GLOBAL ELEMENT ANALYSIS OF TRANSFORMER LEAKAGE FIELD AND TANK LOSSES

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Summary

A method based on variational principles is described for the computation of the leakage field in power transformers. The eddy-currents in the tank are taken into account. Two-dimensional models of both translational symmetry and axisymmetry are considered. The solution is obtained in the form of a double series with the coefficients calculated by simple formulas. Some flux plots are given to illustrate the results.

Introduction

An accurate knowledge of the leakage field of a transformer permits the computation of many factors essential from the point of view of transformer design. Reactances, short-circuit forces, eddy-current losses in the windings are easily calculated once the magnetic field in the vicinity of the windings is known. At the same time, the numerical prediction of eddy-current losses in the tank, in clamping plates and in other structural elements requires the determination of the electromagnetic field in these metal parts and of the magnetic field in their neighbourhood with the effect of their eddy-currents taken into account.

The simplest methods for the prediction of certain related factors, such as reactances, radial forces and eddy-current losses in the windings assume a unidirectional, axial magnetic field in the windings. The neglection of the radial field, however, does not permit even the approximation of axial forces and tank losses among others. Moreover, in case of unusual winding arrangements the accuracy is very low.

The leakage pattern can be calculated by dividing the windings into elementary current-conducting filaments and obtaining the resultant field by integrating the differential fields stemming from these. Boundary conditions may then be taken into account by images. The disadvantage of this method is its huge computational cost because of the need to take a large number of images into account and to carry out numerical integration of functions with singularity.

Three basic methods have emerged so far combining fair accuracy and relative ease of computation.

One of them was introduced by Roth [13], and it yields the solution of the problem as a double Fourier series with simple formulas given for the coefficients. The appeal of the method lies in the closed form of the solution, in the lack of need for solving large systems of simultaneous equations, and in the possibility of improving accuracy by simply including new terms in the Fourier series. Unfortunately, attempts to generalize the method to the axisymmetric case resulted in an extremely complicated procedure [14]. Another drawback is that no way is apparent to take the eddy-currents in the tank into account.

Another possibility to compute transformer leakage phenomena is the application of the method of finite differences. This numerical procedure permits the approximate solution of many problems of electrodynamics. It has been applied to transformer leakage problems without eddy-current calculations [4], but also to computations involving the tank losses [16]. The method is capable of treating axisymmetric problems. In contrast to Roth's procedure, the solution of a large system of simultaneous equations is necessary at the application of finite differences. The magnetic field is obtained as a set of potential values at discrete points in the studied region, thus lacking the advantage of a closed form solution.

Finally, a successful approach to transformer leakage problems has been the method of finite elements. Based on variational principles, this method can also be applied to a very broad class of electromagnetic problems [7]. Efficient programs have been developed for the analysis of transformer leakage phenomena by finite elements [1, 15]. The treatment of the axisymmetric case presents no difficulties. The inclusion of the tank in the analysis is also possible, eddy-current problems have been successfully solved by finite elements [2, 6], although not ones in connection with transformer leakage fields. The manner in which the solution is obtained and its form are similar to those at the method of finite differences, but the method of finite elements tends to produce more accurate results with the same number of unknowns [9].

The aim of the present paper is to suggest a method based on variational principles which, however, resembles Roth's procedure. It also yields the solution in the form of a Fourier series like Roth's result, but it can be easily generalized to the axisymmetric case without making the computation impractically complicated. The effects of the eddy-currents in the tank are also accounted for. The method to be presented is a global element method (the term is due to Delves and Hall [8]) which can be derived both by Galerkin techniques [5, 11] and by Ritz's method [3]. A model of translational symmetry will be used since this was felt the best two-dimensional approximation for taking the tank into account, but the possibility of extension to the axisymmetric and even to the three-dimensional problem will be pointed out.

