TEMPERATURE FIELDS AND THERMAL STRESS FIELDS IN RADIATION SHIELDINGS

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Introduction

The purpose of radiation shieldings is to reduce the intensity of radiation emitted by radioactive sources to a level which will allow safe operation for the personnel behind the shielding. The energy of radiation absorbed will be transformed into heat, so that—depending on the local distribution of absorption—heat sources arranged very heterogeneously within the body of the shielding will develop. Most shieldings (e.g. in primary circuits of nuclear power plants) are initially operated at high temperatures, which will rise further due to the internal heat sources, resulting in an inhomogeneous temperature distribution. Since no thermal expansion of the material is allowed to take place, stresses will be called into play in the body of the shielding.

Nuclear reactors being the most powerful radiation sources in industrial practice, their shielding is of vital importance. The crucial significance of the amounts of heat developed shall be illustrated by the following estimation. The heat output of the PWR reactor type WWER-440 is 1375 MW. Some per cents, let us assume 5% of this amount of heat will be absorbed in the shielding. Half of the amount will be developed within the internal surface layer of the shielding, in a thickness of about 0.1 m and a total volume of 3.5 m^3. Hence the average thermal source density within this layer is around 10 MW/m^3, whereas the furnace load in coaldust-fired power plants is usually below 1 MW/m^3.

Heat evolution in the shielding [1, 2]

In nuclear power plants only neutron and gamma radiation are of any practical importance. Their properties—regarding heat evolution—differ in some aspects. The *neutron reactions* involved are

— elastic scattering, [reaction (n, n)]: heat evolved at the site of the reaction;

— reaction of a charged particle, i.e. (n, p), (n, α) etc.: heat evolved at the site of the reaction;

— non-elastic scattering [(n, n' + γ)]: heat evolved at the site of the reaction and at the sites of the reactions of the photon emitted;

— activating neutron capture [(n, γ)]: heat evolved at the sites where the reactions of the photon emitted take place.

In the latter two processes it should also be considered that the gamma photons will eventually be emitted after a significant delay.

It is characteristic for *gamma processes* that heat is evolved at the site of the reaction in all three major reactions: photoelectric absorption, Compton scattering and pair formation.

In principle it appears a simple task to calculate the reactions in which heat is evolved at the site of the reaction. One must integrate the product of flux density, reaction cross section and energy transmitted in one reaction over the respective energy spectrum:

$$H_n(\mathbf{r}) = \int_0^\infty \Phi_n(E_n, \mathbf{r})\Sigma_n(E_n, \mathbf{r})h_n(E_n)\,\mathrm{d}E_n$$

$$H_\gamma(\mathbf{r}) = \int_0^\infty \Phi_\gamma(E_\gamma, \mathbf{r})\Sigma_\gamma(E_\gamma, \mathbf{r})h_\gamma(E_\gamma)\,\mathrm{d}E_\gamma$$

In practice, however, computation is difficult, since the functions flux density *vs.* energy and cross section *vs.* energy which are at disposal should be considered as fairly correct approaches only. The task is all the more difficult, because large-size shieldings are being made almost exclusively of concrete, and this material requires great accuracy of computation, owing to its low thermal conductivity and to material characteristics being largely dependent on temperature. To calculate the temperature field, the heat source distribution must be integrated twice by site, and hence a moderate error in the heat source distribution will result in a large error in the temperature field. Further integration required to calculate the thermal stress field will cause a further increase in the error of calculation.

To follow the non-elastic scattering of neutrons and their activation capture is more difficult, in principle too: neutron and gamma processes must be handled simultaneously. (From the solution of the neutron task, the secondary gamma source distribution can be obtained, and this will provide the gamma thermal source distribution. This is how e.g. the well-known SABINE3 shielding code proceeds [3, 4]).

In Table 1, the composition of some concretes used in shielding is listed; the composition is calculated from the data in [5] and [6]. In Table 2, the macroscopic cross section data of the components listed in Table 1 are grouped according to the above-mentioned reactions: for neutron reactions, calculated from the corresponding data in [7, 8, 9, 10], for gamma reactions according to [1] and [11]. For a given reaction, the macroscopic effective cross section will be obtained as the sum of the products of the partial densities and the corresponding cross sections.

