CALCULATION OF BRAKING FORCE IN EDDY CURRENT BRAKES

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, Received March 2, 1982 Presented by Prof. Dr. I. VÁGÓ

Introduction

The eddy current braking of linear motors is usually required for obtaining moving force — velocity characteristics optimal from the point of view of a particular drive. This necessitates a braking device with prescribed characteristics which can be attained with the aid of braking poles of definite dimensions and of pole flux-density of appropriate value. This paper presents a procedure for the calculation of braking force at poles of given dimensions and at given flux-density. In the course of the solution, Ritz process based on variational principles is presented for the calculation of stationary conductive fields excited by motional induction, and a technique is formulated for treating singular excitations which improves the convergence of the numerical procedure. An alternative method based on an infinite number of images for the solution of the problem is also presented thus permitting the examination of the Ritz process. Finally, the results of the measurements carried out to check the calculations are presented.

1. Modelling of the problem, derivation of the describing equations

The scheme of the studied arrangement is shown in Fig. 1. A plate of width 2d, thickness s and conductivity σ moves between the poles excited by direct current at a uniform velocity v. The task is to determine the braking force acting on the plate. To this end, the eddy currents resulting from motional induction have to be calculated. The following simplifying presumptions are made for the solution:

The flux-density \mathbf{B}_0 of the magnetic field generated by the exciting coils is taken to be perpendicular to the plate and of constant magnitude B_0 under the poles, and zero outside the poles. The reaction of the eddy currents on the pole's flux-density is neglected, thus the electric field is solely induced by the fluxdensity \mathbf{B}_0 . In the direction perpendicular to the surface of the plate uniform



Fig. 1. Scheme of eddy current braking of linear motor

current distribution is assumed. The conductivity σ is considered to be constant and the longitudinal dimension of the plate to be infinite.

As a consequence of the simplifying presumptions, the electric field of eddy currents can be discussed in the coordinate system of the poles as a stationary conductive field generated by the impressed field intensity $\mathbf{E}_i =$ $= \mathbf{v} \times \mathbf{B}_0$ constant in time. In this coordinate system the current distribution and the vector \mathbf{E}_i of the impressed field intensity seem to be constant. The process is linear, thus the electric field belonging to one pair of poles will only be discussed in the following. The Maxwell equations describing the phenomenon are

$$\operatorname{div} \mathbf{J} = 0 \tag{1.1}$$

$$\operatorname{curl} \mathbf{E} = \mathbf{0} \tag{1.2}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i) \tag{1.3}$$

J, E and \mathbf{E}_i denote the vectors of the field generated by one pair of poles. The coordinate system is chosen as shown in Fig. 1. In this system, $\mathbf{v} = v\mathbf{a}_x$, $\mathbf{B}_0 = -B_0\mathbf{a}_z$, thus $\mathbf{E}_i = B_0v\mathbf{a}_y$ under the poles and zero elsewhere. Since uniform current distribution has been presumed in direction z, the problem can be discussed as a two-dimensional one in the plane xy. The following equation is derived for the scalar potential introduced as $\mathbf{E} = -\text{grad } \varphi$:

$$\Delta \varphi = \operatorname{div} \mathbf{E}_i \tag{1.4}$$

where \varDelta is the planar Laplace operator.

In the following, relative coordinates and dimensionless, relative quantities are introduced to simplify the numerical calculations and to obtain results independent of concrete dimensions. The relative coordinates are

$$\xi = \frac{x}{d} \tag{1.5}$$

$$\eta = \frac{y}{d} \tag{1.6}$$

In the system of relative coordinates the original region is modified as shown in Fig. 2, and it does not vary on the variation of geometrical dimensions which only affects the relative pole-dimensions a' and b'. The definitions of relative quantities are:



Fig. 2. The studied region in the system of relative coordinates

The relative potential and field intensity:

$$\varphi'(\xi,\eta) = \frac{\varphi(x,y)}{E_i d} \tag{1.7}$$

$$\mathbf{e} = -\operatorname{grad} \varphi' \tag{1.8}$$

The relative value of the impressed field intensity is $\mathbf{e}_i = \frac{\mathbf{E}_i}{E_i}$, thus

$$\mathbf{j} = \mathbf{e} + \mathbf{e}_i \tag{1.9}$$

where j denotes the relative current density.