The model of the transformer

An exact calculation of transformer leakage phenomena requires a three-dimensional model to be used. This, however, involves a much more costly computation than the current two-dimensional models whose accuracy is lower but so far seems sufficient for design purposes. The two-dimensional model employed in the calculation to be presented is derived as one of the axial sections indicated by dotted lines in Fig. 1 on the top-view outline of a three-limbed core transformer. The sections are perpendicular to the tank wall and cross through the axis of the core and the windings. The model is thought to be infinitely long in the direction normal to the cross section which is shown in Fig. 2 with the necessary dimensions indicated.

Further simplifying assumptions are made to derive the model shown in Fig. 3:

(a) The gaps between the yoke and the tank lid and bottom are disregarded to obtain a rectangular region.

(b) The permeability of the core is thought to be infinite.

(c) Similar assumption is made for the tank lid and bottom which are distant enough from the windings to carry no appreciable eddy-currents.

(d) The tank wall is taken to be infinitely thick. This is a good approximation in view of the fact that the thickness of the wall is several times greater than the skin depth of iron at power frequencies.

(e) The saturation of iron in the tank wall will not be taken into account.

(f) The current distribution in the windings is assumed to be uniform and known.

A Cartesian system of coordinates has also been indicated in Fig. 3. The number of windings in the model is arbitrary and will be denoted by w. (In Fig. 3, $w=2$).

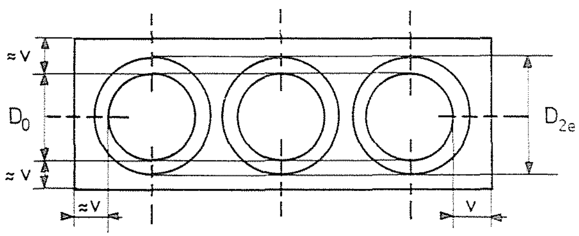

Fig. 1

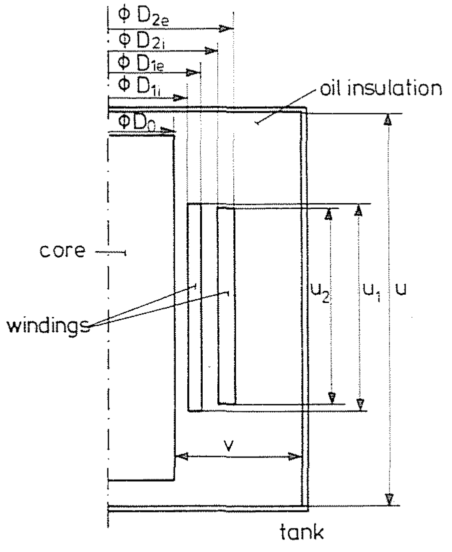

Fig. 2

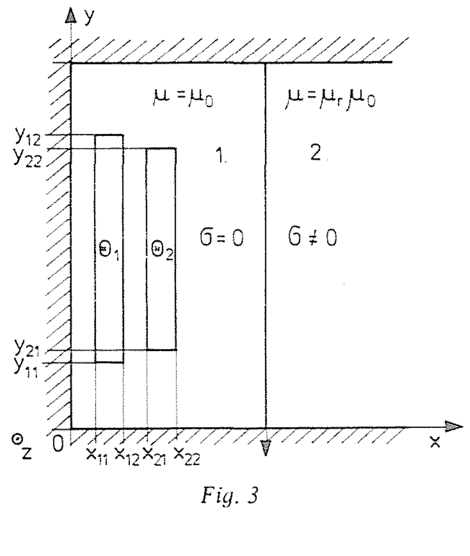

Fig. 3

The studied region is divided into two subregions, one denoted by 1 containing the windings and the insulation, the other denoted by 2 formed by the tank wall. The complex amplitudes of the z-directed vector potential functions $\mathbf{A}_1 = A_1 \mathbf{e}_z$ and $\mathbf{A}_2 = A_2 \mathbf{e}_z$ valid in the respective subregions satisfy the following differential equations:

$$-\frac{1}{\mu_0}\left(\frac{\partial^2 A_1}{\partial x^2}+\frac{\partial^2 A_1}{\partial y^2}\right)=J, \tag{1}$$

$$-\frac{1}{\mu}\left(\frac{\partial^2 A_2}{\partial x^2}+\frac{\partial^2 A_2}{\partial y^2}\right)+j\omega\sigma A_2=0 \tag{2}$$

where μ and μ_0 are the permeabilities in the subregions, j is the imaginary unit, ω is the angular frequency and σ is the conductivity of the tank wall. J is the current density:

$$J=\begin{cases}\dfrac{\sqrt{2}\Theta_r}{(x_{r2}-x_{r1})(y_{r2}-y_{r1})} & \text{in the } r\text{-th winding } (r=1,2,\ldots,w)\\ 0 & \text{outside the windings.}\end{cases} \tag{3}$$

Θ_r denotes the r.m.s. value of the total excitation of the r-th winding.

The boundary conditions

$$\frac{\partial A_1}{\partial y}(y=0)=0, \tag{4}$$

$$\frac{\partial A_1}{\partial y}(y=u)=0, \tag{5}$$

$$\frac{\partial A_1}{\partial x}(x=0)=0, \tag{6}$$

$$\frac{\partial A_2}{\partial y}(y=0)=0, \tag{7}$$

$$\frac{\partial A_2}{\partial y}(y=u)=0, \tag{8}$$

$$\lim_{x\to\infty} A_2=0 \tag{9}$$

and the interface conditions

$$\frac{1}{\mu_0}\frac{\partial A_1}{\partial x}(x=v)=\frac{1}{\mu}\frac{\partial A_2}{\partial x}(x=v), \tag{10}$$

$$A_1(x=v)=A_2(x=v) \tag{11}$$

are to be satisfied at the solution of the problem. The boundary conditions (4)—(8) require the magnetic field to be perpendicular to the infinitely permeable iron parts, (9) specifies the regular behaviour of the vector potential in infinity, while (10) and (11) are the usual interface conditions describing the continuity of the tangential component of field intensity and the normal component of flux density.

The vector potential A_2 describing the electromagnetic field in the tank wall will be sought as

$$A_2=\sum_{k=0}^{\infty} b_k \cos\left(k\pi\frac{y}{u}\right)e^{-p_k(x-v)};\ p_k^2=\left(\frac{k\pi}{u}\right)^2+j\omega\mu\sigma,$$

$$\mathrm{Re}(p_k)>0. \tag{12}$$

This expansion satisfies the differential equation (2) and the boundary conditions (7)—(9) at any choice of the coefficients b_k.

Let us initially assume the vector potential A_2 to be known, i.e. the coefficients b_k in (12) to be fixed. Now, the differential equation (1), the boundary conditions (4)—(6) and the interface condition (10) constitute a boundary value problem for the magnetic field in subregion 1. The solution of this problem will be determined by global approximation. This solution, naturally, depends upon the parameters b_k ($k=1, 2, \ldots$). If these are selected so that the interface condition (11) is satisfied, the magnetic field in both subregions will have been determined.

Global element formulation

The boundary value problem formulated for subregion 1 calls for the calculation of the stationary magnetic field in a bounded region Ω with the current density in Ω and the tangential component of magnetic field intensity on the boundary S given, i.e. the differential equation

$$\frac{1}{\mu_0}\,\mathbf{curl}\,\mathbf{curl}\,\mathbf{A}=\mathbf{J} \tag{13}$$

is to be solved in Ω with the inhomogeneous Neumann type boundary condition

$$\frac{1}{\mu_0}\,\mathbf{curl}\,\mathbf{A}\times\mathbf{n}=\mathbf{h}. \tag{14}$$

$\mathbf{A}$ is the vector potential, $\mathbf{J}$ is the current density, $\mathbf{n}$ is the outer normal of S and $\mathbf{h}=\mathbf{H}_t\times\mathbf{n}$ with $\mathbf{H}_t$ denoting the tangential component of the field intensity on S.