Table 1

Chemical composition of concretes for radiation shielding (kg/m^3)

Component	Ordinary	Iron additive	Ferro-phosphoric	Magnetite	Baryte	Serpentinitic
1 Hydrogen	23.0	19.5	22.6	24.8	15.2	28.3
6 Carbon	2.3	—	—	—	—	—
8 Oxygen	1216.7	345.7	320.4	1333.4	1102.0	1081.3
11 Sodium	36.8	—	—	—	4.6	—
12 Magnesium	4.6	7.7	5.7	31.7	13.5	345.4
13 Aluminium	77.8	19.5	16.3	22.0	20.2	32.3
14 Silicon	775.1	54.0	41.3	98.3	26.2	414.5
15 Phosphorus	—	—	1265.4	5.2	—	—
16 Sulfur	—	2.9	—	—	353.1	—
19 Potassium	29.9	—	—	—	—	—
20 Calcium	101.2	235.0	173.6	223.6	160.9	171.6
25 Manganese	—	20.7	—	—	2.4	—
26 Iron	32.2	5195.0	2954.7	2261.0	311.9	126.6
56 Barium	—	—	—	—	1490.0	—
Total	2300.0	5900.0	4800.0	4000.0	3500.0	2200.0

The contribution of the individual components to total heat evolution can be estimated from weighting the cross sections of each reaction by the corresponding fluxes. The computation, e.g., from the external surface data of the reactor vessel in the case of the reactor WWER-440 [12] will yield a neutron flux consisting of

— thermal neutrons: $2.2 \times 10^{15}\ \mathrm{m^{-2}\ s^{-1}}$
— epithermal neutrons (0.4 eV to 0.4 MeV): $1.2 \times 10^{15}\ \mathrm{m^{-2}\ s^{-1}}$
— fast neutrons (0.7 to 2.5 MeV): $2.2 \times 10^{14}\ \mathrm{m^{-2}\ s^{-1}}$

and a gamma flux

— (converted to 1 MeV): $2.5 \times 10^{16}\ \mathrm{m^{-2}\ s^{-1}}$.

Table 2
Macroscopic effective cross sections for 1 kg/m³ density (m⁻¹)

Component	Neutron reactions						Total gamma cross sect. 1 MeV
	Elastic scattering		Activating capture		Charged particle emission	Nonelastic scattering	
	thermal	1 MeV	thermal	epithermal	fission spectrum		
1 Hydrogen	1.21	2.45E-1*	1.98E-2	8.90E-3	0	0	1.26E-1
6 Carbon	2.36E-2	1.28E-2	2.01E-5	7.52E-3	1.84E-4	6.68E-5	6.35E-2
8 Oxygen	1.41E-2	1.62E-2	6.70E-7	1.72E-5	2.50E-8	1.02E-5	6.36E-2
11 Sodium	8.12E-3	9.22E-3	1.40E-3	8.12E-4	5.94E-6	1.21E-3	6.09E-2
12 Magnesium	8.67E-3	7.90E-3	1.56E-4	9.24E-5	3.15E-6	8.47E-4	6.28E-2
13 Aluminium	3.13E-3	6.85E-3	5.38E-4	5.47E-4	1.06E-5	8.14E-4	6.13E-2
14 Silicon	4.72E-3	6.86E-3	3.43E-4	1.98E-4	6.91E-5	4.98E-4	6.34E-2
15 Phosphorus	6.81E-3	6.22E-3	3.69E-4	1.56E-4	7.00E-5	0	6.17E-2
16 Sulfur	5.64E-3	2.72E-3	9.77E-4	1.20E-3	3.55E-8	0	6.35E-2
19 Potassium	3.08E-3	3.20E-3	3.19E-3	1.74E-3	2.95E-7	1.42E-4	6.19E-2
20 Calcium	4.51E-3	4.81E-3	6.61E-4	2.81E-3	3.65E-8	0	6.36E-2
25 Manganese	3.73E-3	2.52E-3	1.46E-2	1.53E-2	7.32E-7	0	5.82E-2
26 Iron	1.23E-2	2.73E-3	2.73E-3	1.50E-3	1.06E-3	7.29E-4	5.96E-2
56 Barium	3.51E-3	2.89E-3	5.26E-4	6.15E-3	1.17E-8	1.10E-5	5.70E-2

