ANALOG-DIGITAL DEVICES FOR PARAMATER ESTIMATION OF THE TRANSFER FUNCTION

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Summary

In this paper comparatively simple method is presented for identifying linear and nonlinear dynamic units. It is based on the analysis of steady-state response and makes use of the sequential integrating procedure. Analog-digital devices needed for realizing this method are described. It is shown that use of the microprocessor made it possible to continuously contol the elements in the function control systems.

1. Introduction

The design of control systems requires information about structure and parameters of individual control elements. To this aim adaptive models offer an efficient method [1]. Two problems may be distinguished such as structure identification and parameter identification. In the first problem the structure of the transfer function of controlled unit is a priori unknown and could be defined as a result of adaptation of the model structure. In the second case, previous theoretical and experimental knowledge is relived on to make an assumption about a certain structure of transfer function and then the coefficients of this may be obtained by adapting the model parameters.

In this paper the problem of parameter identification of the controlled element is considered, and the relevant analog-digital modelling devices are described. It is known that the Step Response methods are very often applied in practice [2]. The precision of these methods especially depends on the type of input test signals. In [3] a method using test signal as time-power function

$$X(t)=\alpha_k t^k, \qquad k=0, 1, 2, \ldots,$$

has been suggested. However, it has some shortcomings. First, it is necessary that free oscillations in the control unit are ended before the output signal has

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reached its limit value, a condition difficult to provide already for $k \geqq 2$ without carrying out several corrections of coefficients α_k.

Second, even an insignificant deviation of test signal from the time-power form provides considerable errors of parameter estimation.

The method to be presented applies a periodic test signals of arbitrary form and has only one constraint: the transient response must end in a finite time τ after the arise of the input test signal [4].

2. Method of parameter estimation

This method may be applied for identification of the aperiodic unit described by the linear transfer function

$$H(s) = \frac{b_0}{1+a_1 s+ \ldots +a_n s^n} = \frac{b_0}{R(s)} . \tag{1}$$

The estimation of transfer function coefficients is based on the limit theorem of the Laplace transform. Unknown parameters are defined in $n+1$ steps. In each step such a test signal is entered to satisfy following condition:

$$\begin{aligned} &\lim_{t \to \infty} x(t) = x_\infty = \text{const}\,, \\ &\lim_{t \to \infty} y(t) = y_\infty = \text{const}\,, \end{aligned} \tag{2}$$

where $y(t)$ — output signal of the unit. In the "zeroth" step we obtain coefficient b_0:

$$K_0 = \frac{y_\infty}{x_\infty} = b_0 . \tag{3}$$

Further, consider expression

$$U_1(t) = \int_0^t (K_0 x(t) - y(t))\, dt\,, \tag{4}$$

assuming that K_0 is known.

The corresponding Laplace transform is

$$U_1(s) = \frac{K_0}{s} X(s) - \frac{1}{s} Y(s) = \left(\frac{K_0}{sH(s)} - \frac{1}{s}\right) Y(s)$$

and the limit theorem gives coefficient K_1 defined as

$$K_1 = \lim_{t\to\infty} \frac{U_1(t)}{y(t)} = \lim_{s\to 0}\left(\frac{K_0}{sH(s)} - \frac{1}{s}\right). \qquad (5)$$

Nothing that $K_0 = b_0$, after substituting (1) into (5):

$$K_1 = \lim_{s\to 0}\left(a_1 + \sum_{i=2}^{n} a_i s^{i-1}\right) = a_1 .$$

In general, in the m-th step following recursion expression can be written:

$$U_m(t) = \int_0^t (U_{m-1}(t) - K_{m-1}y(t))\,\mathrm{d}t =$$
$$= \int_0^t\left(-K_{m-1}y + \int_0^t\left(-K_{m-2}y + \ldots + \int_0^t (K_0x - y)\,\mathrm{d}t \ldots\right)\mathrm{d}t\right)\mathrm{d}t . \qquad (6)$$

Therefore, the Laplace transform is

$$U_m(s) = \frac{K_0}{s^m H(s)} - \frac{1 + \sum_{i=1}^{m-1} K_i s^i}{s^m}\, Y(s) .$$

As $K_i = a_i$ for $i = 0, 1, \ldots, m-1$, we have a general formula for a_m:

$$K_m = \lim_{t\to\infty} \frac{U_m(t)}{y(t)} = \lim_{s\to 0} \frac{R(s) - 1 - \sum_{i=1}^{m-1} a_i s^i}{s^m} = a_m . \qquad (7)$$

Thus, the algorithm for parameter estimation includes the sequential procedure, containing integration of the differences between output signal y(t) and output signal from preceding integrator, and computation of the ratio of the steady-state value $U_{m\infty}$ to y_∞.