The global elements approximation to the solution of (13)—(14) is sought as

$$\mathbf{A}\approx\mathbf{A}_n=\sum_{p=1}^{n} a_p\boldsymbol{\varphi}_p \tag{15}$$

where $\boldsymbol{\varphi}_p$ ($p=1, 2, \ldots$) are elements of a function set entire in an appropriate sense and a_p ($p=1, 2, \ldots, n$) are numerical parameters. These can either be determined by the application of Ritz's method [3] to the functional

$$W=\frac{1}{2}\int_{\Omega}\frac{1}{\mu_0}(\mathbf{curl}\ \mathbf{A})^2 d\Omega-\int_{\Omega}\mathbf{J}\mathbf{A}d\Omega-\oint_{S}\mathbf{h}\mathbf{A}dS, \tag{16}$$

or by directly applying the Bubnov—Galerkin method to the equation including both the differential equation (13) and the boundary condition (14):

$$\Theta_{\Omega}\frac{1}{\mu_0}\mathbf{curl}\ \mathbf{curl}\ \mathbf{A}+\delta_S\frac{1}{\mu_0}\mathbf{curl}\ \mathbf{A}\times\mathbf{n}=\Theta_{\Omega}\mathbf{J}+\delta_S\mathbf{h} \tag{17}$$

Θ_{Ω} is the function equal to 1 in Ω and vanishing outside it while δ_S is a distribution concentrated on S [5]. Both procedures yield the following set of simultaneous equations for the column vector $\mathbf{a}$ formed by the coefficients a_k ($k=1, 2, \ldots, n$):

$$\boldsymbol{D}\mathbf{a}=\mathbf{d} \tag{18}$$

with the elements of the square matrix $\boldsymbol{D}$ and column vector $\mathbf{d}$ obtained as

$$D_{pq}=\int_{\Omega}\frac{1}{\mu_0}\mathbf{curl}\ \boldsymbol{\varphi}_p\ \mathbf{curl}\ \boldsymbol{\varphi}_q d\Omega,\ \ p, q=1, 2, \ldots, n, \tag{19}$$

$$d_p=\int_{\Omega}\mathbf{J}\boldsymbol{\varphi}_p d\Omega+\int_{S}\mathbf{h}\boldsymbol{\varphi}_p dS,\ \ p=1, 2, \ldots, n. \tag{20}$$

The solution of (18) substituted into (15) yields a global element approximation to the vector potential function solving the boundary value problem (13)—(14).

In the present problem Ω is the rectangular, planar region $\{0<x<v; 0<y<u\}$, $\mathbf{J}$ is given by (3) and (4)—(6), (10) and (12) yield $\mathbf{h}=h\mathbf{e}_z$:

$$h=\begin{cases} 0 \quad \text{on } y=0,\ y=u,\ x=0, \\ \dfrac{1}{\mu}\dfrac{\partial A_2}{\partial x}(x=v)=-\sum\limits_{k=0}^{\infty} b_k\dfrac{p_k}{\mu}\cos\left(k\pi\dfrac{y}{u}\right) \quad \text{on } x=v. \end{cases} \tag{21}$$

The approximation (15) is chosen as the double Fourier expansion:

$$A_1=\sum_{i=0}^{\infty}\sum_{k=0}^{\infty} a_{ik}\cos\left(i\pi\frac{x}{v}\right)\cos\left(k\pi\frac{y}{u}\right), \tag{22}$$

i.e.

$$\varphi_p=\cos\left(i\pi\frac{x}{v}\right)\cos\left(k\pi\frac{y}{u}\right),\ \ i, k=0, 1, 2, \ldots. \tag{23}$$

