THE SUBTERRANEAN ELECTROMAGNETIC FIELD OF POWER TRANSMISSION LINES

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Introduction

The electromagnetic field of a power transmission line produces disturbing signals in cables planted, e.g. for telecommunicational purposes, in earth. The disturbing signal is mainly influenced by the (longitudinal) component of the electric field intensity parallel with the axis of the transmission line. The calculation of this component is discussed in the following.

The electromagnetic field of a power transmission line can be obtained by superposition from the field of an earth-return line. Therefore, the calculation of an earth-return transmission line—a system consisting of a horizontal conductor and earth—will be discussed. The earth is presumed to be bounded by a plane, its conductivity σ_e, permittivity ε_e and permeability μ_0 are assumed

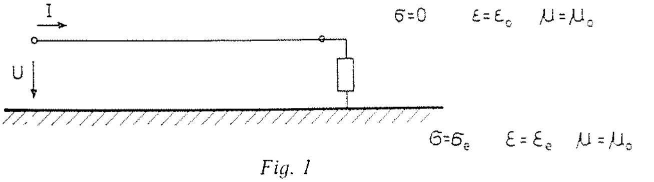

Fig. 1

to be constant, while air is considered to be ideal isolator. The cylindrical conductor of conductivity σ_c and radius a stretches parallel with the surface of earth at a height h, is surrounded by air and carries a sinusoidal current of angular frequency ω and complex amplitude I (Fig. 1).

General formulas of electromagnetic field

The electromagnetic field results from a Sommerfeld surface wave-guide and from an additional field caused by the presence of earth.

The electromagnetic field over earth is composed of the superposition of three fields: the TM field of the conductor current without earth, the TE and

TM fields due to the presence of earth, while the field in earth is combined of TE and TM fields.

The TM mode electric and magnetic field intensities are expressed with the aid of Hertz-vector $\mathbf{\Pi}^e$ as follows:

$$\mathbf{E} = \text{grad div } \mathbf{\Pi}^e - k^2 \mathbf{\Pi}^e , \tag{1}$$

$$\mathbf{H} = (\sigma + j\omega\varepsilon) \text{ rot } \mathbf{\Pi}^e , \tag{2}$$

where σ is the conductivity, ε the permittivity, μ the permeability of the medium, k is the propagation coefficient and

$$k^2 = (\sigma + j\omega\varepsilon) j\omega\mu \tag{3}$$

The conductor current gives rise to a Sommerfeld surface wave. The Hertz-vector $\mathbf{\Pi}_c^e$ corresponding to this TM mode wave is of direction z in the cylindrical coordinate system attached to the form of the conductor, and its amplitude is

$$\mathbf{\Pi}_c^e = \frac{C}{g^2} H_0^{(1)}(gr) e^{\pm \gamma z + j\omega t} , \tag{4}$$

where

$$C = \frac{I}{2\pi a} \frac{g}{j\omega\varepsilon_0} \frac{1}{H_1^{(1)}(ga)} . \tag{5}$$

$H_0^{(1)}(gr)$ and $H_1^{(1)}(gr)$ are zero and first order Hankel-functions of the first kind,

$$g^2 = \gamma^2 - k^2 , \tag{6}$$

and γ is the propagation coefficient in direction z. The negative sign of γ in (4) corresponds to a wave travelling in direction z, while the positive sign to one propagating in direction $-z$. In the following, the equations of the wave travelling in direction z will be written.

In air

$$k^2 = k_0^2 = j\omega\mu_0 j\omega\varepsilon_0 , \tag{7}$$

and at the determination of g the root with $\text{Im } g > 0$ should be chosen.

To write the additional field in air due to the presence of earth the Cartesian coordinate system shown in Fig. 2 is chosen. To satisfy the boundary conditions, the solution is written in the form of a Fourier-integral. Taking the symmetry of the arrangement into account, only even functions of x are

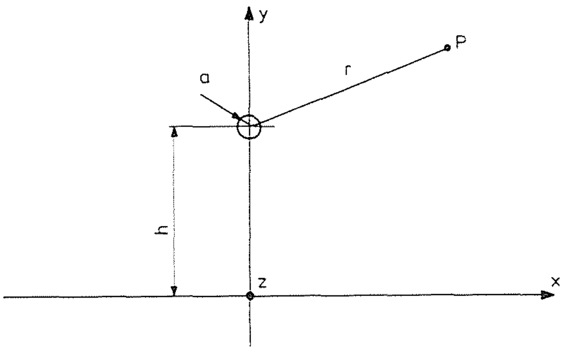

Fig. 2

considered at the solution. The TM mode Hertz-vector $\mathbf{\Pi}_0^e$ is of direction z, and its amplitude is:

$$\Pi_0^e = \int_0^{+\infty} G^e(\alpha) \cos(\alpha x) \, e^{-y\sqrt{\alpha^2-g^2}} \, e^{j\omega t - \gamma z} \, d\alpha \,, \tag{8}$$

where $\sqrt{\alpha^2 - g^2}$ is of nonnegative real part, and the Fourier amplitude function $G^e(\alpha)$ is determined with the aid of the boundary conditions. Writing $\mathbf{\Pi}_0^e$ instead of $\mathbf{\Pi}^e$, (1) and (2) yield the electric and magnetic field intensities of the additional TM field.