This method may be extended for parameter estimation of dynamic elements with nonlinear static characteristic. In this case the input test $x(t)$ and output $y(t)$ are related differential by related

$$a_n y^{(n)} + \ldots + a_1 y' + y = b_0(x) \cdot x(t) .$$

The Laplace transform is:

$$Y(s)R(s) = \Psi(s)$$

where

$$\Psi(s) = L[b_0(x(t)) \cdot x(t)] ,$$

$$R(s) = \sum_{i=1}^{n} a_i s^i .$$

In knowledge of function $K_0 = b_0(x)$ it is possible to form $U_1(t)$ by analogy to (4):

$$U_1(s) = \left(\frac{1}{s} \frac{\Psi(s)}{Y(s)} - \frac{1}{s} \right) Y(s) . \tag{8}$$

Inserting $R(s) = \dfrac{\Psi(s)}{Y(s)}$ into (8) gives

$$K_1 = \lim_{s \to 0} \left(\frac{1}{s} R(s) - \frac{1}{s} \right) = a_1 .$$

Sequential integration of the differences as in (6) gives estimates of all the coefficients a_i of polynomial $R(s)$, describing, together with function $b_0(x)$, the behaviour of the nonlinear dynamic unit.

3. Analog-digital devices

The device for determining the coefficients of linear transfer function (1) is shown in Fig. 1.

It includes input and output signal converters C_1 and C_2, adaptive inertialess models $M_0, M_1, \ldots, M_n$ and integrators $I_1, I_2, \ldots, I_n$. The gates $G_1, G_2, \ldots, G_n$ in each step connect the corresponding adder-subtractor AS to control model block CMB.

It had been noted that input and output signals must satisfy condition (2). Practically, the process can be considered as steady state in time τ if the following requirement is satisfied:

$$|y^{(k)}(t)| \leqq \varepsilon , \qquad \text{where} \qquad t \geqq \tau ; \quad k = 1, \ldots, n ;$$

and ε is given in compliance with the requirements of accuracy.

Hence, expression (7) can be used in the form:

$$K_m = \frac{U_m(\tau)}{y(\tau)} = a_m .$$

In the "zeroth" step in time τ the CMB controls the steady-state gain K_0 of the

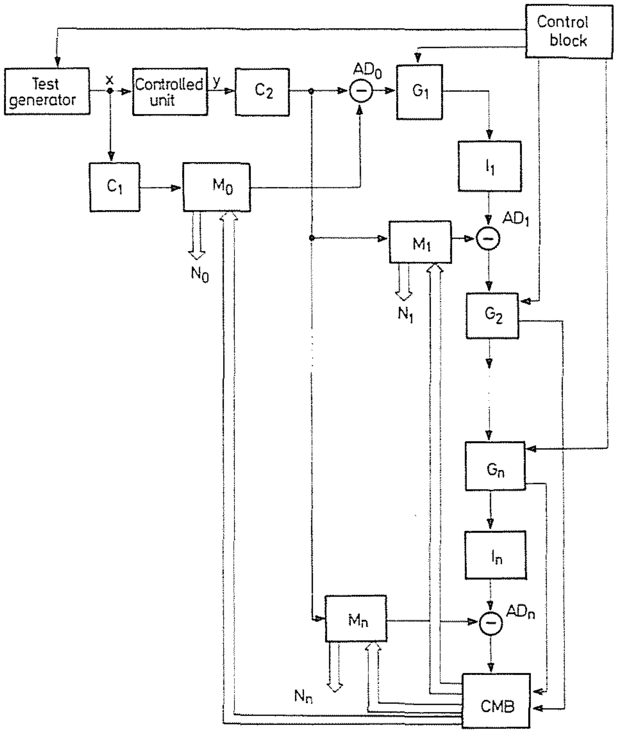

Fig. 1. Flowchart of the device for parameter estimation of linear units

inertialess model M_0 to equal b_0. In all subsequent steps K_0 will be fixed in model M_0.

In the first step, corresponding to (4), the difference from output of the adder-subtractor AS_0 is integrated and the CMB controls the gain K_1 in model M_1 so that in time τ it becomes

$$K_1 = \frac{U_1(\tau)}{y(\tau)} = a_1 .$$

The process of adaptation of the coefficients continues until all a_m will be defined.

As an adaptive inertialess model it is convenient to use the operational amplifier with controlled digital resistor (CDR) [5]. The flowchart of such a model is shown in Fig. 2. Control digital code from CMB determines the switching of the weighted resistors $r_1, \ldots, r_L$ in feedback of amplifier. Usually the weights of these resistors are chosen as binary: $r_k = 2^k r_0$.

In each step the CMB controls resistance of the corresponding CDR_m, where m-number of steps, so that in time τ the output signal of the AS_m becomes