The set of trigonometrical functions (23) is entire in the sense required by the global element method [11]. Although its elements satisfy a homogeneous Neumann boundary condition at $x=v$, the expansion is capable of approximating the solution of the boundary value problem much like ordinary Fourier series converge to functions with discontinuous derivatives. The advantage of the choice (22) is that the matrix $\boldsymbol{D}$ in (18) is diagonal due to the orthogonal properties of trigonometrical functions. Therefore, each equation in (18) contains only one of the parameters a_{ik}, and using (19), (20), (21), (23) this equation is of the form:

$$a_{ik}\frac{uv}{4\mu_0}\left[\delta_i\left(\frac{k\pi}{u}\right)^2+\delta_k\left(\frac{i\pi}{v}\right)^2\right]=I_{ik}-(-1)^i b_k\frac{p_k u}{2\mu}\delta_k,\qquad i,k=0,1,2,\ldots \tag{24}$$

where

$$\delta_j=\begin{cases}2 & \text{if } j=0,\\ 1 & \text{if } j\neq 0\end{cases} \tag{25}$$

and

$$I_{ik}=\int_0^v\int_0^u J\cos\left(i\pi\frac{x}{v}\right)\cos\left(k\pi\frac{y}{u}\right)dy\,dx=$$

$$=\sum_{r=1}^{w}\frac{\sqrt{2}\Theta_r}{(x_{r2}-x_{r1})(y_{r2}-y_{r1})}\alpha_{ri}\beta_{rk}, \tag{26}$$

$$\alpha_{ri}=\begin{cases}x_{r2}-x_{r1} & \text{if } i=0,\\ \dfrac{\sin\left(i\pi\dfrac{x_{r2}}{v}\right)-\sin\left(i\pi\dfrac{x_{r1}}{v}\right)}{\dfrac{i\pi}{v}} & \text{if } i>0\end{cases};$$

$$\beta_{rk}=\begin{cases}y_{r2}-y_{r1} & \text{if } k=0,\\ \dfrac{\sin\left(k\pi\dfrac{y_{r2}}{u}\right)-\sin\left(k\pi\dfrac{y_{r1}}{u}\right)}{\dfrac{k\pi}{u}} & \text{if } k>0.\end{cases} \tag{27}$$

It is evident from (24) that a_{00} ($i=0$ and $k=0$) cannot be determined. However, this corresponds to a constant term in the vector potential and is therefore irrelevant.

In order to satisfy the interface condition (11), $x=v$ is substituted into (12) and (22). Comparing the two series, (11) yields

$$\sum_{i=0}^{\infty} a_{ik}(-1)^i = b_k; \quad k=0, 1, \ldots . \tag{28}$$

The set of equations (24), (28) can be solved individually at each value of i and k. The solution is:

$$b_k = \frac{\displaystyle\sum_{i=0}^{\infty} (-1)^i \frac{4\mu_0 I_{ik}}{uv\left[\delta_i\left(\frac{k\pi}{u}\right)^2 + \delta_k\left(\frac{i\pi}{v}\right)^2\right]}}{1 + \displaystyle\frac{2p_k}{\mu_r v}\sum_{i=0}^{\infty} \frac{1}{\delta_i\left(\frac{k\pi}{u}\right)^2 + \delta_k\left(\frac{i\pi}{v}\right)^2}}, \quad k=0, 1, \ldots, \tag{29}$$

$$a_{ik} = 4\mu_0 \frac{I_{ik} - (-1)^i b_k \frac{p_k u}{2\mu}\delta_k}{uv\left[\delta_i\left(\frac{k\pi}{u}\right)^2 + \delta_k\left(\frac{i\pi}{v}\right)^2\right]}, \quad i, k=0, 1, \ldots . \tag{30}$$

The leakage field of the transformer has thus been obtained in a double Fourier series form (22) with the coefficients given in closed form in (30). It is worth noting that if the eddy-currents in the tank wall are neglected, i.e. $\mu \to \infty$ in (30), the result is identical to that of Roth [13].