The TE field due to the presence of earth is derived from the magnetic Hertz-vector $\mathbf{\Pi}^m = \Pi^m \mathbf{e}_z$ as

$$\mathbf{E} = -j\omega\mu \operatorname{rot} \mathbf{\Pi}^m \,, \tag{9}$$

$$\mathbf{H} = \operatorname{grad} \operatorname{div} \mathbf{\Pi}^m - k^2 \mathbf{\Pi}^m \,. \tag{10}$$

The Hertz-vector above earth is:

$$\Pi_0^m = \int_0^{+\infty} G^m(\alpha) \sin(\alpha x) e^{-y\sqrt{\alpha^2-g^2}} \, e^{j\omega t - \gamma z} \, d\alpha \,. \tag{11}$$

The meaning and the way of determination of $G^m(\alpha)$ are similar to those of $G^e(\alpha)$. The x-dependence of Π_0^m has been chosen to ensure that the symmetry of the electromagnetic field is similar to that of the TM mode.

The electromagnetic field in earth

The electromagnetic field in earth can also be written as a sum of a TM and a TE mode. The Hertz-vector corresponding to the TM mode is of direction z, and its amplitude is

$$\Pi_e^e = \int_0^{+\infty} F^e(\alpha)\cos(\alpha x)\, e^{y\sqrt{\alpha^2-f^2}}\, e^{j\omega t-\gamma z}\,\mathrm{d}\alpha\,, \tag{12}$$

where the meaning and the way of determination of $F^e(\alpha)$ are similar to those of $G^e(\alpha)$, and

$$f^2=\gamma^2-k_e^2\approx -k_e^2 \tag{13}$$

$$k_e^2=j\omega\mu_0\sigma_e \tag{14}$$

$\mathbf{\Pi}_e^e$ yields the TM mode electric and magnetic fields in earth according to (1) and (2), by writing $\mathbf{\Pi}_e^e$ instead of $\mathbf{\Pi}^e$, σ_e instead of $\sigma+j\omega\varepsilon$ and k_e instead of k.

The Hertz-vector $\mathbf{\Pi}_e^m$ corresponding to the TE mode is also of direction z with its amplitude:

$$\Pi_e^m = \int_0^{+\infty} F^m(\alpha)\sin(\alpha x) e^{y\sqrt{\alpha^2-f^2}}\, e^{j\omega t-\gamma z}\,\mathrm{d}\alpha\,. \tag{15}$$

The meaning and the way of determination of $F^m(\alpha)$ are also similar to those of $G^e(\alpha)$. The TE mode electric and magnetic fields are hence determined according to (9) and (10).

Four independent boundary conditions relate to the components of the electric and magnetic fields on the surface of earth. In the following, the continuity of E_z, H_x, H_y and H_z will be used. Writing the field components as Fourier integrals, the equations for the Fourier coefficients yield:

$$-g^2G^e(\alpha)+f^2F^e(\alpha)=\frac{C}{\pi}P(\alpha)\,, \tag{16}$$

$$-j\omega\varepsilon_0\alpha G^e(\alpha)+\sigma_e\alpha F^e(\alpha)-\gamma\left(\sqrt{\alpha^2-g^2}+\right.$$

$$\left.+\frac{g^2}{f^2}\sqrt{\alpha^2-f^2}\right)G^m(\alpha)=\frac{C}{\pi}\alpha\frac{j\omega\varepsilon_0}{g^2}P(\alpha)\,, \tag{17}$$

$$j\omega\varepsilon_0\sqrt{\alpha^2-g^2}\,G^e(\alpha)+\sigma_e\sqrt{\alpha^2-f^2}\,F^e(\alpha)+\gamma\alpha\left(1-\frac{g^2}{f^2}\right)G^m(\alpha)=$$

$$=\frac{C}{\pi}\frac{j\omega\varepsilon_0}{g^2}\sqrt{\alpha^2-g^2}\,P(\alpha)\,, \tag{18}$$

where

$$P(\alpha)=\int_{-\infty}^{+\infty} H_0^{(1)}(g\sqrt{x^2+h^2})\cos(\alpha x)\,\mathrm{d}x=$$

$$=\frac{2}{j}\frac{e^{-h\sqrt{\alpha^2-g^2}}}{\sqrt{\alpha^2-g^2}}\,. \tag{19}$$

