# THE USE OF SERRAPHIL FUNCTIONS IN THE PERFORMANCE CALCULATION OF ASYNCHRONOUS MACHINES

GY. TŐKE and G. NÁDASY*

Department of Electric Machines,
Technical University, H-1521 Budapest

## Summary

Parasitic torques in asynchronous machines are caused by the variation of the air gap permeance due to the slots and by the m.m.f. harmonics produced by the distributed windings. In practice these torques are calculated by means of finite Fourier series. By the help of serraphil and serramorph functions the influence of the slots and the distributed windings on the air gap field and the dependence of the elements from the rotor position in the torque matrix can be described in a closed form. In this way the resolution of the voltage and mechanical equations by computer become relatively simple. It is possible to take the saturation into consideration, too.

## Introduction

It is necessary to know the variation in space and time of the air-gap field in order to calculate both the steady state and the transient electrical and mechanical characteristics of asynchronous machines. The influence on the air-gap field of the space distribution of the stator and rotor windings, the pulsation of the air-gap permeance owing to the slots can be described approximately by means of Fourier series [1, 2, 3, 4]. No higher accuracy can be achieved taking more terms of the series into account, as, because of the Gibbs phenomenon, it is impossible to describe these step functions precisely by means of Fourier series. Taking more terms of the series into account, beyond a certain limit the computation time becomes very long and the physical picture of the processes is lost. This is why the investigation is often restricted to the fundamental harmonic. However, it is fairly difficult to take into account the influence of the harmonics additionally in such a physical-mathematical model.

The air-gap field of the asynchronous machine can be described in a closed form with the help of serraphil functions and it is possible to take into consideration the variation in space and time of the permeance, the

* United Electric Machine Works (EVIG)

nonlinearity of the magnetic circuit, the distribution of the windings. In the following the main types of serraphil functions and their application in connection with electrical machines will be shown.

## The serraphil function and its combinations

Functions with saw-tooth like diagrams are called serraphil-type functions. According to our knowledge first it was K. A. Turban [5] who first pointed out the possible use of this function in electricity. In connection with electrical machines it was used by Z. Pagano and A. Perfetto [6] as well as H. Rausch and W. Freise [7].

In Fig. 1.a a set of serraphil functions can be seen. The analytical expression is:

$$\mathrm{Ser}\,(\alpha)=\frac{2}{\pi}\,\mathrm{Arctg}\,\frac{A\cdot\sin\alpha}{1+A\cos\alpha}$$

In this equation Arctg refers to the main value of the arctg function. The serraphil function has an $\alpha=2\pi$ periodicity and the amplitude and the shape of the function depend on the choice of the coefficient "$A$". If $A=1$ the expression describes a saw tooth (serramorph) function. In the range of $A\in[0, 1]$ the serraphil function can be differentiated and integrated.

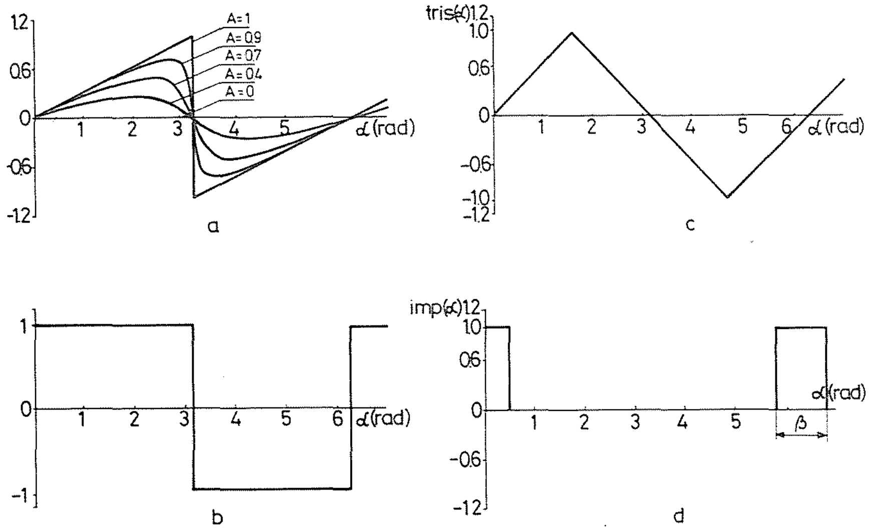



*Fig. 1*

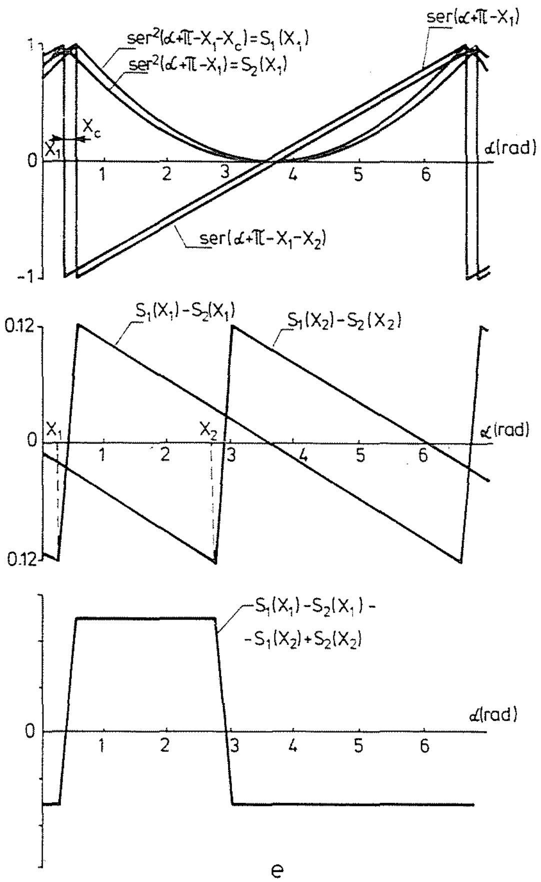



*Fig. 1*

By combining these functions other step functions occurring in electricity can also be described analytically, in closed form. Their names come from the abbreviation in English, expressing their shapes.