Extension to the axisymmetric case

If an axisymmetric model is preferred for the transformer leakage calculation, the above global element procedure is easily modified to meet this requirement. In general, the application of the global element method necessitates the simultaneous solution of as many equations as there are terms in the approximating expansion. The orthogonal choice of the trial functions in the translationally symmetric model has permitted to split this system into independent equations. The procedure for the axisymmetric case, which is roughly outlined in the following, carries this decomposition to a lesser extent but still retains the simplicity of the procedure given for the translationally symmetric model.

The model of Fig. 3 is used again, but the coordinates x are now replaced by r, and the coordinates y by z. The radius of the core limb is denoted by r_0.

The differential equations corresponding to (1) and (2) now refer to φ-directed vector potentials and are of the forms

$$-\frac{1}{\mu_0}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_1}{\partial r}\right)+\frac{\partial^2 A_1}{\partial z^2}\right]=J\,, \tag{31}$$

$$-\frac{1}{\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_2}{\partial r}\right)+\frac{\partial^2 A_2}{\partial z^2}\right]+j\omega\sigma\, A_2=0\,. \tag{32}$$

The boundary and interface conditions are analogous to (4)—(11), but are expressed in cylindrical coordinates.

The vector potential A_2 in the tank is now of the form

$$A_2=\sum_{k=0}^{\infty} b_k \cos\left(k\pi\frac{z}{u}\right) H_0^{(1)}(p_k r)\,;\; p_k^2=-\left(\frac{k\pi}{u}\right)^2-j\omega\mu\sigma\,,$$
$$\operatorname{Im}(p_k)>0 \tag{33}$$

where $H_0^{(1)}$ is the zero order Hankel function of the first kind. Similarly to (12), this expansion satisfies the differential equation (32) and the boundary conditions corresponding to (7)—(9) at any choice of b_k.

The approximation in subregion 1 is now chosen as

$$A_1=\sum_{i=1}^{n}\sum_{k=0}^{\infty} a_{ik}\, g_i(r)\cos\left(k\pi\frac{z}{u}\right) \tag{34}$$

with $g_i(r)$ $(i=1, 2, \ldots)$ constituting a function set entire in the appropriate sense (e.g. trigonometrical, cylindrical, hyperbolical functions or orthogonal polynomials) [11]. Now, the matrix $\boldsymbol{D}$ in (18) is not entirely diagonal, but it is made up of an infinite number of n order square matrices resting on its main diagonal. Thus, (18) is split into independent matrix equations for each value of k:

$$\boldsymbol{D}_k\mathbf{a}_k=\mathbf{d}_k\,;\; k=0, 1, \ldots \tag{35}$$

where $\mathbf{a}_k$ denotes the column vector of the coefficients $a_{1k}, a_{2k}, \ldots, a_{nk}$, and in accordance with (19) and (20):

$$D_{kpq}=u\pi\int_{r_0}^{r_0+v}\left[\left(\frac{k\pi}{u}\right)^2 r g_p(r)\, g_q(r)+\delta_k\frac{1}{r}\frac{d(rg_p)}{dr}\frac{d(rg_q)}{dr}\right]dr\,,$$
$$p, q=1, 2, \ldots, n\,; \tag{36}$$

$$d_{kp}=2\pi\int_{r_0}^{r_0+v}\int_0^u J\,g_p(r)\cos\left(k\pi\frac{z}{u}\right)r\,dr\,dz+$$

$$+b_k\frac{u\pi}{2\mu}\delta_k\left[g_p(r)\frac{d}{dr}rH_0^{(1)}(p_k r)\right]_{r=r_0+v}. \tag{37}$$

The continuity of the vector potential is equivalent to

$$\sum_{i=1}^{n} a_{ik}\,g_i(r_0+v)=b_k\,H_0^{(1)}(p_k(r_0+v)). \tag{38}$$

Now, (35) and (38) yield a system of simultaneous linear equations for the $n+1$ unknowns $\mathbf{a}_k$ and b_k. Its solution for each value of k yields the coefficients in the expansions (33) and (34). Thus, the global element procedure has been extended to the axisymmetric case.

