METROLOGY PROBLEMS IN TESTING DEFORMATIONS OF ELASTIC SOLIDS

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Summary

Deformations of elastic solids are normally tested by determining the stress-strain condition at the given point from specific strain values measured in three defined directions at given surface points of the solid.

This theoretically settled method comprises, nevertheless, a score of practical difficulties and sources of error.

A recent possibility to eliminate these sources of error is to apply laser optic instruments. Research has been done on the construction of equipment to this aim at the High School of Mechanics and Automation, Kecskemét, Department of Machine Production Engineering.

One of the most frequent measurement methods in testing deformations of elastic solids applies strain gauges. In this method, the specimen is assured to be homogeneous, isotropic, and to obey Hooke's law. Specific strain values ε_a, ε_b and ε_c are measured in three directions starting from surface point 0 of the specimen (Fig. 1). Let directions with unit vectors $\bar{n}_a$, $\bar{n}_b$ and $\bar{n}_c$ include a given angle φ. Let tangent plane with coordinates (xz) fit point 0, having unit vector $\bar{j}$ as normal. Elementary cube describing stress state at point 0 is seen in Fig. 1.

Let us determine strain and stress matrices $\bar{\bar{U}}_0$ and $\bar{\bar{\emptyset}}_0$, resp., belonging to 0.

$$\bar{\bar{U}}_0 = \begin{bmatrix} \varepsilon_x & 0 & \frac{1}{2}\gamma_{xz} \\ 0 & \varepsilon_y & 0 \\ \frac{1}{2}\gamma_{zx} & 0 & \varepsilon_z \end{bmatrix}, \quad \bar{\bar{\emptyset}}_0 = \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{zx} & 0 & \sigma_z \end{bmatrix}$$

$$\bar{n}_a = \bar{i} n_x + \bar{k} n_z = \bar{i} \sin \varphi + \bar{k} \cos \varphi,$$

$$\bar{n}_c = -\bar{i} n_x + \bar{k} n_z = -\bar{i} \sin \varphi + \bar{k} \cos \varphi,$$

$$\varepsilon_a = \bar{n}_a^* \bar{\bar{U}}_0 \bar{n}_a .$$

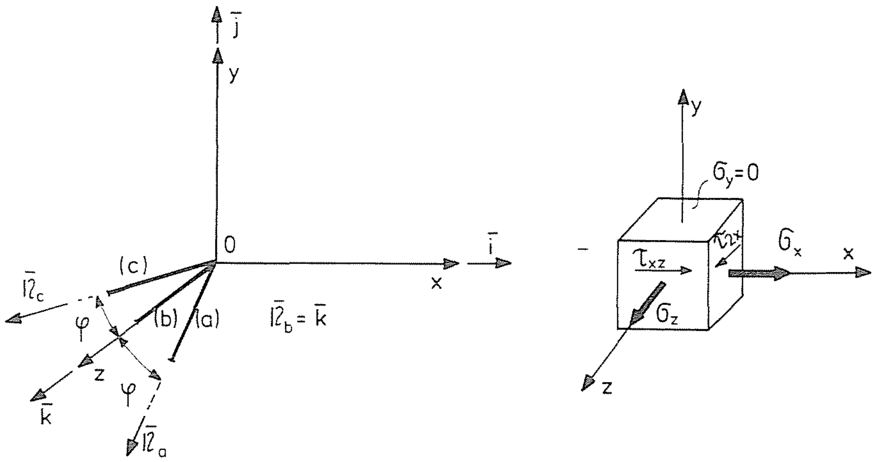

Fig. 1

$$\bar{\bar{U}}_0 \bar{n}_a = \begin{bmatrix} \varepsilon_x & 0 & \frac{1}{2}\gamma_{xz} \\ 0 & \varepsilon_y & 0 \\ \frac{1}{2}\gamma_{zx} & 0 & \varepsilon_z \end{bmatrix} \begin{bmatrix} n_x \\ 0 \\ n_z \end{bmatrix} = \begin{bmatrix} \varepsilon_x n_x + \frac{1}{2}\gamma_{xz} n_z \\ 0 \\ \frac{1}{2}\gamma_{zx} n_x + \varepsilon_z n_z \end{bmatrix},$$

$$\varepsilon_a = [n_x \quad 0 \quad n_z] \begin{bmatrix} \varepsilon_x n_x + \frac{1}{2}\gamma_{xz} n_z \\ 0 \\ \frac{1}{2}\gamma_{zx} n_x + \varepsilon_z n_z \end{bmatrix}.$$

$$\varepsilon_a = \varepsilon_x n_x^2 + \frac{1}{2}\gamma_{xz} n_x n_z + \frac{1}{2}\gamma_{zx} n_x n_z + \varepsilon_z n_z^2.$$