The symmetrical square function (Fig. 1.b) is the difference of two displaced serramorph functions:

$$\text{res}(\alpha)=\text{ser}(\alpha)-\text{ser}(\alpha-\pi) \quad (\textit{re}\text{ctangular } \textit{s}\text{inus})$$

The symmetrical triangular function (Fig. 1.c) is the difference of the squares of two displaced serramorph functions:

$$\text{tris}(\alpha)=\text{ser}^2\left(\alpha+\frac{\pi}{2}\right)-\text{ser}^2\left(\alpha-\frac{\pi}{2}\right) \quad (\textit{tri}\text{angular } \textit{s}\text{inus})$$

The impulse function of positive unit amplitude, width $\beta$ and periodicity $2\pi$ (Fig. 1.d) is:

$$\operatorname{imp}(\alpha)=\frac{1}{2}\left[\operatorname{ser}\left(\alpha-\frac{\beta}{2}+\pi\right)-\operatorname{ser}\left(\alpha+\frac{\beta}{2}+\pi\right)+2\operatorname{ser}(\beta)\right]$$

The latter can be used advantageously to describe the supply voltage (e.g. of an inverter) that consists of step functions. In the next chapters asymmetrical trapezoidal functions will be used. The different steps of writing this function are shown in Fig. 1.e.

## The M.M.F. distribution and the airgap permeance

The m.m.f. of the distributed winding can be obtained by the superposition of the instantaneous m.m.f. values of the coils.

In Fig. 2.a the cross section of a coil of $N_t$ turn and $i$ current is shown. The slots are at an $x_1$ and/or $x_2$ angle from $\alpha=0$ of the coordinate system and the slot opening is $x_c$ in radians. If the slot current is assumed to be distributed uniformly along $x_c$ the distribution of the m.m.f. produced by the coil can be seen in Fig. 2.b. It consists of trapezoidal sections and its average value is zero. The m.m.f. distribution can be described analytically in a closed form by the help of serraphil functions:

$$\vartheta_1(\alpha,t)=\frac{\pi}{4}\frac{N_t i(t)}{x_c}\left[\operatorname{ser}^2(\alpha-x_c-x_1+\pi)-\operatorname{ser}^2(\alpha-x_c-x_2+\pi)+\right.$$
$$\left.+\operatorname{ser}^2(\alpha-x_c+\pi)-\operatorname{ser}^2(\alpha-x_1+\pi)\right]$$

If he slot current is concentrated in the centre of the slot opening, the m.m.f. distribution of the coil will be as shown in Fig. 2.c. Its analytical expression is:

$$\vartheta_1(\alpha,t)=\frac{N_t i(t)}{2}\left[\operatorname{ser}(\alpha+\pi-x_2)-\operatorname{ser}(\alpha+\pi-x_1)\right]$$

The m.m.f. of the total winding can be obtained by the superposition of the m.m.f. values of the coils. An example of this can be seen in Fig. 3.a which shows the developed diagram of one phase winding of a three phase, short pitched winding. The data of the winding are: $Z_1=36, p=2, s/\tau_p=7/9, q=3$, the slot pitch in electrical degrees: $\alpha_v=180°/9$, number of turns of one coil: $N_t$, the instantaneous value of the phase current: $i$. The analytical form of the m.m.f. distribution:

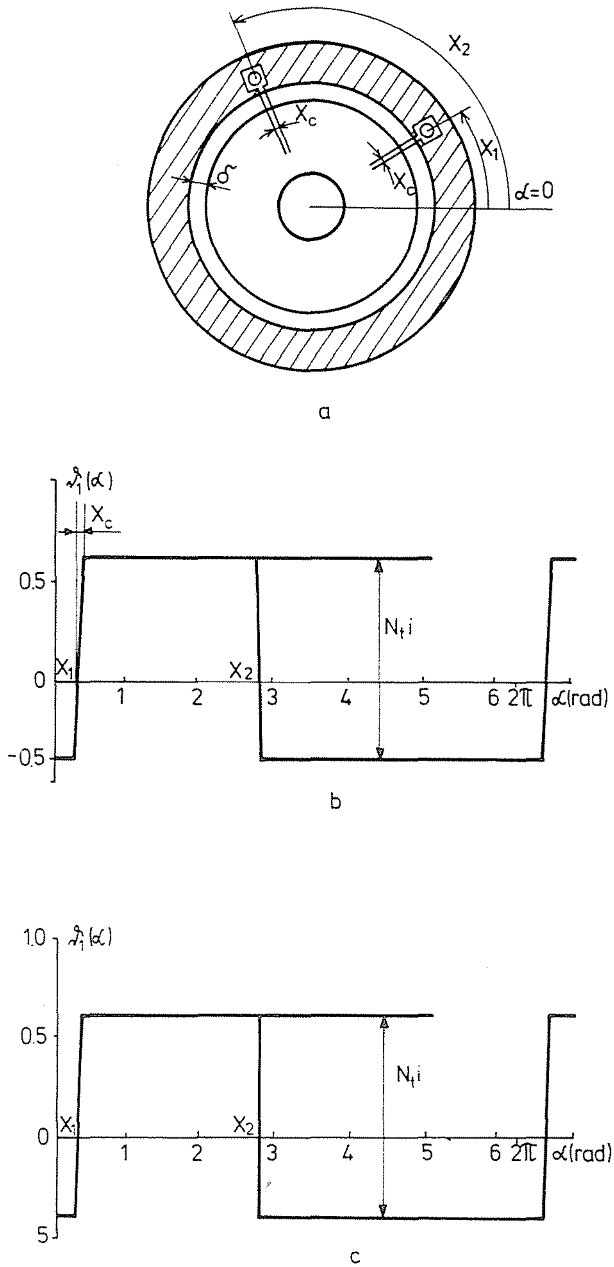

X2
Xc
X1
δ
Xc
α=0
a
ϑ1(α)
Xc
0.5
0
-0.5
X1
X2
Nt i
1
2
3
4
5
6
2π
α(rad)
b
1.0
ϑ1(α)
0.5
0
5
X1
X2
Nt i
1
2
3
4
5
6
2π
α(rad)
c