The method can be further developed to treat a three-dimensional model with the whole transformer contained in a rectangular tank. Triple series are then used for the approximation and the orthogonal choice of functions in one or more variables reduces the amount of work needed to obtain the expansion coefficients. Further investigations are being carried out in this respect.

Tank losses

The eddy-current loss in the transformer tank can be easily obtained once the expansion (12) is known for the vector potential in the tank. Naturally, the two-dimensional model of Fig. 3 can only yield a loss per unit length in the direction perpendicular to the plane of the model. Although a strong variation of the field in this direction is expected, a reasonable approximation to the total loss may be obtained by the assumption that the per unit loss is the same in other cross sections facing the windings. This approximation has been used in [10] and found to give good agreement with test results (Table 1). Accordingly, the transformer tank loss is calculated from the p.u. loss p as

$$P=nD_{2e}p \tag{39}$$

where D_{2e} is the diameter of the outer winding and n is the number of wall sections facing the winding. (For the three-limbed core transformer of Fig. 1, $n=8$.)

The p.u. loss is conveniently calculated with the aid of the Poynting vector:

$$p=\frac{1}{2}\operatorname{Re}\int_S(\mathbf{E}\times\mathbf{H}^*)\,d\mathbf{S}=-\frac{1}{2}\operatorname{Re}\int_0^u E_z(x=v)\,H_y^*(x=v)\,dy. \tag{40}$$

Table 1

p.u. loss computed by the method of [16]	1.43 kW/m
p.u. loss obtained by (42)	1.55 kW/m
total loss measured, as given in [10]	14.4 kW
total loss according to (39)	17.8 kW

S is here the part of the tank surface with unit depth in the z direction, * denotes the complex conjugate of the quantities marked and E_z and H_y are complex amplitudes. The field intensities are easily calculated from the vector potential as

$$\mathbf{E} = -j\omega \mathbf{A}_2, \quad \mathbf{H} = \frac{1}{\mu} \, \mathbf{curl}\, \mathbf{A}_2 , \tag{41}$$

thus (40) yields after substituting (12) and (41), and performing the integration:

$$p = \frac{\omega}{\mu} \frac{u}{8} \sum_{k=1}^{\infty} |b_k|^2 \, \mathrm{Im}\,(p_k) \tag{42}$$

The value of the p.u. loss is thus given by a closed formula easily evaluated from the data of the transformer. A good agreement has been found with the p.u. loss computed by the method of [16] for the particular transformer of reference [10] (Table 1). The latter calculation had been carried out by the method of finite differences. The measured total loss has been derived in [10] as the difference in short circuit losses measured with the windings in the tank and with the tank removed. This method is bound to give a low value for the loss, since the eddy-current losses in the structural elements other than the tank are somewhat decreased by the presence of the tank.

Eddy-current losses in the windings, transformer reactances, forces acting on the windings are also easily calculated from the leakage field as shown e.g. in [1], [15] and [12].

Flux plots

To further illustrate the method, a few flux plots are given in Figs. 4—6 for Rabins' case 2 [12] which has been investigated by several methods in [1] and [15]. Translational symmetry has been assumed.

In Fig. 4, the tank wall has been taken to be of conductivity $\sigma = 7 \cdot 10^6$ A/Vm and relative permeability $\mu_r = 100$, and the double Fourier series has been truncated at the sixth harmonic in both variables.

