ELECTRICAL NETWORK MODELLING OF D. C. MACHINES AND THEIR REGULATING DEVICES

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Summary

Direct current electrical machines and their regulating devices can be modelled by electrical networks, so-called equivalent circuits. In the method presented the machine and the regulator are substituted by one electrical network permitting to determine the transient behaviour of the equipment. The computational procedure described the current limiting and error signal limiting in the regulating circuit to be taken into account.

Network model of direct current electrical machines

The general network models or equivalent circuits of electrical machines have been detailed in previous works by the author [1, 2]. These models substitute the mechanical transients in the machine by electrical network transients making use of electrodynamical analogies.

The network model of a direct current machine with its main flux taken to be constant is shown in Fig. 1.

This model can be considered as a two-port with one port corresponding to the terminals of the rotor (u_r and i_r denote the rotor voltage and current), and the other so-called "mechanical port" representing the shaft (i_ω is the angular velocity of the shaft and u_{Tm} is the external torque acting on the shaft). The other symbols are: R_r and L_r are the resistance and inductance of the rotor, R_K is resistance characterising mechanical losses, and L_J the inductance associated with the inertia, M is the parameter corresponding to the torque factor $c\Phi$. Definitions of R_K, L_J and M are presented in previous publications by the author [1, 2].

The Kirchhoff equations

$$u_r = R_r i_r + L_r \frac{di_r}{dt} + M i_\omega$$

$$u_{Tm} = R_K i_\omega + L_J \frac{di_\omega}{dt} - M i_r \qquad (1)$$

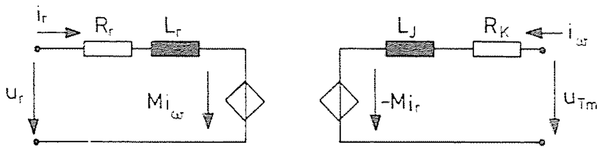

Fig. 1

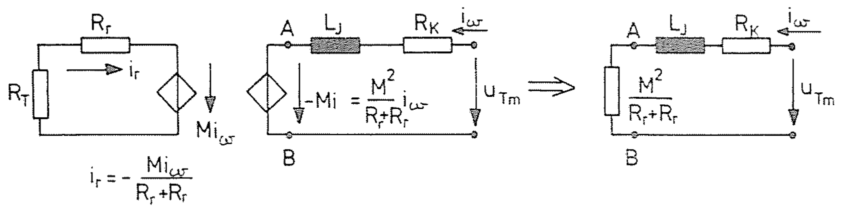

Fig. 2

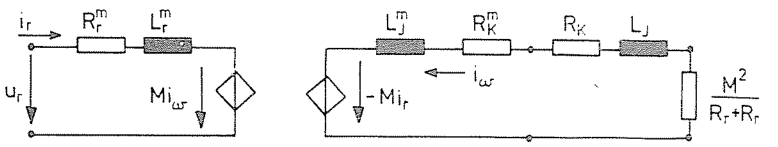

Fig. 3

written for the network in Fig. 1 describe the D. C. machine whether it operates as a motor or a generator. The controlled sources ensuring the coupling between the circuits of the two-port (acting as gyrators) simulate the electromechanical energy conversion.

Assume that a generator loaded by resistance R_t is examined, with L_r negligible. In this case, the network in Fig. 2a is obtained where the circuit connected to terminals $A-B$ can be substituted by a single resistor as shown in Fig. 2b. Thus, the armature circuit of the generator may be considered as a resistance from the mechanical circuit aspect. In case the generator loaded by resistance R_t is driven by a D. C. motor, joining the mechanical ports (this corresponds to the coupling of the shaft) yields the network shown in Fig. 3 (subscript m refers to the parameters of the motor).

Model of the regulating elements

Operational amplifiers are often applied in regulating devices. Operational amplifiers realized by integrated circuits may be considered as ideal (zero input current and infinite amplification) at a fairy approximation. The model of an ideal operational amplifier is the nullor [3] shown in Fig. 4 complete with the input circuit and feed-back elements.

The elements providing proportional amplification are modelled by controlled sources. The controlled voltage-source referred to in this paper is shown in Fig. 5.

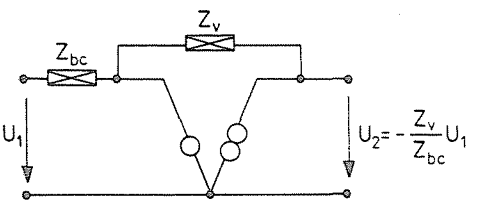

Fig. 4

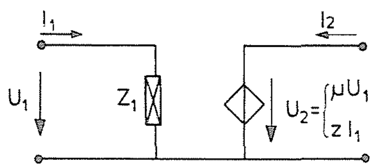

Fig. 5

Model of a regulated D. C. drive

The analysis of a regulated D. C. drive has been carried out with the aid of the models presented in the previous items. The circuit diagram of the equipment examined is shown in Fig. 6.

The direct current motor M is fed by a controlled rectifier. Ignition is controlled by signal y_1, therefore the rectifier is taken to be a nonlinear voltage source controlled by signal y_1. The motor is loaded by a direct current generator G. The signals proportional to the motor speed and current are fed back after amplification to the speed control and current control equipment. The control devices are $R-C$ feedback operational amplifiers which, due to their limited dynamic range, also limit signals y_1 and y_2.