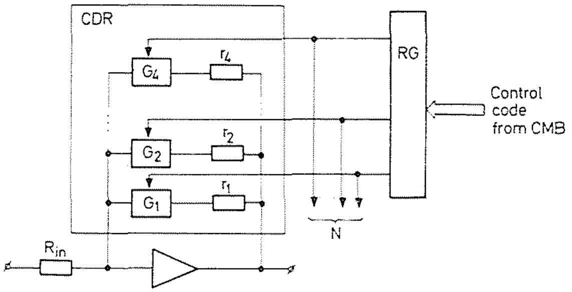

Fig. 2. The inertialess model with controlled gain

equally zero. Thus, the model coefficient is

$$K_m(\tau)=a_m \,.$$

Note that all the defined K_i, $i=1, \ldots, m$ get fixed for the next steps. During the work on the outputs of model registers the codes N_k occur related with parameters of the dynamic unit by expressions

$$b_0 = \frac{C_1 R_{\max}}{C_2 R_{\text{in}} R_{\max}} N_0\,; \tag{9}$$

$$a_k = \frac{R_{\max}}{R_{\text{in}} N_{\max}} N_k \prod_{j=1}^{k} T_j\,, \qquad k=1, \ldots, n$$

where C_1, C_2 — coefficients of converters C_1 and C_2;
$R_{\max}$ maximum resistance of CDR;
R_{in} input resistor of the operational amplifier;
N_k, $N_{\max}$ — output and maximum digital codes of model M_k;
T_j — time constant of integrator I_j.

The algorithm for parameter estimation of the nonlinear dynamic element may be realized by means of the analog-digital device shown in Fig. 3. Here only part of the device to determine nonlinear static characteristic is reproduced, the other part is similar to that in Fig. 1.

This device is completed by the analog-digital converter ADC, memory RAM and multiplexer. Unlike the preceding algorithm the "zeroth" step now consists of L cycles, in which the test signal generator provides $x(t)$ with distinct levels of the steady-state value $x_1(\tau)$, $1=1, 2, \ldots, L$.

Hence, in time τ the steady-state gain of model M_0 for this cycle is found to be

$$K_{01} = \frac{y_1(\tau)}{x_1(\tau)}$$

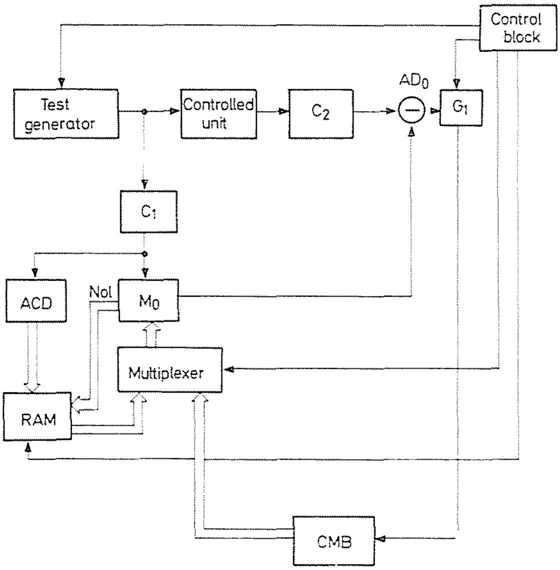

Fig. 3. The device for parameter estimation of nonlinear units

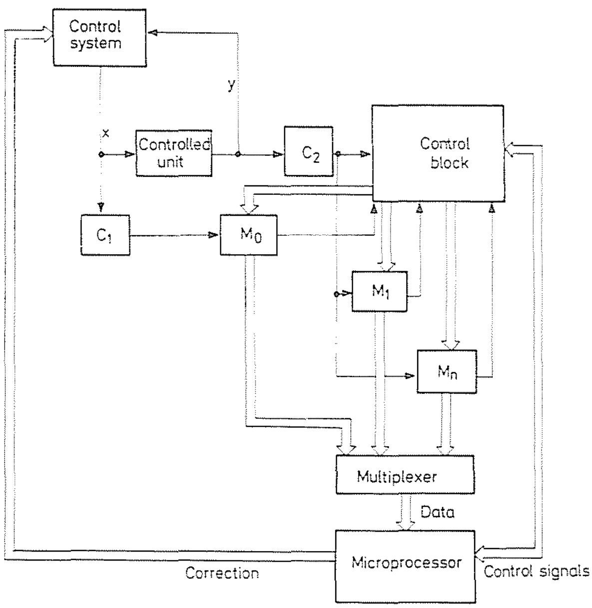

Fig. 4. Flochwart of microprocessor-based parameter estimation system

and corresponding digital code N_{01} is stored into cell of RAM, according to address depending on $x_1(\tau)$.

The ADC determines this address as digital code

$$A_1 = \rfloor \gamma x_1(\tau) \lfloor$$

where γ — scale coefficient of ADC.

Thus, after L cycles RAM stores L values of the nonlinear static characteristic $b_0(x)$. In the next steps RAM is in read mode and the multiplexer connects control input of the model M_0 with output of RAM. The other parameters a_m of the dynamic unit are determined as described in Section 2.

For many applications it is necessary to determine the parameters of controlled unit not only once but continually, to watch its variations in the control process. Presented devices could be used to this aim without any changes in the control system. It is possible, because input and output signals of unit are not sharply delimited (2).

For continuous parameter estimation during exploitation the control system completed by the microprocessor in Fig. 4 may be recommended. The control block analyses signals x and y for selecting the instant τ where these signals reach steady state.

At this time the microprocessor analyses by computing the controlled unit parameters, the stability conditions, accuracy of the control process and other performances, and then makes the correction of parameters and structure of the control system.

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