*Fig. 2*

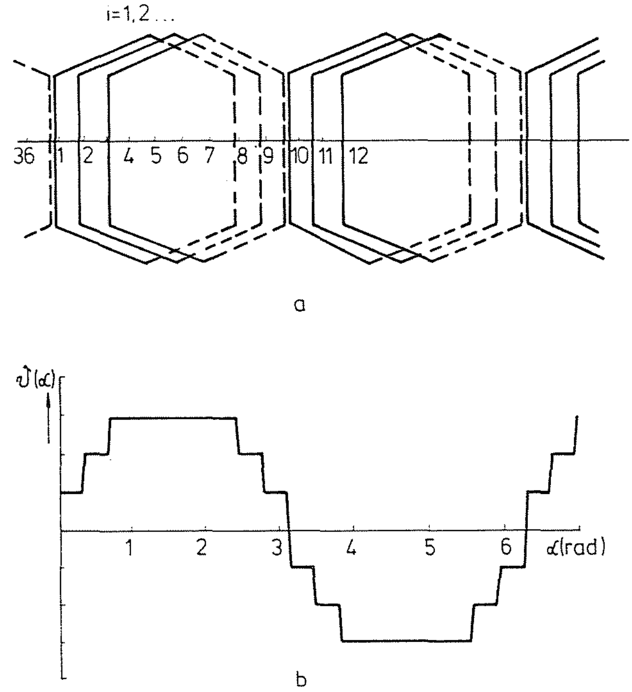
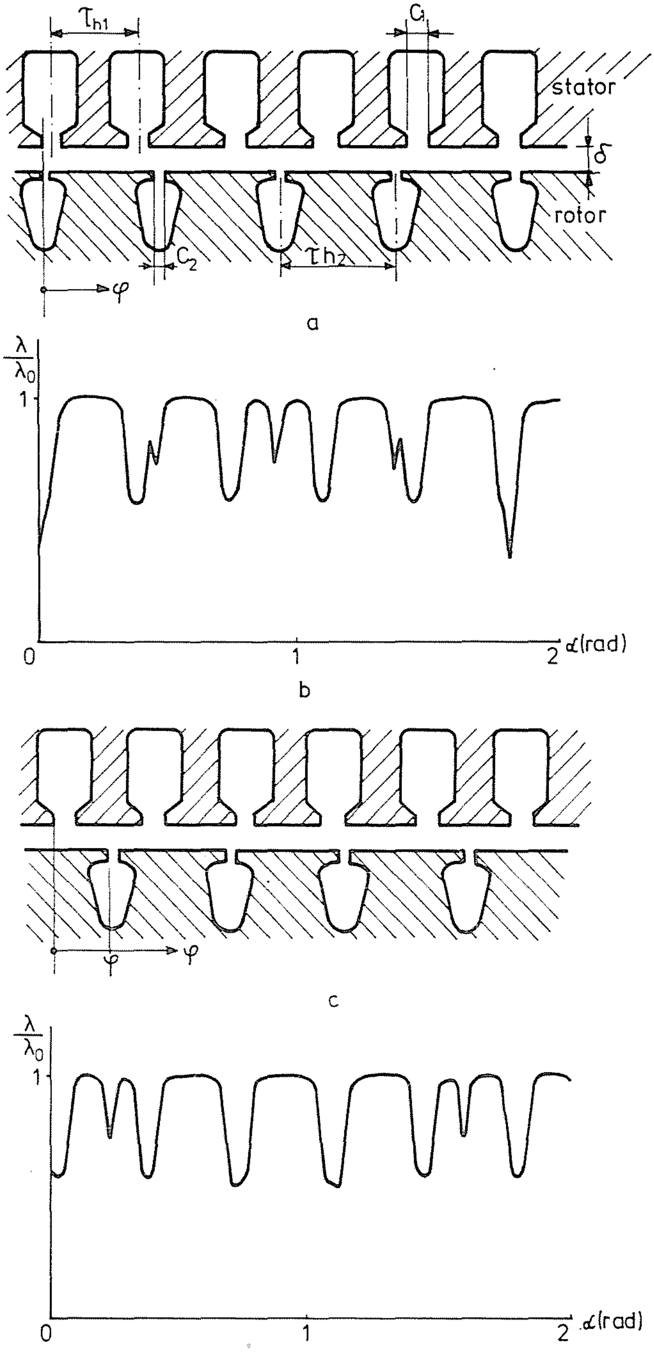



*Fig. 3*

$$\vartheta(\alpha, t) = N_t i(t) \left\{ \text{res}(\alpha) + \frac{1}{2} [\text{res}(\alpha - \alpha_v) - \text{res}(\alpha - 8\alpha_v) + \text{res}(\alpha - 2\alpha_v) - \text{res}(\alpha - 7\alpha_v)] \right\}$$

On the basis of this expression the m.m.f. distribution of the winding can be easily calculated or plotted by computer (Fig. 3.b). The permeance of the air gap section of the magnetic circuit depends on the length of the air gap, the slot pitch and the slot opening. Both the stator and the rotor of the asynchronous machine are slotted so permeance depends on the position of the rotor angle position as well.