* 2.45E-1 $=2.45\times10^{-1}$

Thermal neutrons do not transmit energy *directly* to the shielding (thermal equilibrium). Nor do epithermal neutrons play any significant role in *direct* energy transmission since

$$h_n=\frac{A}{(A+1)^2}E_n\,,$$

the energy transmitted will decrease steeply with increasing mass numbers A of the scattering nucleus, and will amount, in the case of concretes, to an average value of only some percents. It may therefore be seen from the magnitude of the above data that *heat evolution within the shielding originates mainly from gamma radiation.*

The objective in shielding dimensioning for the *attenuation of radiation* is an accurate determination of the radiation *emerging* from the shielding, that is, an accurate result shall be obtained at great thicknesses, *low fluxes*. It is of secondary importance whether the model describes the radiation field within the shielding correctly. (The correction, e.g., of the straightahead gamma scattering—simple exponential attenutation—by a buildup factor [1, 13] gives results suited only to characterize the conditions *behind* the shielding.) On the other hand, in dimensioning from the *thermal engineering view*, the reactions in the *interior* of the shielding, close to the radiation source, yielding *high fluxes*, are of major importance. To reconcile these differing requirements is a difficult task touching the problem of process modelling.

The consequences of inaccuracies will be demonstrated by the following example. The thermal conductivity of concrete is only a fraction of that of steel, so that a temperature distribution similar to that in steel will result in concrete at much lower radiation source intensities. *Thermal shield* is a specially directed part of protecting vessel-type reactors (e.g. PWR, BWR). Its objective is to reduce the stresses in the internal layers of the vessel wall and the concrete shielding by attenuating the neutron and gamma radiation of the reactor. Owing to the uncertainties of the source data and to the neglections applied in the model, in a given case the thermal source density to be expected

— at the internal surface of a 0.1 m thick steel thermal shield is 200 ± 80 MW/m^3,

— at its external surface 7 ± 3 MW/m^3.

Let us consider the two extreme cases: for 280 and 10 MW/m^3 and 120 and 4 MW/m^3, respectively, the difference is so great that the *surplus* thermal strain at the internal surface is about 78 N/mm^2 in the first case, a value half of the hot creep! Maximum temperature at a depth of 0.06 m is about 35 °C higher than the surface temperature.

The temperature at the internal surface of the shielding of the WWER-440 reactor is about 100 °C higher than the temperature of the primary circuit. For this reason this part of the shielding is made of a particular, heat-resistant (serpentinitic) concrete [12]. The accuracy of the calculations must ensure that the temperature of this layer will not rise above the allowed value, which is only about 25 °C higher than the calculated value.

Calculation of the temperature field

The general form of the differential equation for the temperature field is the so-called temperature-dependent thermal conductivity equation:

$$\frac{1}{c\rho} \nabla(\lambda \nabla T) - \frac{\partial T}{\partial t} = - \frac{H}{c\rho} \tag{1}$$

In the general case the task can be solved numerically only. Owing to the very low thermal conductivity of concrete, spatial integration can be performed with very dense gridding only, and for this reason the computer requirement is extremely high, even in the steady-state case (e.g. a memory of 1 MByte [14]). The step of integration with respect to time is also very small, in conformity with the spatial grid, and therefore the computation takes a very long time.

For these reasons it seemed indicated to develop a fast program with low memory requirement for approach calculations by simplifying the task. For the steady state, if the temperature dependence of thermal conductivity is neglected, Eq. (1) will assume the following form:

$$\Delta T = -\frac{H}{\lambda}. \tag{2}$$

In the case of simple geometry and simple heat source distribution, this latter differential equation can be solved *analytically* too, and the program must only list the solution function in a table.