The same case is shown in Fig. 5 with terms up to the tenth harmonic included in the approximation. Comparison of the two plots indicates that the

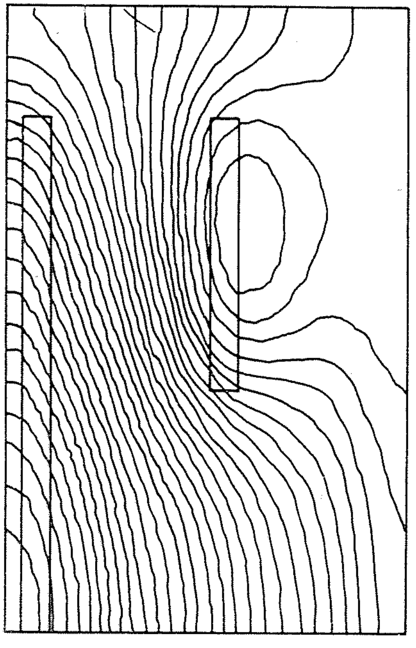

Fig. 4

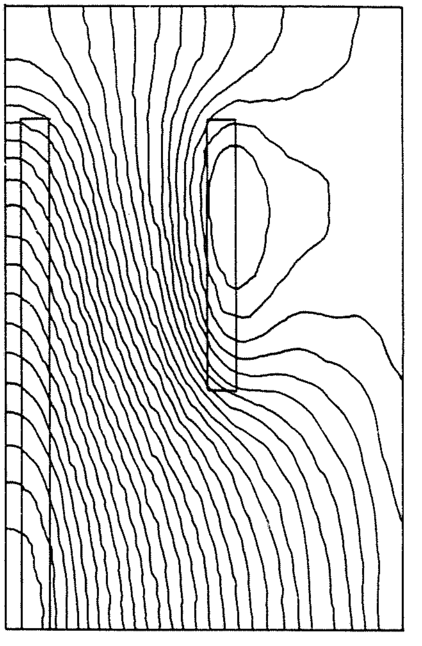

Fig. 5

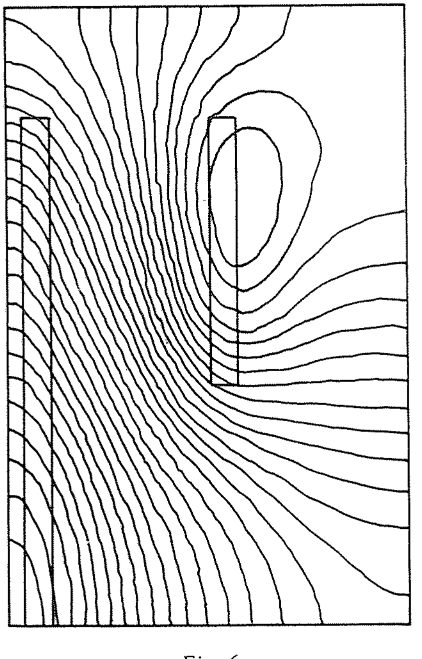

Fig. 6

convergence of the double Fourier series obtained by the global element method is satisfactory even as regards field quantities. Naturally, an even better convergence is found in integral quantities like reactances or losses.

For the sake of comparison, Fig. 6 shows the same transformer leakage field with the tank wall assumed to have infinite permeability. The flux plots given in [1] and [15] refer to this case. The number of harmonics included in the approximation shown in Fig. 6 is six, i.e. a 48 term series has been used for the vector potential. The effort needed to obtain the leakage field seems to be much less than in the case of the finite element and image techniques mentioned in [1] and [15], and the agreement of the plot with those obtained by the latter is satisfactory.

Conclusions

A numerical method yielding analytical results has been developed for the analysis of the leakage field of power transformers. Eddy-currents in the tank have been taken into account. The calculations are based on a two-dimensional model, and special considerations are introduced to obtain the tank loss in the whole transformer. This loss is derived in the form of a rapidly converging infinite series easily computed from the data of the transformer. The computed results compare well with measured ones. Flux plots have been included and compared with previous ones obtained by other authors.

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