In Fig. 4.a a section of the slotted stator and rotor of the previous motor is shown. The dimensions are: slot numbers of the stator $Z_1 = 36$, of the rotor $Z_2 = 28$, $\delta = 0{,}55$ mm, $\tau_{h1} = 13{,}28$ mm, $c_1 = 3$ mm, $\tau_{h2} = 16{,}95$ mm, $c_2 = 1$ mm. The relative position of the rotor to the stator is marked by the coordinate $\varphi$ and its value in the configuration in Fig. 4.b is zero. The mean value of the permeance can be calculated from Carter's coefficients of the stator and the rotor. A



τh1
C1
stator
δ
rotor
C2
τh2
φ
a
λ/λ0
1
0
1
2
α(rad)
b
φ
φ
c
λ/λ0
1
0
1
2
α(rad)
d


*Fig. 4*

number of authors have dealt with the description of the angle dependence of the permeance [1, 2, 3, 4, 7]. Among these, theoretical investigations based on conformal transformation and experimental work with Hall-elements and conducting paper can be found. On this basis the permeance can be approximately described by means of serraphil functions as follows:

$$\lambda(\alpha,\varphi)=\Lambda_0\left\{1+\frac{1}{4}\left[\operatorname{ser}\pi\left(1-\frac{c_1}{\tau_{h1}}\right)-\operatorname{ser}\pi\left(1+\frac{c_1}{\tau_{h1}}\right)+\operatorname{ser}\pi\left(1-\frac{c_2}{\tau_{h2}}\right)-\right.\right.$$
$$-\operatorname{ser}\pi\left(1+\frac{c_2}{\tau_{h2}}\right)-\operatorname{ser}\left(Z_1\alpha-\frac{c_1}{\tau_{h1}}2\pi\right)+\operatorname{ser}(Z_1\alpha)-$$
$$\left.\left.-\operatorname{ser}\left((Z_2(\alpha-\varphi)-\left(\frac{c_2}{\tau_{h2}}\frac{2\pi}{Z_2}\right)\right)+\operatorname{ser}((Z_2(\alpha-\varphi))\right]\right\}$$

In the equation $\Lambda_0=\mu_0/\delta$ is the permeance of the smooth air gap between the non slotted stator and rotor. The first five terms of the expression describe the mean value of the permeance, the further four describe the dependence of the permeance from $\alpha$ and $\varphi$ which are measured in the same coordinate system. On the basis of the given stator and rotor slot numbers as well as the dimensions of the air gap and the slots, the air gap permeance was calculated from the previous expression. The figure drawn by the computer in case of $\varphi=0$ is shown in Fig. 4.b.

Fig. 4.c shows that the rotor is turned by $\tau_{h2}/2$ in the positive direction of $\varphi_1$ compared to its basic position. The permeance distribution belonging to this position is shown in Fig. 4.d. Previously, as examples, the m.m.f. of one phase winding of a three phase machine and the air gap permeance of a both side slotted machine were described by the help of serraphil functions. This

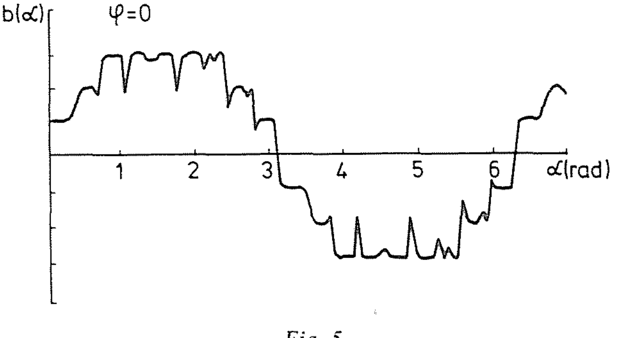



*Fig. 5*

m.m.f. produces an air gap induction as follows:

$$b(\alpha, \varphi, t) = \vartheta(\alpha, t) \cdot \lambda(\alpha, \varphi)$$

The distribution of the air gap induction can be seen in Fig. 5 in case of $\varphi = 0$ and $N_t i = 1$.

## The voltage and torque equation

The air-gap field in the asynchronous machine is produced by the sum of the stator and rotor m.m.f. The air-gap induction of the machine with a total of $t$ number of coils in the stator and the rotor can be calculated as follows:

$$b(\alpha, \varphi, t) = \sum_{j=1}^{t} \vartheta_j(\alpha, t) \cdot \lambda(\alpha, \varphi)$$

If there is no skew, the flux of coil $K$ is:

$$\Phi_k(\alpha, \varphi, t) = \frac{Dl_i}{2} \int_{x_{k1}}^{x_{k2}} b(\alpha, \varphi, t) d\alpha$$

where $D$ is the air-gap diameter, $l_i$ is the ideal length of the armature, $x_{k2} - x_{k1}$ the distance of the coil sides in radians. From the known flux $\Phi_k$ the flux linkage $\psi_{mk}$ of coil $k$ can be calculated. Calculating the total flux linkage $\psi_k$ also the leakage fluxes must be taken into account (slot and end winding leakage). The terminal voltage of the $k$ coil is $U_k$ and the voltage equation:

$$U_k = R_k i_k + \frac{d\Psi_k}{dt}$$

In case of a cage rotor the voltage equation of the $n$-th loop, using the notation of Fig. 6:

$$0 = i_n \mathbf{R}_{nh} - (i_{n-1} + i_{n+1}) \mathbf{R}_r + \frac{d\Psi_n}{dt}$$

In these equations $\mathbf{R}_k$ is the resistance of the $k$-th coil, $\mathbf{R}_{nh}$ is the loop resistance and $\mathbf{R}_r$ is the resistance of one bar. The voltage equations in matrix form are:

$$\begin{bmatrix} U_s \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_R \end{bmatrix} \begin{bmatrix} i_s \\ i_R \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_s \\ \Psi_R \end{bmatrix}.$$

In this equation $U_s$ is the vector composed from the phase voltages, $i_s$ from the phase currents of the stator and $i_R$ is the vector of the rotor loop currents. $\mathbf{R}_s$