The simplifications chosen: the computations consider a model in which the geometry of the shielding is either an infinite plate or an infinite cylindrical shell; the heat source distribution is approached in the solution of Eq. (2) either by a polynomial or by an exponential series.

The integration constants of the solution function depend on the boundary conditions (Fig. 1). In conformity with the cases important in practice, constant temperature is envisaged at the surface directed towards the radiation source: when $x=0$ or $r=r_0$, resp., $T=T_0$. At the external surface of the shielding we can choose between two kinds of boundary conditions: when $x=x_1$ or $r=r_1$, resp., then

— as boundary condition of the first kind, constant temperature can be stipulated: $T=T_1$; or else

— as boundary conditions of the third kind, constant heat transfer coefficient α and constant ambient temperature T_∞ ("temperature at an infinite distance") can be stipulated.

Plate and polynomial approach

The equation of thermal conductivity is

$$\frac{d^2T}{dx^2} = -\frac{1}{\lambda}\sum_{i=0}^{n} H_i x^i$$

The solution of the the equation is

$$T = -\frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i x^{i+2}}{(i+1)(i+2)} + C_1 x + C_2$$

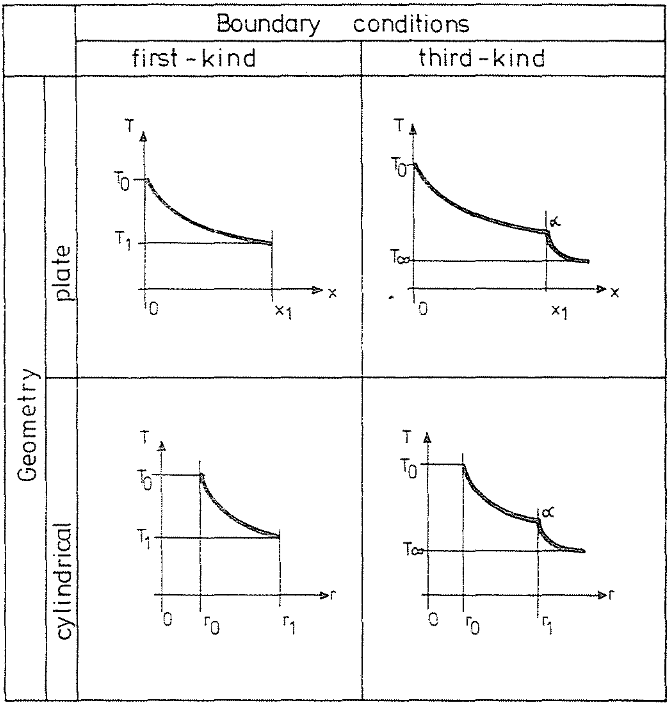

Fig. 1

The integration constants, in the case of the boundary condition of the first kind, are

$$C_1 = \frac{1}{x_1}\left(T_1 - T_0 + \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i x_1^{i+2}}{(i+1)(i+2)}\right)$$

$$C_2 = T_0$$

The integration constants, in the case of the boundary conditions of the third kind, are

$$C_1 = \frac{(T_\infty - T_0)\alpha + \frac{\alpha}{\lambda}\sum_{i=0}^{n}\frac{H_i x_1^{i+2}}{(i+1)(i+2)} + \sum_{i=0}^{n}\frac{H_i x_1^{i+1}}{i+1}}{\alpha x_1 + \lambda}$$

$$C_2 = T_0$$

Plate and exponential series approach

The equation of thermal conductivity is

$$\frac{d^2T}{dx^2} = -\frac{1}{\lambda}\sum_{i=1}^{n} H_i \exp(\mu_i x)$$

The solution of the equation is

$$T = -\frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}\exp(\mu_i x) + C_1 x + C_2$$