In view of $\gamma_{xz} = \gamma_{zx}$, and $\varepsilon_z = \varepsilon_b$,

$$\varepsilon_a = \varepsilon_x n_x^2 + \varepsilon_b n_z^2 + \gamma_{xz} n_x n_z,$$

$$\varepsilon_c = \varepsilon_x n_x^2 + \varepsilon_b n_z^2 - \gamma_{xz} n_x n_z,$$

$$\varepsilon_a + \varepsilon_c = 2\varepsilon_x n_x^2 + 2\varepsilon_b n_z^2,$$

Relationships above yield ε_x and γ_{xz}:

$$\varepsilon_x = \frac{\varepsilon_a + \varepsilon_c - 2\varepsilon_b n_z^2}{2n_x^2}, \quad \gamma_{xz} = \frac{\varepsilon_a - \varepsilon_x n_x^2 - \varepsilon_b n_z^2}{n_x n_z}$$

Element ε_y of strain matrix is obtained from the relationship between $\bar{\bar{\Phi}}_0$ and $\bar{U}_0$:

$$\bar{\bar{\Phi}}_0 = 2G\left(\bar{U}_0 + \frac{U_I}{m-2}\bar{\bar{E}}\right)$$

where G is the modulus of elasticity in torsion, m the Poisson's ratio of the tested elastic solid, U_I is the first scalar invariant of the strain matrix, $\bar{\bar{E}}$ is the unit matrix.

$$\bar{\bar{\Phi}}_0 = \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{zx} & 0 & \sigma_z \end{bmatrix} = 2G\left\{\begin{bmatrix} \varepsilon_x & 0 & \frac{1}{2}\gamma_{xz} \\ 0 & \varepsilon_y & 0 \\ \frac{1}{2}\gamma_{zx} & 0 & \varepsilon_z \end{bmatrix} + \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{m-2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$\sigma_y = 0 = 2G\left(\varepsilon_y + \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{m-2}\right),$$

$$\varepsilon_y = -\frac{\varepsilon_x + \varepsilon_z}{m-1}.$$

Thus, all elements of $\bar{U}_0$ and $\bar{\bar{\Phi}}_0$ can be determined. Sources of error in the presented method are:

1. The material is not always homogeneous and isotropic, so G and m values may change.
2. Strain gauge tags are not "point-like".
3. Tag sticking defects.
4. Heat compensation errors.
5. Electric contact defects.
6. Difficulties of accommodating wiring in dynamic measurements.

All these imposed to develop some new method likely to get rid of the listed sources of error.

Laser optic measurement methods of deformometry

A research team at the Department of Machine Production Engineering of the High School of Mechanics and Automation, Kecskemét, has been concerned with laser optic measurement methods under the guidance of Prof. Dr. Benedek Molnár, Head of Department. This work, shared by the Author in 1982, will be reported on in the following.

The research work was expected to find a method for determining the deformation (displacement, rotation) of a surface point of an elastic solid, irrespective of material characteristics, by means of a measuring instrument of relatively simple handling, reliable under service conditions, and requiring as little computation as possible.

These requirements argued for laser optic methods where, beside of being independent of material characteristics, no errors due to contact defects of electric wiring, and for dynamic measurements, no difficulties of accommodating wires occur. Scheme of operation of an experimental measuring instrument is seen in Fig. 2.

The measuring instrument lends itself to sensing $1/2\gamma$ turn of a surface point 0 of the elastic solid. For a $1/2\gamma$ turn of the measuring mirror, the laser beam incident on the mirror fastened at 0 is reflected at an angle γ. The reflected beam is directed to an auxiliary mirror and hence to a screen. Known values are distances l_1, l_2, angle φ of the auxiliary mirror, and the measured distance A of the point of incidence of the laser beam on the screen (Fig. 2).

Unknowns are angle γ, angle α included by the reflected laser beam and the auxiliary mirror plane, as well as values m and y describing the position of the point of incidence of the laser beam on the auxiliary mirror (Fig. 2).