$G^e(\alpha)$, $G^m(\alpha)$ and $F^e(\alpha)$ can be determined from these equations. To calculate the propagation coefficient, the expression of $G^e(\alpha)$ is necessary:

$$G^e(\alpha)=-\frac{1}{g^2}\frac{C}{\pi}T(\alpha)\,P(\alpha)\,, \tag{20}$$

and the approximate expression

$$T(\alpha)\approx 1-\frac{2k_0^2}{g^2f^2}(\sqrt{\alpha^2-g^2}\sqrt{\alpha^2-f^2}-\alpha^2) \tag{21}$$

can be obtained for $T(\alpha)$.

The electromagnetic field in earth can be derived from the Hertz-vectors written. The z component of the electrical field is calculated as:

$$E_z=f^2\int_0^{+\infty} F^e(\alpha)e^{y\sqrt{\alpha^2-f^2}}\,e^{j\omega t-\gamma z}\cos(\alpha x)\,\mathrm{d}\alpha\,. \tag{22}$$

Using (16) and (18)

$$E_z=\frac{4C}{j\pi}\frac{k_0^2}{g^2f^2}e^{j\omega t-\gamma z}\int_0^{+\infty} e^{-h\sqrt{\alpha^2-g^2}}\,e^{y\sqrt{\alpha^2-f^2}}\left(\sqrt{\alpha^2-f^2}-\right.$$

$$\left.-\frac{\alpha^2}{\sqrt{\alpha^2-g^2}}\right)\cos(\alpha x)\,\mathrm{d}\alpha\,,\qquad y\leqq 0 \tag{23}$$

is obtained. An algorithm for the evaluation of the impropriate integral appearing here is presented in the following.

Algorithm

The following factors have been taken into account at the preparation of the algorithm for the numerical evaluation of the expression (23).

In the practical cases examined by us $|ga|<10^{-4}$, and thus the small argument approximation

$$H_1^{(1)}(ga)\approx\frac{2j}{\pi ga}\qquad |ga|\ll 1 \tag{24}$$

for the Hankel-function can be used in the expression (4) of the constant C depending on excitation.

The relationship

$$e^{-h\sqrt{\alpha^2-g^2}}\approx e^{-h\alpha} \tag{25}$$

can be employed in the integrand of (23), since if $\alpha^2<10g^2$ for practical values $e^{-h\sqrt{\alpha^2-g^2}}\approx 1\approx e^{-h\alpha}$. If however $\alpha^2\geqq 10g^2$, then $\sqrt{\alpha^2-g^2}\approx\alpha$.

Using these, (23) can be written as follows:

$$E_z\approx-\frac{I}{\pi\sigma_e}e^{j\omega t-\gamma z}\int\limits_0^{+\infty}e^{-h\alpha}\,e^{y\sqrt{\alpha^2-f^2}}\cos(\alpha x)\left(\sqrt{\alpha^2-f^2}-\right.$$

$$\left.-\frac{\alpha^2}{\sqrt{\alpha^2-g^2}}\right)\mathrm{d}\alpha\,. \tag{26}$$

The evaluation of (26) has been carried out by numerical integration with the following considerations. If

$$\alpha^2>n|f^2|,\qquad n>10 \tag{27}$$

then

$$\sqrt{\alpha^2-\mathrm{f}^2}\approx\alpha\,, \tag{28}$$

and since $|\mathrm{f}^2|\gg|g^2|$,

$$\frac{\alpha^2}{\sqrt{\alpha^2-\mathrm{g}^2}}\approx\alpha \tag{29}$$

is a good approximation. Therefore, if (27) holds:

$$\sqrt{\alpha^2-f^2}-\frac{\alpha}{\sqrt{\alpha^2-g^2}}\approx 0\,. \tag{30}$$

The other factor in the integrand is well approximated by $e^{-(h+y)\alpha} \cdot \cos(\alpha x)$ in case (27) holds, and the latter tends to zero exponentially if α increases.