The integration constants, in the case of the boundary condition of the first kind, are

$$C_1 = \frac{1}{x_1}\left(T_1 - T_0 + \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}(\exp(\mu_i x_1) - 1)\right)$$

$$C_2 = T_0 + \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}$$

The integration constants, in the case of the boundary conditions of the third kind, are

$$C_1 = \frac{\sum_{i=1}^{n}\frac{H_i}{\mu_i}\exp(\mu_i x_1) + \left[\frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}(\exp(\mu_i x_1) - 1) + T_1 - T_0\right]\alpha}{\alpha x_1 + \lambda}$$

$$C_2 = T_0 + \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}$$

Cylinder and polynomial approach

The equation of thermal conductivity is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{1}{\lambda}\sum_{i=0}^{n} H_i r^i$$

The solution of the equation is

$$T = C_1 \ln\left(\frac{r}{r_1}\right) + C_2 - \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2} r^{i+2}$$

The integration constants, in the case of the boundary condition of the first kind, are

$$C_1 = \frac{\left[T_0 + \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2}r_0^{i+2}\right] - \left[T_1 + \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2}r_1^{i+2}\right]}{\ln\left(\frac{r_0}{r_1}\right)}$$

$$C_2 = T_1 + \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2}r_1^{i+2}$$

In the case of third-kind boundary conditions, the integration constants will be obtained by solving the following linear system of equations:

$$\begin{bmatrix} \ln\left(\frac{r_0}{r_1}\right) & 1 \\ \frac{\lambda}{r_1} & \alpha \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2}r_0^{i+2} + T_0 \\ \sum_{i=0}^{n}\frac{H_i}{i+2}r_1^{i+1} + \frac{\alpha}{\lambda}\sum_{i=0}^{n}\frac{H_i}{(i+2)^2}r_1^{i+2} + \alpha T_\infty \end{bmatrix}$$

Cylinder and exponential series approach

The equation of thermal conductivity is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = -\frac{1}{\lambda}\sum_{i=1}^{n} H_i \exp(\mu_i r)$$

The solution of the equation is

$$T = \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}\left[E_1(\mu_i r) - \exp(\mu_i r)\right] + C_1 \ln\left(\frac{r}{r_1}\right) + C_2$$

In the solution function E_1 is the first-kind integral-exponential function (cf. *Appendix*). The integration constants, in the case of first-kind boundary conditions, are

$$C_1 = \frac{T_0 - \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}\left[E_1(\mu_i r_0) - \exp(\mu_i r_0)\right]}{\ln\left(\frac{r_0}{r_1}\right)} - \frac{T_1 - \frac{1}{\lambda}\sum_{i=1}^{n}\frac{H_i}{\mu_i^2}\left[E_1(\mu_i r_1) - \exp(\mu_i r_1)\right]}{\ln\left(\frac{r_0}{r_1}\right)}$$

$$C_2 = T_1 - \frac{1}{\lambda} \sum_{i=1}^{n} \frac{H_i}{\mu_i^2} [E_1(\mu_i r_1) - \exp(\mu_i r_1)]$$

In the case of third-kind boundary conditions, the integration constants will be obtained by solving the following linear system of equations:

$$\begin{bmatrix} \ln\left(\frac{r_0}{r_1}\right) & 1 \\ \frac{\lambda}{r_1} & \alpha \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\lambda} \sum_{i=1}^{n} \frac{H_i}{\mu_i^2} [E_1(\mu_i r_0) - \exp(\mu_i r_0)] \\ \alpha T_\infty - \sum_{i=1}^{n} \frac{H_i}{\mu_i} \left(\frac{1}{\mu_i r_1} - 1\right) \exp(\mu_i r_1) - \frac{\alpha}{\lambda} \sum_{i=1}^{n} \frac{H_i}{\mu_i^2} [E_1(\mu_i r_1) - \exp(\mu_i r_1)] \end{bmatrix}$$

Calculation of the stress field

The "thermal stresses" are caused by the force which hinders the dimensional changes of the body due to temperature changes. This force may be an external force (a typical example is a bar clamped at both ends, like e.g. a rail), or else—in the case of locus-dependent stress fields—the constraint exerted by other parts of the body itself.