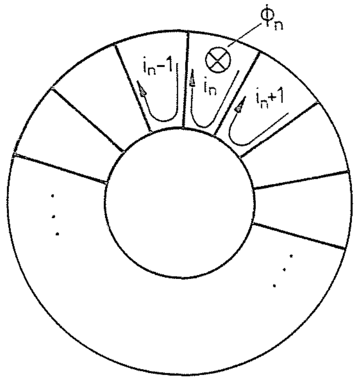



*Fig. 6*

and $\mathbb{R}_R$ is the resistance matrix of the stator and/or the rotor. $\Psi_s$ and $\Psi_R$ are vectors composed from the flux linkages of the stator and/or the rotor windings. Supposing a magnetic linearity the equation of the flux linkages is:

$$\begin{bmatrix} \Psi_s \\ \Psi_R \end{bmatrix} = \mathbb{L} \begin{bmatrix} i_s \\ i_R \end{bmatrix}$$

where $\mathbb{L}$ is the inductance matrix which contains all the self and mutual inductances of the winding system. Both the self and the mutual inductances are complicated functions of the rotor position because of the distributed windings and the slotted stator and rotor. The inductances can be described analytically in closed form by means of serraphil functions. For example the self inductance of the 2nd elementary coil ($i=2$) in Fig. 3.a as a function of the rotor position is:

$$L_{s2}(\varphi) = \frac{Dl\,\mu_0 N_t^2}{2\delta}\left[\left(1+\frac{1}{4}\left(\operatorname{ser}\left(\pi-\frac{c_1\pi}{\tau_{h1}}\right)-\operatorname{ser}\left(\pi+\frac{c_1\pi}{\tau_{h1}}\right)+\operatorname{ser}\left(\pi-\frac{c_2\pi}{\tau_{h2}}\right)\right.\right.\right.$$

$$\left.\left.-\operatorname{ser}\left(\pi+\frac{c_2\pi}{\tau_{h2}}\right)\right)\right)\frac{4}{\pi}\cdot(2\operatorname{ser}(\pi)^2-\operatorname{ser}(X_2+\pi-X_1)^2-\operatorname{ser}(X_1+\pi-X_2)^2+$$

$$+\frac{\pi}{8}(\operatorname{ser}(Z_1X_2)^2/Z_1+\operatorname{ser}(Z_2X_2-Z_2\varphi)^2/Z_2-\operatorname{ser}\left(Z_1X_2-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-$$

$$-\operatorname{ser}\left(Z_2X_2-Z_2\varphi-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\Big)\frac{1}{2}(\operatorname{ser}(\pi)-\operatorname{ser}(X_2+\pi-X_1))-$$

$$-\frac{\pi}{8}\left(\operatorname{ser}(Z_1X_1)^2/Z_1+\operatorname{ser}(Z_2X_1-Z_2\varphi)^2/Z_2-\operatorname{ser}\left(Z_1X_1-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-\right.$$

$$\left.-\operatorname{ser}\left(Z_2X_1-Z_2\varphi-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\right)\frac{1}{2}(\operatorname{ser}(X_1+\pi-X_2)-\operatorname{ser}(\pi))\Bigg].$$

The variation of the $L_{s2}$ inductance is shown in Fig. 7. The self-inductance of the loop formed by two neighbouring bars can be calculated similarly. For example, Fig. 8 shows how the $L_{R1}$ self inductance of the rotor coil formed by the bars 1 and 2 in Fig. 4.a depends on the rotor position.

The mutual inductance between the coils can also be described by means of serraphil functions. For example the mutual inductance $L_{12}$ between the two former coils is:

$$L_{12}(\varphi)=\frac{Dl\,\mu_0 N_t}{2\delta}\Bigg[\Bigg(1+\frac{1}{4}\left(\operatorname{ser}\left(\pi-\frac{c_1\pi}{\tau_{h1}}\right)-\operatorname{ser}\left(\pi+\frac{c_1\pi}{\tau_{h1}}\right)+\operatorname{ser}\left(\pi-\frac{c_2\pi}{\tau_{h2}}\right)\right.$$

$$\left.\left.-\operatorname{ser}\left(\pi+\frac{c_2\pi}{\tau_{h2}}\right)\right)\right)\cdot\frac{4}{\pi}\left(\operatorname{ser}\left(\varphi+\frac{8\pi}{7}-X_2\right)^2-\operatorname{ser}\left(\varphi+\frac{8\pi}{7}-X_1\right)^2-\right.$$

$$\left.-\operatorname{ser}(\varphi+\pi-X_2)^2+\operatorname{ser}(\varphi+\pi-X_1)^2\right)+\frac{1}{2}\left(\operatorname{ser}\left(\varphi+\frac{8\pi}{7}-X_2\right)-\right.$$

$$\left.-\operatorname{ser}\left(\varphi+\frac{8\pi}{7}-X_1\right)\right)\cdot\frac{\pi}{8}\left(\operatorname{ser}\left(Z_1\left(\varphi+\frac{\pi}{7}\right)\right)^2\Big/Z_1+\operatorname{ser}\left(Z_2\frac{\pi}{7}\right)^2\Big/Z_2-\right.$$

$$\left.-\operatorname{ser}\left(Z_1\left(\varphi+\frac{\pi}{7}\right)-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-\operatorname{ser}\left(Z_2\left(\frac{\pi}{7}\right)-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\right)-$$

$$-\frac{1}{2}(\operatorname{ser}(\varphi+\pi-X_2)-\operatorname{ser}(\varphi+\pi-X_1))\cdot\frac{\pi}{8}(\operatorname{ser}(Z_1\varphi)^2/Z_1-$$

$$\left.-\operatorname{ser}\left(Z_1\varphi-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-\operatorname{ser}\left(-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\right)\Bigg]$$

The variation of $L_{12}$ as a function of the rotor position is shown in Fig. 9. All elements of the inductance matrix can be found by the introduced calculation method. Substituting $\varphi=\varphi_0+\omega t$ ($\omega$=constant) the voltage equation can be solved numerically [9] and so the current of each coil can be calculated.

The current of a stator phase winding and the current of a rotor bar as a function of time are shown in Fig. 10.a and Fig. 10.b for the asynchronous machine in the previous example. The supply voltage (3 × 380 V, 50 Hz) and the load ($s=0{,}045$) are nominal values.