The relative current flowing through a curve l' of the plane $\xi \eta$ is:

$$i = \int_{I'} \mathbf{j}(\mathbf{a}_{\zeta} \times \mathbf{d}\mathbf{l}') \tag{1.10}$$

 \mathbf{a}_{ζ} is the unit vector normal to the plane $\zeta \eta$. The relative power loss due to the two pairs of poles is:

$$p = 4 \int_{0}^{1} \int_{-\infty}^{\infty} |\mathbf{j}(\xi, \eta) - \mathbf{j}(\xi + 2c', \eta)|^2 \, \mathrm{d}\xi \, \mathrm{d}\eta$$
(1.11)

c' is here the relative value of the dimension c. In formula (1.8) as well as in the following the differential operators relate to relative coordinates. The relationships between the original and relative quantities are:

$$\mathbf{E} = E_i \mathbf{e} \tag{1.12}$$

$$\mathbf{J} = \sigma E_i \mathbf{j} \tag{1.13}$$

$$P = s\sigma d^2 E_i^2 p = s\sigma d^2 B_0^2 v^2 p \tag{1.14}$$

If the image of the curve l' is l in the plane xy, the current flowing through a cross section of the plane determined by the curve l is:

$$I = sd\sigma E_i i \tag{1.15}$$

(1.4) yields the equation

$$\Delta \varphi' = \operatorname{div} \mathbf{e}_i \tag{1.16}$$

for the relative potential function. The following boundary conditions should be prescribed on the boundary of the region for the determination of the potential function:

$$\frac{\partial \varphi'}{\partial \eta} = 0 \quad \text{at } \eta \pm 1 \tag{1.17}$$

$$\lim_{\xi \to \pm \infty} \mathbf{e} = \mathbf{0} \quad \text{for } -1 \le \eta \le 1 \tag{1.18}$$

In accordance with the definition of \mathbf{e}_i and the relationship $\mathbf{E}_i = B_0 v \mathbf{a}_y$, the right-hand side of (1.16) is zero except on the lines $-a' \le \le \xi \le a', \eta = \pm b'$, and it is singular along these lines. Let us exclude the singular locations from the region. Thus, the Laplace equation

$$\Delta \varphi' = 0 \tag{1.19}$$

is obtained for the potential function. However, the normal component of current density and the potential function have to be continuous along the lines excluded from considerations. This means the following conditions for φ' and e:

$$\lim_{\eta \to \pm b' = 0} \varphi'(\xi, \eta) = \lim_{\eta \to \pm b' \neq 0} \varphi'(\xi, \eta); \quad -a' \le \xi \le a'$$
(1.20)

$$\lim_{\eta \to b' = 0} e_{\eta}(\xi, \eta) - \lim_{\eta \to b' = 0} e_{\eta}(\xi, \eta) = -1; \quad -a' \le \xi \le a'$$
(1.21)

$$\lim_{\eta \to -b' = 0} e_{\eta}(\xi, \eta) - \lim_{\eta \to -b' + 0} e_{\eta}(\xi, \eta) = 1; \quad -a' \le \xi \le a'$$
(1.22)

So, the normal component of field intensity has a discontinuity of unit value along the singular lines.

It is known from the theory of electrostatic field that discontinuity of the normal component of the displacement vector or, in homogeneous media, of the normal component of the field intensity appears along surfaces with surface charge density. In accordance with the analogy between the stationary conductive field and the static electric field, the potential function φ' can be considered to be excited by surface charges of density

$$v = \varepsilon_0 \cdot l \left[\frac{V}{m} \right]$$

along the line $-a' \leq \xi \leq a', \eta = b'$ and

$$v = -\varepsilon_0 \cdot l \left[\frac{V}{m} \right]$$

along the line $-a' \leq \xi \leq a'$, $\eta = -b'$ and of infinite length in the direction perpendicular to the plane $\xi\eta$, with the boundary conditions (1.17) and (1.18), provided the permittivity of the medium in the analogous static field is taken to be ε_0 . Consequently, the potential field sought is excited by a singular arrangement of charges.

In the following, the relative potential, field intensity and power loss will be determined with the aid of the analogous static electric field.