The calculation of the stress field is necessary, above all, in reactor vessels (very high heat source density, high stresses even in the isothermal case, owing to the high pressures of operation), in thermal shieldings (very high heat source density) and in concrete shieldings of nuclear reactors (high heat source density, unfavourable thermal and mechanical properties of concrete).

If Hooke's law is valid, the elongation per unit length in the interior of a material, in terms of the coordinate system corresponding to the main stress directions, can be calculated, if the main stresses and thermal expansion are known:

$$\varepsilon_i = \frac{1}{E}(\sigma_i - \sigma_j - \sigma_k) + (T - T_\infty)\beta \tag{3}$$

Elongation per unit length ε being the ratio of the dislocation of the end point of a given length to its original length, and dislocation, if no constraint were acting on the body, being the integral by locus of $(T - T_\infty)\beta$, the main stresses can readily be calculated from Eq. (3). Buckling of the cylindrical shells being out of question in our tasks, one may assume, based on their symmetry, that the thermal expansion of the internal layers is constrained by the external layers, and the constraining force will be characterized by the average temperature weighted by cross sectional area:

$$\overline{T(r)} = \frac{\int_{r_0}^{r} 2\pi r\, T(r)\, dr}{\int_{r_0}^{r} 2\pi r\, dr}$$

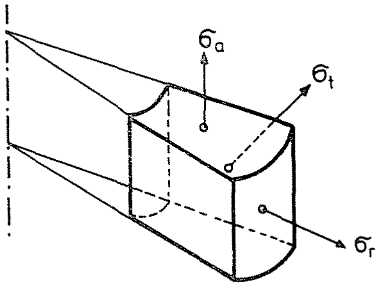

Fig. 2

Consequently, the three main stresses (as indicated in the coordinate system presented in Fig. 2), can be expressed by the following equations:

$$\sigma_r = \frac{\beta E}{1-\nu}\left[\left(1-\left(\frac{r_0}{r}\right)^2\right)\frac{\overline{T(r_1)}}{2} - \overline{T(r)}\right]$$

$$\sigma_t = \frac{\beta E}{1-\nu}\left[\left(1-\left(\frac{r_0}{r}\right)^2\right)\frac{\overline{T(r_1)}}{2} - \overline{T(r)} + T(r)\right]$$

$$\sigma_a = \frac{\beta E}{1-\nu}\left[\overline{T(r_1)} - \overline{T(r)}\right]$$

Plate-shaped shieldings may be considered as extreme cases of cylinder superficies, and therefore the above formulas can be used for their description, at the transition boundary $r_0/r_1 \to 1$.

Thermal stresses in the shielding

In the general case, it is not Eq. (2), but Eq. (1) that will describe the temperature field developing in the shielding. The solution of the task will be obtained by numerical computations fitted for the given task. None the less, qualitative conclusions can also be made from the above-discussed simplified model.

There is no general rule as to what are the operating conditions at which the shielding is subjected most of all to thermal stresses. The major aspects of the studies might be the following:

(i) if the shielding is thick and hence heat source strength will be important in a part of it only, the temperature field in the external parts will be similar to the case when heat sources are absent in the shielding. Close to the radiation source the temperature of the shielding will be significantly higher, consequently important thermal stress should be expected in the vicinity of the internal surface.

(ii) if the shielding is thin, both the internal and the external surface can be dangerous.

(iii) Close to the internal surface, the tangential stress σ_t and the axial stress σ_a are compressive stresses, close to the external surface they are tensile stresses. With materials like *concrete* where these stress properties are differing, it is necessary to provide for the required loadability by means of suitable design of the structure.

(iv) In the non-steady state the temperature field will be distorted, and in particular, this distortion will be different at the start and at the shut-down [1, 15]. In these periods exceedingly high thermal stresses may arise.