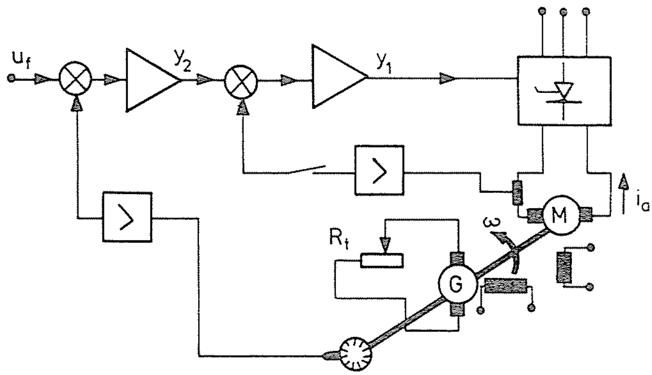

Fig. 6

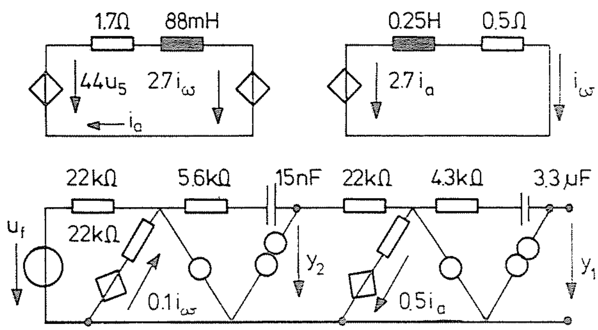

Fig. 7

The network model of the equipment shown in Fig. 7 can be constructed of the models given in items 2 and 3.

The examination of the model has been carried out with the aid of state equations. The normal form of state equations is known to be [3]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{2}$$

where $\mathbf{x}$ and $\dot{\mathbf{x}}$ denote the state variables and their time-derivatives, $\mathbf{u}$ the excitation matrix, $\mathbf{A}$ and $\mathbf{B}$ are matrices, characterizing the network. The output voltages of the operational amplifiers (limited variables) can be written as

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{3}$$

In case one of the amplifiers is driven to its limit,

$$y_i = y_{i\,\max} \tag{4}$$

independent of the values of the state variables, and (2) may be replaced by:

$$\dot{\mathbf{x}} = \mathbf{A}_0\mathbf{x} + \mathbf{E}y_m + \mathbf{B}_0\mathbf{u} \tag{5}$$

for computing the changes in the state variables. Matrix **E** can be determined from the structure of the network examined. It is noted that, since

$$\begin{aligned} \mathbf{A} &= \mathbf{A}_0 + \mathbf{EC} \\ \mathbf{B} &= \mathbf{B}_0 + \mathbf{ED} \end{aligned} \tag{6}$$

Eq. (5) can be used even if the amplifiers are not driven to the limit.

According to the above considerations, the state variables have been computed step by step, by the following procedure:

1. Setting of initial conditions.
2. Computation of variables **y** according to (3).
3. Checking limit conditions: if $|y_i| > y_{im}$, then

$$y_i = y_{i\,\max} = \operatorname{sign}(y_i) \cdot y_{im} \tag{7}$$

4. Computation of the changes in the state variables by the method of finite differences, according to (5)
5. Computation of state variables, return to 2.

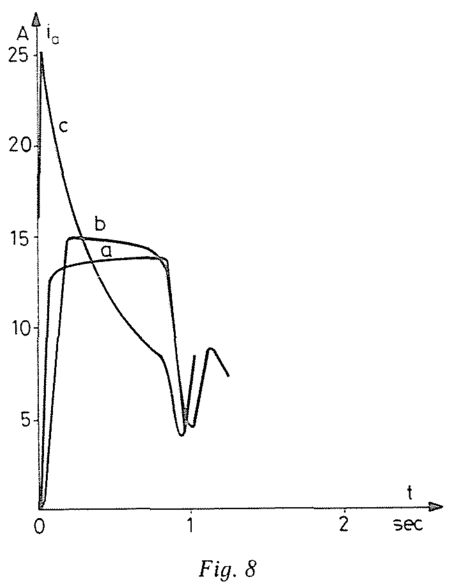

Fig. 8

Results

The results of model calculations and of measurements on the equipment are shown in Figs 8 and 9 (curve a: computed result, curve b: measured values). In any cases the diagrams represent a process to starting the idle machines by abruptly changing the speed control signal u_f in a unit step. The motor current and speed computed by ignoring the limited ranges of the amplifiers are also plotted in the diagrams (curve c).

The measurements have been carried out at the Department of Electrical Machines, Technical University, Budapest on the equipment installed there.

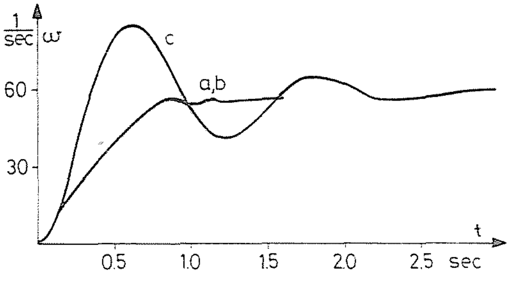

Fig. 9

References

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2. SEBESTYÉN, I.: Network models of D. C. machines. (In Hungarian.) Elektrotechnika, *68*, 310 (1980).
3. VÁGÓ, I.: Application of graph theory to electrical networks. (In Hungarian.) Budapest, Műszaki Könyvkiadó. 1976.

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