2. Solution by variational method

The Ritz process based on variational principles serves for the numerical solution of boundary value problems of certain types [2, 3]. Let us presume that the operator equation described by

$$-\Delta u = f \tag{2.1}$$

in a region V bounded by the surface $S = S_1 + S_2$ and by the homogeneous mixed boundary conditions

$$u|_{S_1} = 0$$
 (2.2)

$$\left. \frac{\partial u}{\partial n} \right|_{S_2} = 0 \tag{2.3}$$

are to be solved. In (2.1) u and f are functions defined in the region V and $\frac{\partial u}{\partial n}$ denotes the derivative of u in the normal direction. According to Ritz process, the approximate solution is sought as an expansion

$$u_n = \sum_{k=1}^n a_k \varphi_k \tag{2.4}$$

The functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ are linearly independent, and they are elements of a function-set entire in the energy space of the operator [3]. The coefficients a_k are obtained as the solution of a set of linear equations:

$$\sum_{k=1}^{n} [\varphi_k, \varphi_j] a_k = (f, \varphi_j), \quad j = 1, 2, \dots, n.$$
(2.5)

In this expression $[\varphi_k, \varphi_j]$ denotes the energy product of the functions φ_k and φ_i which is defined in our case as [2, 3]:

$$\left[\varphi_k, \varphi_j\right] = \int_V \operatorname{grad} \varphi_k \operatorname{grad} \varphi_j \, \mathrm{d}V \,. \tag{2.6}$$

The right-hand side of (2.5) is the scalar product defined in the region V of the functions f and φ_i :

$$(f, \varphi_j) = \int_V f \varphi_j \,\mathrm{d}V \tag{2.7}$$

It can be shown that the sequence of the approximate solutions thus obtained tends to the solution of the operator equation in the energy norm which means that

$$\lim_{n \to \infty} \int_{V} \operatorname{grad}^{2} \left(u_{n} - u_{0} \right) = 0 \tag{2.8}$$

where u_0 is the exact solution.

The convergence of the variational method depends upon the nature of the exciting function. The convergence is expected to be slower if the exciting function is discontinuous or, or as in our case, is singular. To improve the convergence of the numerical solution the singular component is extracted from the potential function sought. To this end, the relative potential function φ' is sought as the sum of three functions:

$$\varphi'(\xi,\eta) = u_s(\xi,\eta) + \psi(\xi,\eta) + v(\xi,\eta) \tag{2.9}$$

Here, $u_s(\xi, \eta)$ is the potential excited by the surface charges in free space, $\psi(\xi, \eta)$ is an arbitrary function with at least two continuous derivatives complying with the symmetry conditions on φ' and having a derivative in direction η along the lines $\eta = \pm 1$ which is of equal absolute value and opposite sign as the same of the function u_s . $v(\xi, \eta)$ is the unknown function whose determination is possible with the aid of the equations relating to φ' .

The function u_s can be obtained by elementary methods:

$$u_{s}(\xi,\eta) = \frac{1}{4\pi} \left(\beta \ln \frac{\beta^{2} + \delta^{2}}{\beta^{2} + \gamma^{2}} - \alpha \ln \frac{\alpha^{2} + \delta^{2}}{\alpha^{2} + \gamma^{2}} \right) + \frac{1}{2\pi} \left(\delta \operatorname{Arctan} \frac{\beta}{\delta} - \gamma \operatorname{Arctan} \frac{\beta}{\gamma} - -\delta \operatorname{Arctan} \frac{\alpha}{\delta} + \operatorname{Arctan} \frac{\alpha}{\gamma} \right)$$
(2.10)

Here, $\alpha = \xi - a'$, $\beta = \xi + a'$, $\gamma = \eta - b'$, $\delta = \eta + b'$. The derivative of the function u_s in direction η on the lines $\eta = \pm 1$ is

$$g(\xi) = \frac{1}{2\pi} \left(\operatorname{Arctan} \frac{\alpha}{\varepsilon} - \operatorname{Arctan} \frac{\beta}{\varepsilon} - \operatorname{Arctan} \frac{\alpha}{\kappa} + \operatorname{Arctan} \frac{\beta}{\kappa} \right)$$
(2.11)

where $\varepsilon = 1 - b'$, $\kappa = 1 + b'$. Thus a function complying with the conditions is e.g.