Let us introduce the denotation

$$\sqrt{|f^2|} = p = \sqrt{\omega \mu_0 \sigma_e}\,. \tag{31}$$

Hence (26) can be written as

$$E_z \approx -\frac{I}{\pi \sigma_e} \sum_{i=0}^{\infty} \int_{inp}^{(i+1)np} e^{-h\alpha}\, e^{y\sqrt{\alpha^2+jp^2}} \cos(\alpha x) \left(\sqrt{\alpha^2+jp^2} - \frac{\alpha^2}{\sqrt{\alpha^2-g^2}} \right) \mathrm{d}\alpha\,. \tag{32}$$

The sum appearing here converges very fastly. Namely, according to the above considerations, its first term approximates it well. Therefore to evaluate the integral numerically, E_z is computed with the aid of a partial sum of the series:

$$E_z = -\frac{I}{\pi \sigma_e} \sum_{i=0}^{k} \int_{inp}^{(i+1)np} e^{-h\alpha}\, e^{y\sqrt{\alpha^2+jp^2}} \cos(\alpha x)\, v \left(\sqrt{\alpha^2+jp^2} - \frac{\alpha^2}{\sqrt{\alpha^2-g^2}} \right) \mathrm{d}\alpha\,. \tag{33}$$

The integrals in the terms of the series are evaluated by Simpson's formula:

$$K_i = \int_{inp}^{(i+1)np} e^{-h\alpha}\, e^{y\sqrt{\alpha^2+jp^2}} \cos(\alpha x) \left(\sqrt{\alpha^2+jp^2} - \frac{\alpha^2}{\sqrt{\alpha^2-g^2}} \right) \mathrm{d}\alpha =$$

$$= \int_{\alpha_{1i}}^{\alpha_{2i}} [F_r(\alpha) + jF_s(\alpha)]\, \mathrm{d}\alpha \approx$$

$$\approx \frac{\alpha_{2i} - \alpha_{1i}}{3N} \sum_{m=0}^{N} B(m)\, [F_r(\alpha_m) + jF_s(\alpha_m)]\,, \tag{34}$$

where N is even,

$$B(m)=\begin{cases} 1 & \text{if } m=0 \text{ or } m=N \\ 2 & \text{if } m \text{ is even and } m\neq 0,\ m\neq N \\ 4 & \text{if } m \text{ is odd} \end{cases} \tag{35}$$

$$\alpha_m = m\frac{\alpha_{2i}-\alpha_{1i}}{N}+\alpha_{1i} \tag{36}$$

To compute the integrals in (34) the value of N is doubled until the relative difference of two successive approximating sums is less than an error limit given in advance.

Thus, with the introduction of K_i, (33) can be written as

$$E_z = -\frac{I}{\pi\sigma_e}\sum_{i=0}^{k} K_i\,. \tag{37}$$

To attain a desired accuracy, the value of k cannot be given without the knowledge of the geometrical dimensions. Namely, the speed of convergence depends on the values of x, y and h, k is chosen to satisfy the condition

$$\frac{|K_k|}{\left|\sum_{i=0}^{k-1} K_i\right|}<\delta\,, \tag{38}$$

with δ being an error limit given in advance.

Numerical results

A program has been written on the basis of the above algorithm on a desk calculator type EMG 666. In the following, the amplitude of the z-component of the electric field intensity is presented in a few points x, y with

$$I=10^3 A; \qquad a=6\text{ mm};$$

$$\sigma_e=0.01\,\frac{S}{m}; \qquad \sigma_0=0$$

for $h=1$m and $h=6$m and frequencies $f=50$ Hz and $f=2$ kHz. The values of series impedance Z_s and parallel admittance Y_p of an earth-return transmission line known from previous calculations have been employed.

With $n=50$ and $\delta=10^{-3}$ the condition (38) was satisfied at $k=2$, $k=3$. The results are the following:

$f=50$ Hz	$h=1$ m				$h=6$ m				
x/m / $-y/m$	1	20	200	1000	1	20	200	1000	
0	0.413	0.247	0.107	0.0287	0.316	0.244	0.107	0.0287	V/m
0.5	0.396	0.246	0.107	0.0287	0.316	0.244	0.107	0.0287	V/m
1.0	0.382	0.246	0.107	0.0287	0.307	0.243	0.107	0.0287	V/m
1.5	0.370	0.246	0.107	0.0287	0.303	0.242	0.107	0.0287	V/m
$f=2$ kHz	$h=1$ m				$h=6$ m				
x/m / $-y/m$	1	20	200	1000	1	20	200	1000	
0	11.9	5.39	0.823	0.0335	8.32	5.34	0.839	0.0368	V/m
0.5	11.3	5.39	0.821	0.0326	8.12	5.32	0.837	0.0345	V/m
1.0	10.7	5.38	0.820	0.0322	7.94	5.30	0.836	0.0337	V/m
1.5	10.2	5.37	0.818	0.0321	7.77	5.28	0.834	0.0334	V/m

The values at $y=0$ have also been computed with the aid of the relationships valid for above earth. The results of the two calculations coincided with an accuracy of 1 per cent.

Summary

The calculation of the electromagnetic field in air of an overground transmission line is discussed. An improvement of the results found in the literature is presented in this paper, showing the equations of the electromagnetic field in the earth to be written in the form of impropriate integrals. An algorithm for their evaluation and some numerical results are also presented. The procedure can be applied—among others—to calculate the disturbing field arising in cables in earth.

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