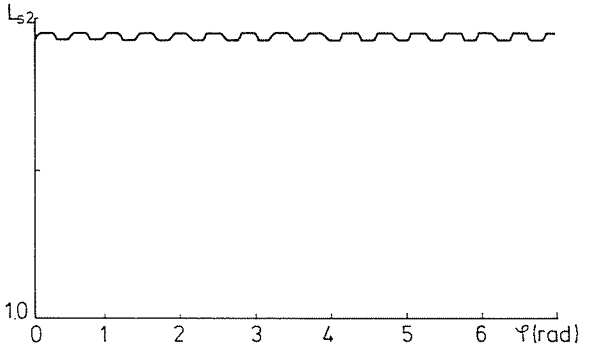



*Fig. 7*

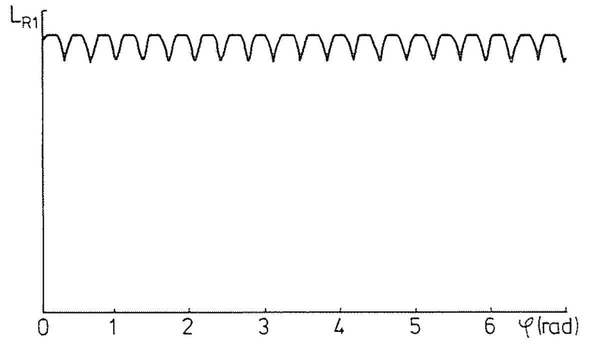



*Fig. 8*

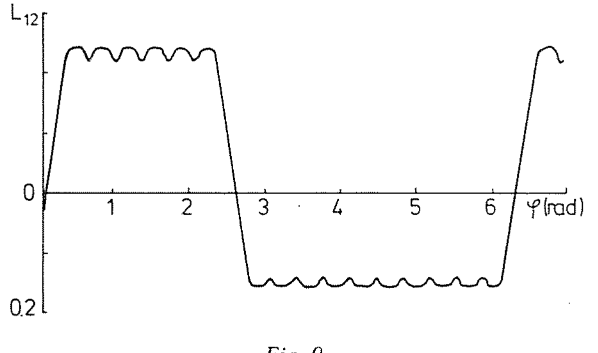



*Fig. 9*

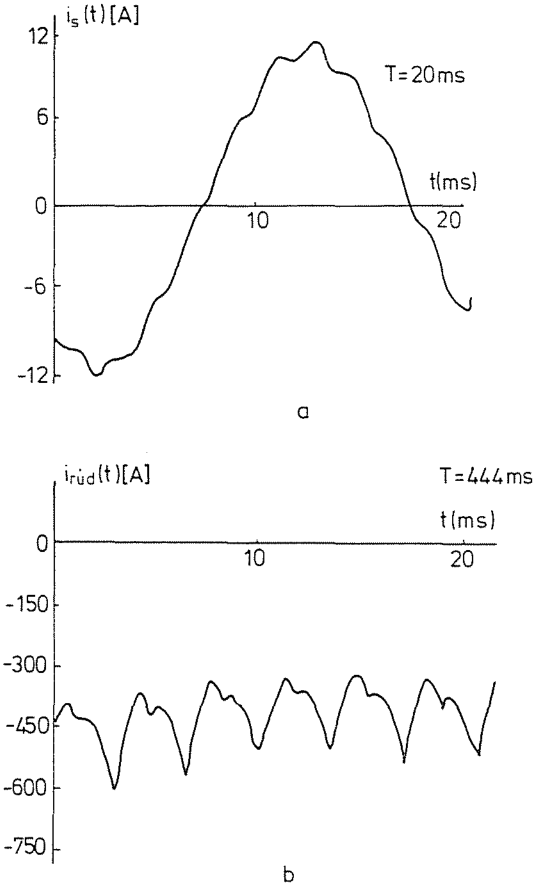



*Fig. 10*

The electromagnetic torque at the given load can be generally calculated from the known currents and the known elements of the inductance matrix [8]:

$$m_v = \frac{1}{2} i^t \mathbf{T} i$$

where $i^t$ is the transpose of the current vector and $\mathbf{T}$ is the torque matrix:

$$\mathbf{T} = \frac{d}{d\varphi}(\mathbf{L}).$$

In order to calculate the torque, the derivatives with respect to the rotor position of the elements of the inductance matrix should be produced first. The derivatives can be given by means of serraphil functions, too. For the stator winding of the previous example:

$$\frac{dL_{12}(\varphi)}{d\varphi}=\frac{Dl\,\mu_0 N_t}{2\delta}\Bigg[\Bigg(1+\frac{1}{4}\Bigg(\mathrm{ser}\left(\pi-\frac{c_1\pi}{\tau_{h1}}\right)-\mathrm{ser}\left(\pi+\frac{c_1\pi}{\tau_{h1}}\right)+\mathrm{ser}\left(\pi-\frac{c_2\pi}{\tau_{h2}}\right)-$$

$$-\mathrm{ser}\left(\pi+\frac{c_2\pi}{\tau_{h2}}\right)\Bigg)\Bigg)\cdot\frac{\pi}{4}\left(\mathrm{ser}\left(\varphi+\frac{8\pi}{7}-X_2\right)^2-\mathrm{ser}\left(\varphi+\frac{8\pi}{7}-X_1\right)^2-\right.$$

$$\left.-\mathrm{ser}\,(\varphi+\pi-X_2)^2+\mathrm{ser}\,(\varphi+\pi-X_1)^2\right)+\frac{1}{2}\left(\mathrm{ser}\left(\varphi+\frac{8\pi}{7}-X_2\right)-\right.$$

$$\left.-\mathrm{ser}\left(\varphi+\frac{8\pi}{7}-X_1\right)\right)\cdot\frac{\pi}{8}\left(\mathrm{ser}\left(Z_1\left(\varphi+\frac{\pi}{7}\right)\right)^2\Big/Z_1+\mathrm{ser}\left(Z_2\frac{\pi}{7}\right)^2\Big/Z_2-\right.$$

$$-\mathrm{ser}\left(Z_1\left(\varphi+\frac{\pi}{7}\right)-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-\mathrm{ser}\left(Z_2\left(\frac{\pi}{7}\right)-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\Bigg)-$$

$$-\frac{1}{2}\Big(\mathrm{ser}\,(\varphi+\pi-X_2)-\mathrm{ser}\,(\varphi+\pi-X_1)\Big)\cdot\frac{\pi}{8}\Big(\mathrm{ser}\,(Z_1\varphi)^2/Z_1-$$

$$\left.-\mathrm{ser}\left(Z_1\varphi-\frac{2\pi c_1}{\tau_{h1}}\right)^2\Big/Z_1-\mathrm{ser}\left(-\frac{2\pi c_2}{\tau_{h2}}\right)^2\Big/Z_2\right)\Bigg]$$