$$\psi = -\eta g(\xi) \tag{2.12}$$

In accordance with the conditions on φ' and the properties of u_s and ψ , the Laplace–Poisson equation

$$-\Delta v = \Delta \psi = -\eta \frac{d^2 g(\xi)}{d\xi^2}$$
(2.13)

is obtained for the unknown function v, and the derivative of v in direction η should be zero along the lines $\eta = \pm 1$. The Ritz process is applied to determine the function v. Since the exciting function in (2.13) has an infinite number of derivatives and $\cdot v$ is to satisfy homogeneous boundary conditions, the convergence of the numerical procedure is fast. Due to the symmetry of the region shown in Fig. 2, the potential function φ' is even in ξ and odd in η if the potential on the line $\eta = 0$ is chosen zero. Since u_s and ψ also have this property, v must have it as well. According to [2], the approximate solution in the region infinite in direction ξ is sought in the form

$$v_{n,m} = \sum_{k=0}^{m} \sum_{l=0}^{n} a_{kl} \sin \frac{(2k+1)\pi\eta}{2} \cdot \frac{\cos\left(2l \operatorname{Arctan} \xi\right)}{\sqrt{1+\xi^2}}$$
(2.14)

The elements of the matrix in the set (2.5) are in accordance with (2.6):

$$A_{kl,ij} = \int_{0}^{1} \sin \frac{(2k+1)\pi\eta}{2} \sin \frac{(2i+1)\pi\eta}{2} d\eta \cdot$$

$$\int_{0}^{\infty} \frac{[2l\sin(2l\operatorname{Arctan}\xi) + \xi\cos(2l\operatorname{Arctan}\xi)][2j\sin(2j\operatorname{Arctan}\xi) + \xi\cos(2j\operatorname{Arctan}\xi)]}{(1+\xi^{2})^{3}} d\xi +$$

$$+ \frac{(2k+1)(2i+1)\pi^{2}}{4} \int_{0}^{1} \cos \frac{(2k+1)\pi\eta}{2} \cos \frac{(2i+1)\pi\eta}{2} d\eta \cdot$$

$$\int_{0}^{\infty} \frac{\cos(2l\operatorname{Arctan}\xi)\cos(2j\operatorname{Arctan}\xi)}{1+\xi^{2}} d\xi \qquad (2.15)$$

The elements on the right-hand side are according to (2.7):

$$b_{kl} = -\int_{0}^{l} \eta \sin \frac{(2k+1)\pi\eta}{2} \,\mathrm{d}\eta \cdot \\ \cdot \int_{0}^{\infty} \frac{\mathrm{d}^{2}\mathbf{g}(\xi)}{\mathrm{d}\xi^{2}} \frac{\cos\left(2l\operatorname{Arctan}\xi\right)}{\sqrt{1+\xi^{2}}} \,\mathrm{d}\xi$$
(2.16)

It has been utilized that it is sufficient to integrate over a quarter of the region due to symmetry.

In connection with the numerical procedure, it is noted that the terms in the series (2.14) belong to a strongly minimal function-set [2]. This feature ensures that the process is insensitive to the accumulating numerical errors and the matrix of the set of equations is easily inverted. A further advantageous property is that the matrix has non-zero elements in square blocks of order (n+1)(n+1) along the main diagonal only, and particularly in the main diagonal and in the two rows under and over the latter.

In knowledge of the relative potential function φ' , the relative field intensity and relative current density can be calculated and hence (1.11) and (1.14) yield the power loss and braking force $F = \frac{P}{v}$. Finally, it is noted that if the power loss is calculated from u_s only as in (3.3), the loss in a plate of infinite dimensions in both directions is obtained.

3. Solution with the aid of images

The simple geometry permits the calculation of the analogous electric field by an infinite number of images. Namely, the homogeneous Neumann boundary condition along the lines $\eta = \pm 1$ can be satisfied by an infinite number of image charges (Fig. 3). It is sufficient to give the expressions of the relative field intensity which is written as an infinite series:

$$e_{\xi} = \frac{1}{4\pi} \ln \frac{(\alpha^{2} + \delta^{2}) (\beta^{2} + \gamma^{2})}{(\alpha^{2} + \gamma^{2}) (\beta^{2} + \delta^{2})} + \frac{1}{4\pi} \sum_{k=1}^{\infty} (-1)^{k} \left[\ln \frac{(\alpha^{2} + \delta_{k1}^{2}) (\beta^{2} + \gamma_{k1}^{2})}{(\alpha^{2} + \gamma_{k1}^{2}) (\beta^{2} + \delta_{k1}^{2})} - \ln \frac{(\alpha^{2} + \delta_{k2}^{2}) (\beta^{2} + \gamma_{k2}^{2})}{(\alpha^{2} + \gamma_{k2}^{2}) (\beta^{2} + \delta_{k2}^{2})} \right]$$
(3.1)



