectively. Results in this direction are related to the eigenvalues of the direct product (Stéphanos) and to the eigenvalues of hypermatrices (Williamson) whose block are functions $f_{ij}(A)$ of the matrix A. Egerváry extends the results related to the eigenvalues of such hypermatrices to the eigenvectors, and gives a method to determine, for hypermatrices consisting of commutable blocks, the determinant and the adjoint.

The concept of direct product (Kronecker product) proved efficient in the mathematical study of various mechanical systems with regular structures. In lattice dynamics the system of particles forming a regular lattice in their equilibrium state is studied, assuming that each particle is affected by its immediate neighbours only. The knowledge of normal vibrations of finite lattices is of primary importance in the corpuscular theory of matter. Egerváry's method enables to determine the normal vibrations of two- and three-dimensional lattices fixed along the boundary. It yields the following result: the eigenvectors of the two- or three-dimensional lattices are direct products of the eigenvectors of the one-dimensional "edge lattices", and the squares of the frequencies are equal to the sums of squares of the frequencies of the one-dimensional edge lattices.

Applying the results achieved regarding hypermatrices consisting of commutable blocks, Egerváry developed in [74], [77] and [78] a readily programmable method for solving Poisson's differential equation for arbitrary domains.

Among applications of the matrix theory paper [59] should be mentioned; its particular interest is that in order to solve a certain problem of statics, Egerváry obtained systems of linear equations whose coefficient matrices are essentially of the same type as discussed first in Scholtz's and Hunyadi's work. The problem is to get the forces in the bars of a system consisting of three bars forming a triangle and in the bars of a lattice work consisting of six bars forming a tetrahedron, if the system is loaded by a system of forces in equilibrium in the hinges.

[64] and [65] discuss the general theory of stiffened suspension bridges. Egerváry points out that in selecting a suitable mathematical model for suspension bridges, a difference must be made between cable bridges and chain bridges.

A cable bridge consists of an elastic beam (stiffening girder) and a one-dimensional flexible continuum (cable) coupled with a finite number of suspension bars. In the case of a larger number of suspension bars, the system can be replaced by a simpler model in which the cable and the beam are connected with a suspension membrane; this model leads directly to the well-known linear differential equation first established by E. Melan.

In contrast, a chain bridge's chain bars and suspension bars are to be considered rigid (or only longitudinally elastic). The system is replaceable more accurately by a model with a finite number of degrees of freedom, when replacing the load by statically equivalent concentrated forces acting on the lower end points of the suspension bars. This "finitized" model of chain bridges leads to an algebraic system of linear equations in which the number of unknowns is equal to the number of suspension bars. Egerváry established and solved the system of equations of the finitized model of the chain bridge using matrix methods.

Following Egerváry's activity in matrix theory, this branch of science made rapid progress in Hungary. Undoubtedly, the fast spreading of computers added considerably to its actuality and in connection with it the demand for numerical analysis, including numerical methods of linear algebra, — a branch of mathematics formerly rather neglected in our country — attracted the attention of Hungarian mathematicians. It is the merit of Egerváry that he recognized the importance and interest of this domain earlier than anybody else and opened new perspectives to research in this field of mathematics, turning from classical into modern again. In his last papers and lectures he emphasized repeatedly that discrete methods have come into prominence as the result of general use of high speed computers, and that such methods are gaining importance not only within mathematics, but in various applications of mathematics, e.g. in physics, in engineering sciences, in economical sciences etc.

At the present rate of scientific progress fifty years are a long period. The criterion of the true geniuses is the fact that their scientific results do not become obsolete, on the contrary: they act creatively on further research and open up new vistas for arising disciplines. This is demonstrated by König's and Egerváry's brilliant accomplishments achieved fifty years ago which now belong to the classical basis of a novel discipline: the operation research, — and by the scientific results in other fields of mathematics of Jenő Egerváry, who was one of the most comprehensive and versatile personalities of the Hungarian mathematical school.

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