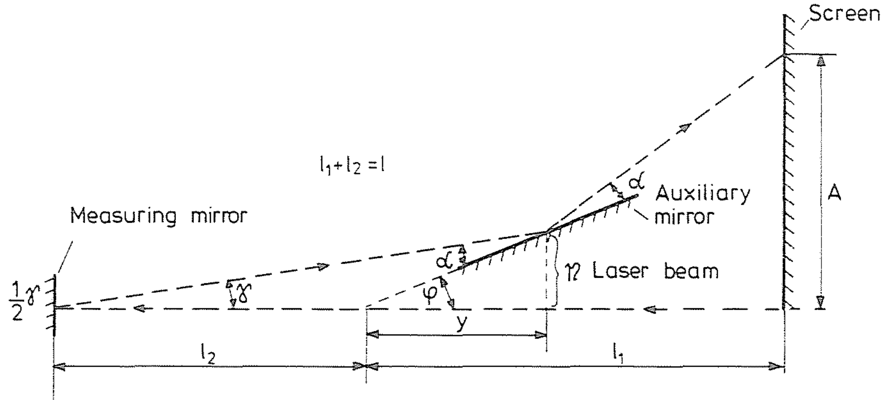

Fig. 2

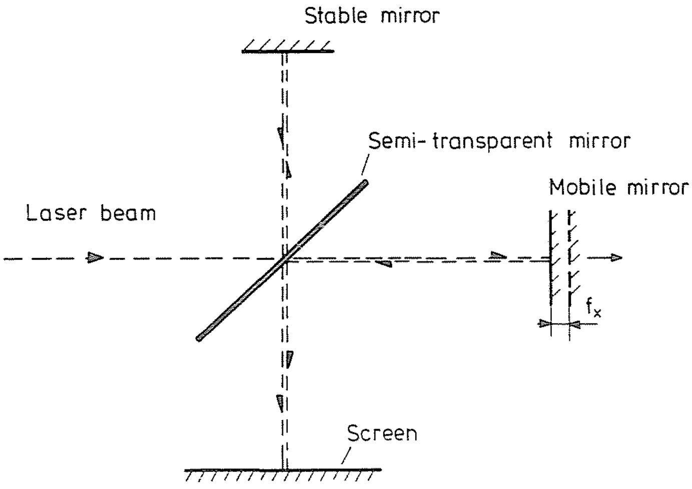

Fig. 3

According to Fig. 2:

1. $\operatorname{tg}\varphi=\frac{m}{y}$, 2. $\operatorname{tg}(\varphi+\alpha)=\frac{A-m}{l_1-y}$,

3. $\operatorname{tg}(\varphi-\alpha)=\frac{m}{l_2+y}$, 4. $\gamma=\varphi-\alpha$.

Solution of the above equation system with four unknowns yields a term for $\operatorname{tg}\alpha$. To ease survey, let us introduce notations:

$$\operatorname{tg}\varphi=a;\ \operatorname{tg}^2\varphi=b;\ a^2=b;\ l_1+l_2=l.$$

With these notations, the expression for $\operatorname{tg}\alpha$ becomes:

$$\operatorname{tg}\alpha=\frac{A(1+b)+l_2(a+ab)-l_1(a+ab)}{A(a+ab)+a^2l_2+l_1b+l}.$$

In the knowledge of α, the wanted $1/2\gamma$ turn is simply:

$$\frac{1}{2}\gamma=\frac{1}{2}(\varphi-\alpha).$$

This method is advantageous by permitting to produce arbitrary A values for rather slight angles γ, by modifying auxiliary mirror angle φ, permitting an arbitrary accuracy for measuring A.

Functioning principle of a measuring instrument for determining displacement f_x of laser beam direction is seen in Fig. 3.

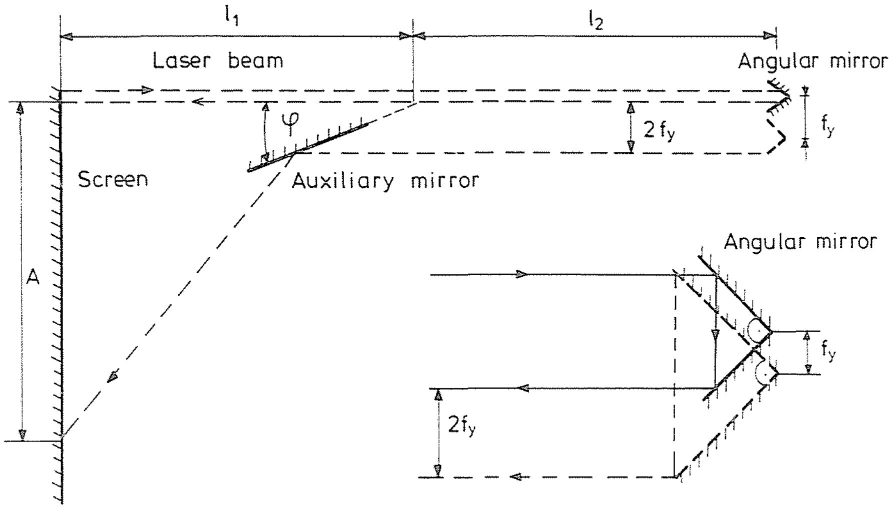

Fig. 4

Incident on a semi-transparent plane mirror, the laser beam partly proceeds without changing its direction, and partly it is reflected in vertical direction. Beam proceeding without direction change gets onto a mobile mirror fastened on the elastic solid. Coherent laser beams coming from the stable and mobile mirrors are incident on the screen with a path difference, as a function of displacement f_x of the mobile mirror, producing interference strips to deduce the f_x value from.

Functioning principle of an instrument for measuring displacement f_y normal to the laser beam direction is seen in Fig. 4. The laser beam is incident on an angular mirror of 90° at a preferential point of the tested elastic solid. In the knowledge of value A measured on the screen, of angle φ of the auxiliary mirror, and of distances l_1 and l_2, the wanted f_y can be determined.

Suitably adjusting the auxiliary mirror angle φ, distance A can be arbitrarily increased, rather proficient for the measurement accuracy.

The described laser optic tests have been made by means of laboratory instruments, no integrated measuring equipment has been assembled and constructed up to now. This is the aim of the next stage of research, justified by promising results of the laboratory experiments.

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