Fig. 3. Satisfaction of the boundary condition with the aid of an infinite number of images

$$e_{\eta} = \frac{1}{2\pi} \left(\operatorname{Arc} \tan \frac{\beta}{\gamma} - \operatorname{Arc} \tan \frac{\alpha}{\gamma} - \operatorname{Arc} \tan \frac{\beta}{\delta} + \operatorname{Arc} \tan \frac{\alpha}{\delta} \right) + \frac{1}{2\pi} \sum_{k=1}^{\infty} (-1)^{k} \left[\operatorname{Arc} \tan \frac{\beta}{\gamma_{k1}} - \operatorname{Arc} \tan \frac{\alpha}{\gamma_{k1}} - \operatorname{Arc} \tan \frac{\beta}{\delta_{k1}} + \operatorname{Arc} \tan \frac{\alpha}{\delta_{k1}} - \operatorname{Arc} \tan \frac{\beta}{\delta_{k2}} - \operatorname{Arc} \tan \frac{\beta}{\delta_{k2}} \right]$$

$$-\operatorname{Arc} \tan \frac{\beta}{\gamma_{k2}} + \operatorname{Arc} \tan \frac{\alpha}{\gamma_{k2}} + \operatorname{Arc} \tan \frac{\beta}{\delta_{k2}} - \operatorname{Arc} \tan \frac{\alpha}{\delta_{k2}} \right]$$

$$(3.2)$$

The meaning of α , β , γ and δ is the same as before and $\delta_{k1} = \eta + 2k + b'$, $\delta_{k2} = \eta + 2k - b'$, $\gamma_{k1} = \eta - 2k - b'$, $\gamma_{k2} = \eta - 2k + b'$. In the knowledge of **e** the power loss can be determined as explained previously, but it is more effective to derive a simpler expression with the aid of Green's theorem:

$$p = 4 \int_{0}^{b'} \eta \left[e_{\xi}(a', \eta) - e_{\xi}(a' + 2c', \eta) \right] d\eta -$$

-4
$$\int_{0}^{b'} \eta \left[e_{\xi}(-a', \eta) - e_{\xi}(-a' + 2c', \eta) \right] d\eta +$$

+4b'
$$\lim_{\eta \to b' + 0} \int_{-a'}^{a'} \left[e_{\eta}(\xi, \eta) - e_{\eta}(\xi + 2c', \eta) \right] d\xi$$
(3.3)

It is evident that single integrals rather than double ones are to be evaluated here.

4. The results of the calculation

According to the above discussion, the relative loss has been determined at different pole dimensions and pole distances. The dependence of the parameters has been examined by keeping the product a'b' and the distance c'-a' constant, and the ratio a'/b' has been varied. The results thus obtained with the aid of images have been plotted in Figs 4.1, 2 and 3. The constant value of the product a'b' means a constant pole surface and, in case of well utilized



Fig. 4.1. Relative power loss vs. the ratio a'/b' at constant pole surface and c'-a'=0



Fig. 4.2. Relative power loss vs. the ratio a'/b' at constant pole surface and c'-a'=0.2

iron, a constant pole-flux. The dimension 2(c'-a') is the distance between the inner edges of the pole pairs. The curves thus indicate the dependence of the relative power loss on the ratio of the pole-width and pole length if the distance between the poles and the pole flux are kept constant.

A dotted line in Fig. 3 indicates the loss at $c' - a' \rightarrow \infty$. This shows that the interaction between the pole pairs is negligible if the relative distance between the inner edges of the pole pairs is greater than 1. Comparison of Figs 4.1, 2 and

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Fig. 4.3. Relative power loss vs. the ratio a'/b' at constant pole surface and c' - a' = 0.5 and $c' - a' \rightarrow \infty$



Fig. 5. Current distribution in the plate

3 reveals that the loss and thus the braking force increase significantly, if the pole pairs are drawn nearer to each other. Therefore, if the aim is to attain a braking force as great as possible, the two poles should be set as near to each other as possible. The minimal distance is determined by the space requirement of the exciting coil and by the leakage between the edges of the two pole pairs which decreases the pole-flux.