(v) Under the effect of radiation the properties of the shielding material will deteriorate at the internal surface (rigidity, decrease of thermal conductivity due to microcracks caused by recrystallization, etc.). Therefore the permissible stresses determined in the design of the shielding should be low.

(vi) Due to the high temperature, concrete may lose the part of its water content which is not chemically bonded, and hence its neutron-attenuating properties might deteriorate dramatically. Suitable design of the structure (e.g. cladding the concrete shielding with steel plate) must provide for protection.

Characterization of the TESTRE program [2]

The TESTRE ("*thermal stress*") program is based on the model discussed in the preceding chapters. Its purpose is to calculate the stress field developing in the shielding. The program involves the case of

— one-dimensional, plane or cylindrical geometry, and

— steady-state thermal conduction.

The simplifications applied are

— independence of thermal conductivity from temperature,

— validity of Hooke's law, and

— no buckling of the cylindrical shielding.

The spatial distribution of the heat sources must be determined in the course of other computations of the shielding, and subsequently its description is approached by

— a polynomial or

— by an exponential series

to compose the input of the program.

The program was made for an IBM 360/40 computer in the FORTRAN IV (F) language. It can be run in the DOS or OS operation system. The memory required is about 50 KByte. The computation time depends on the data, but does not exceed a few minutes (DOS) at any data combination.

List of symbols

A	mass number
c	specific heat
E	modulus of elasticity
E_n	neutron energy
E_γ	gamma energy
h_n	energy transmitted in one neutron reaction
h_γ	energy transmitted in one gamma reaction
H_n	heat source density of neutron reactions
H_γ	heat source density of gamma reactions
r	radius
$\mathbf{r}$	locus vector
r_0	internal radius of cylindrical shielding
r_1	external radius of cylindrical shielding
t	time
T	temperature
T_0	surface temperature of the shielding towards the source
T_1	surface temperature of the shielding away from the source
T_∞	ambient ("infinitely distant") temperature
x	thickness coordinate
x_1	thickness of plate-shaped shielding
α	coefficient of heat transfer
β	coefficient of thermal expansion
ε	elongation per unit length
Φ_n	neutron flux density
Φ_γ	gamma flux density
λ	thermal conductivity
ν	*Poisson*-coefficient
ρ	density
σ	tensile stress
Σ_n	macroscopic effective neutron cross section
Σ_γ	macroscopic effective gamma cross section
∇	*nabla*-operator
Δ	*Laplace*-operator

Appendix

Integral-exponential functions

Integral-exponential functions are defined by the following formula:

$$E_n(x)=x^{n-1}\int_x^{\infty}\frac{\exp(-y)}{y^n}\,dy\,.$$

The recursive rule of their formation is

$$E_n(x)=\frac{1}{n-1}\left[\exp(-x)-xE_{n-1}(x)\right].$$

Due to recursivity, it is sufficient to calculate the improprius integral defining the functions for the case $n=1$. Expressing the exponential function with its series, one obtains

$$E_1(x)=\int_x^{\infty}\sum_{i=0}^{\infty}\frac{1}{y}\frac{(-y)^i}{i!}\,\mathrm{d}y=$$

$$=\int_x^{\infty}\frac{\mathrm{d}y}{y}+\int_x^{\infty}\sum_{i=1}^{\infty}\frac{(-y)^{i-1}}{i!}\,\mathrm{d}y=$$

$$=-\gamma-\ln(|x|)-\sum_{i=1}^{\infty}\frac{(-x)^i}{i!i}\,.$$

In this context, γ is *Euler*'s constant ($\gamma\approx 0.5772156649$).

Special case:

$$E_0(x)=\frac{\exp(-x)}{x}\,.$$

Summary

The heat sources developing in the shieldings of radiation sources are presented. Approaching their locus dependence by a polynomial or by an exponential series, a differential equation for thermal conductivity will be obtained which can be solved analytically. In the knowledge of the temperature field the thermal stress field can be calculated. A computer program termed TESTRE ("*t*hermal *stre*ss") was developed which provides a satisfactory approach of the thermal stress field.

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