The derivative of the mutual inductance between an elementary stator and rotor coil is:

$$\frac{dL_{s2}(\varphi)}{d\varphi}=\frac{Dl\,\mu_0 N_t^2}{2\delta}\Bigg[\frac{1}{2}(\mathrm{ser}\,(\pi)-\mathrm{ser}\,(X_2+\pi-X_1))\Bigg(\Bigg(1+\frac{1}{4}\Bigg(\mathrm{ser}\left(\pi-\frac{c_1\pi}{\tau_{h1}}\right)-$$

$$-\mathrm{ser}\left(\pi+\frac{c_1\pi}{\tau_{h1}}\right)+\mathrm{ser}\left(\pi-\frac{c_2\pi}{\tau_{h2}}\right)-\mathrm{ser}\left(\pi+\frac{c_2\pi}{\tau_{h2}}\right)\Bigg)\Bigg)+\frac{\pi}{8}\Bigg(\mathrm{ser}\,(Z_1X_1)+$$

$$+\mathrm{ser}\,(Z_2(X_2-\varphi))-\mathrm{ser}\left(Z_1X_2-\frac{2\pi c_1}{\tau_{h1}}\right)-\mathrm{ser}\left(Z_2(X_2-\varphi)-\frac{2\pi c_2}{\tau_{h2}}\right)\Bigg)\Bigg)-$$

$$-\frac{1}{2}(\mathrm{ser}\,(X_1+\pi-X_2)-\mathrm{ser}\,(\pi))\cdot\Bigg(\Bigg(1+\frac{1}{4}\Bigg(\mathrm{ser}\left(\pi-\frac{c_1\pi}{\tau_{h1}}\right)-\mathrm{ser}\left(\pi+\frac{c_1\pi}{\tau_{h2}}\right)+$$

$$+\text{ser}\left(\pi-\frac{c_2\pi}{\tau_{h2}}\right)-\text{ser}\left(\pi+\frac{c_2\pi}{\tau_{h2}}\right)\right)+\frac{\pi}{8}\left(\text{ser}\,(Z_1X_1)+\text{ser}\,(Z_2(X_1-\varphi))-\right.$$

$$\left.-\text{ser}\left(Z_1X_1-\frac{2\pi c_1}{\tau_{h1}}\right)-\text{ser}\left(Z_2(X_1-\varphi)-\frac{2\pi c_2}{\tau_{h2}}\right)\right)\right)\Bigg]$$

The diagrams of $dL_{s2}(\varphi)/d\varphi$ can be seen in Fig. 11.a of $dL_{12}(\varphi)/d\varphi$ in Fig. 11.b and $dL_{R1}(\varphi)/d\varphi$ in Fig. 11.c. Substituting $\varphi=\varphi_0+\omega t$ and the previously calculated currents that belong to the given constant load, the electromagnetic torque of the machine can be. The torque vs. time diagram of the former asynchronous machine at the given nominal load is shown in Fig. 12. Both the self and the mutual inductances of the coils are complicated functions of the rotor position because of the distributed stator and rotor windings and the slotted stator and rotor. As the pitch of the stator winding is near to the pole pitch, the self inductances and the mutual inductances between the stator and rotor coils change but slightly in the function of the rotor position. The pitch of the cage rotor is equal to the slot pitch and so the influence of the stator slots is significant. This is why in the case of a single wave supply voltage approximately single wave currents will flow (Fig. 10.a). The bar currents vs. time are shown in Fig. 10.b. The pulsation of torque (Fig. 12) may be explained by the variation in time of the elements of the torque matrix (Fig. 11.a, b, c) and of the bar currents. In the case of a transient process the voltage and mechanical equation must be solved simultaneously. This can only be done numerically even with a much more simple physical-mathematical model than that formerly presented. Though by means of serraphil functions the physical nonlinearity of the parameters can be described in a closed form, nevertheless