In Fig. 5 the current distribution in the plate has been plotted in a particular case. The relative dimensions are a' = 0.358, b' = 0.713, c' = 0.573. The relative current between two adjacent lines of current is 0.055.

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5. Testing measurements

To test the model and the numerical process, measurements have been carried out on an experimental linear motor at the Department of Electrical Machines of the Technical University Budapest. The essential parameters were: a = 22.5 mm, b = 47 mm, c = 40 mm, d = 71.5 mm, s = 1 mm, $\delta_0 = 3 \text{ mm}$, $\sigma = 5.7 \text{ 10}^7 \text{ 1/}\Omega\text{m}$.

To determine the air gap flux density B_0 the flux of the poles has been measured. B_0 was not calculated from the original iron dimensions, since the leakage at the edges causes a part of the flux to run outside the pole edges. The leakage has been examined on the basis of [4] with the presumption of infinite permeability and prismatic poles by Schwarz—Christoffel transformation. According to this, in case of $\delta_0/2a < 5$ and $\delta_0/2b < 5$ a more exact value of B_0 is obtained if the transversal dimension of the poles is modified by 1.3 δ_0 outside and by $0.75 \delta_0$ on the side between the two poles, and the longitudinal dimension by $1.3 \delta_0$ on each side, and the whole flux is related to this surface. The modified dimensions are: a = 25.6 mm, b = 51 mm, c = 41 mm. At these dimensions, the relative power loss was 1.36 by variational method with 14 terms in the approximation and 1.34 by the method of images with 10 pairs of image charges taken into account on both sides. These were found to be between 1.28 and 1.38.

Measurements have been carried out as follows. The plate has been accelerated by a constant force, initially without braking, then at different values of air gap flux density. With the aid of the punch tape attached to the plate and the photo diode mounted on the pole the displacement time function was obtained in a large number of points. A five-degree polynomial was fit on these points by least square method. Hence, the velocity time and acceleration time functions were determined. The frictional force at different velocities was calculated from the acceleration without braking and thus the braking force was obtained from the acceleration of braked motions. The total mass of the

B ₀ [Vs/m ²]	v [m/s]	F _{frie} [N]	F _{br meas.} [N]	F _{br comp.} [N]
0.20	0.9	3.9	14.4	14.1
0.20	0.6	3.9	9.7	9.4
0.405	0.28	2.6	17.2	17.9
0.405	0.68	4.1	44.9	43.6
0.53	0.16	1.8	17.9	17.5
0.53	0.42	3.7	45.3	46.1

 Table 1

 Computed and measured values of braking force

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moving part was 14.7 kg. It was assumed that the acceleration has no effect on the dependence of braking force on velocity. The measured and calculated values of braking force are shown in Table 1. Their comparison shows the maximal difference to be lower than 5°_{0} .

6. Conclusions

A method has been presented in the paper which permits the calculation of braking force in an eddy current brake of a linear motor. The results of the testing measurements indicate that the model and the numerical process describe the phenomena with acceptable accuracy.

As regards the Ritz process presented, it is noted that although its computational requirements are excessive, it is still very effective and can be applied to boundary value problems of far greater complexity than the one solved in this paper.

The most important result of the method described for the treatment of singular excitations is the faster convergence of the numerical process in energy norm, and experience shows the field characteristics computed from the approximate solution to approximate the values obtained by image charges, which can be regarded as exact, even in the vicinity of the singular locations. This is proved by the fact that in case of a sufficient number of terms in the approximation the power loss can be computed from (3.3) instead of (1.11) which is of great numerical significance. Namely, the field intensity is obtained as a finite series and its square appears in the formula of loss, thus a two-variable numerical integration is necessary in (1.11), while only one-variable appear in (3.3). If the Ritz process had been applied directly to singular excitations, formula (3.3) could not have been employed.

Finally, it is noted that computations have been carried out on the desk calculator EMG 666. The computational time necessary to obtain the relative power loss in a particular arrangement was about 4 minutes with the aid of image charges and 15—20 minutes by variational method applying formula (3.3).

Summary

Ritz numerical process for solving variational problems is applied to the calculation of the electric field in an eddy-current brake of a linear motor. An alternative method based on images for the solution is also discussed. The effect of motor dimensions on braking force is examined. The results of the testing measurements carried out to check the calculation are also presented.

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