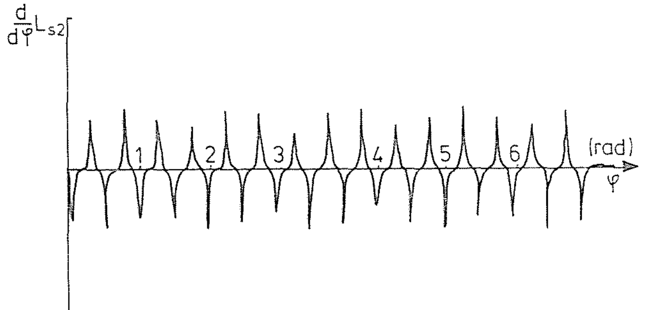



*Fig. 11*

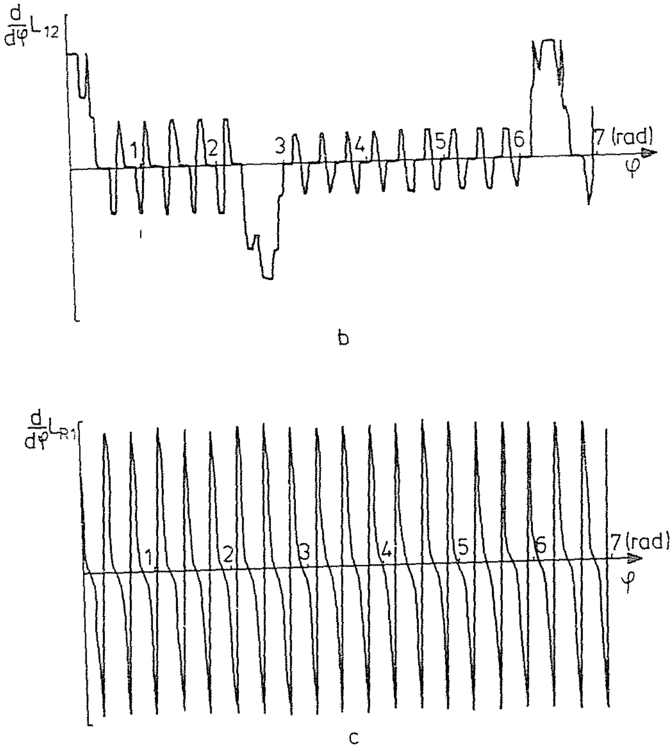

d/dφ L12
1
2
3
4
5
6
7 (rad)
φ
b
d/dφ LR1
1
2
3
4
5
6
7 (rad)
φ
c

*Fig. 11*

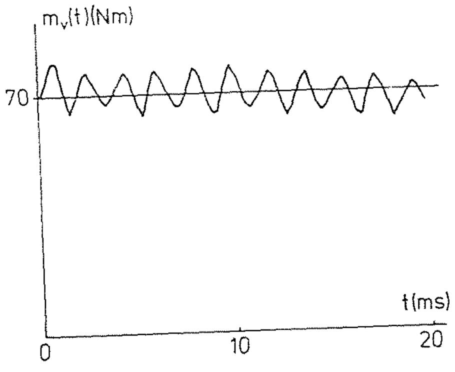

mv(t)(Nm)
70
t(ms)
0
10
20

*Fig. 12*

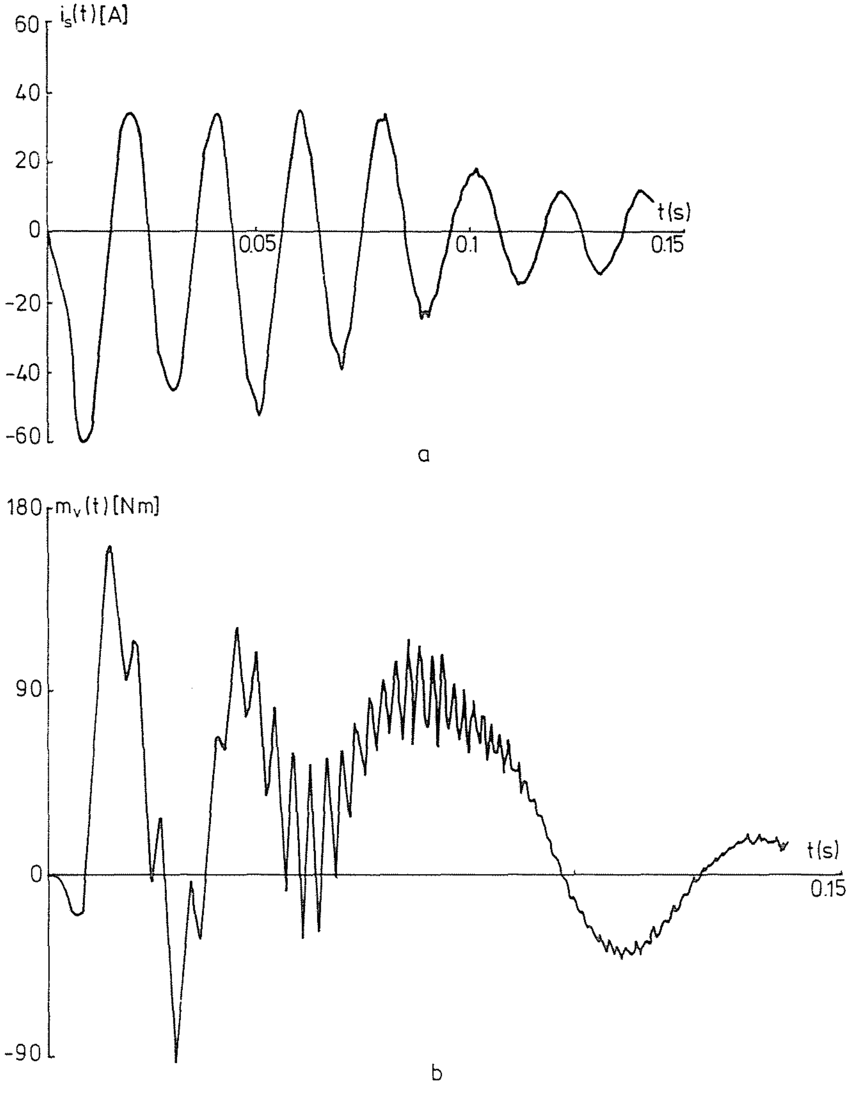



*Fig. 13*

the nonlinear differential equation of the transient process can be solved numerically only. The phase current and the torque of the asynchronous machine of the numerical examples can be seen in Fig. 13.a and Fig. 13.b during the run-up process.

## Conclusions

As a result of the rapid progress of the computational technique it is now possible to investigate the electromagnetic energy conversion process within the machine with the help of combined models composed from elementary parts. One of the possibilities is the use of serraphil functions. By means of these functions the air-gap field distribution, the currents and the torque can be described in a closed form both in the steady state and the transient mode of operation and in the case of nonsinusoidal supply voltage. With the method presented it is easier to take into account the most significant factors (assymetries of the windings, influence of the slotting and of the supply) than with the traditional method, at least from a computational point of view.

It is known that periodic functions can be represented not only by the Fourier method as the sum of a series of sine waves but as the sum of a series of other simple functions e.g. triangular or serramorph functions. The use of such series is very advantageous to describe the step functions that occur in connection with electrical machines. It is now our intention to investigate the influence of asymmetries in the machine and the nonsinusoidal supply.

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Gyula TŐKE 1521 Budapest
Gábor NÁDASY EVIG H-1475 Budapest