PRACTICAL DESIGN OF SAMPLED DATA CONTROL SYSTEMS

By

R. TUSCHÁK

Department of Automation, Technical University, Budapest

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1. Introduction

Some practical aspects of the synthesis of one-loop single output sampled-data control systems with constant parameters will be discussed, restricted to the input-output systems with feedback from the output signal, and not extending to the state-variable feedback.

Sampled data control algorithms have an extended literature and great many design methods have been published such as parameter optimum control, dead beat control, minimum variance control etc. Usually, however, no indication is given on whether this multitude of procedures have a common basis or not, and what is the functioning mechanism the different results rely on. Whatever the designer's tool, an outstanding means of powerful design is a simple model illustrating the operational mechanism of the control loop and the effect of the factors influencing its operation with no complicated formulae. Without such a comprehensive model, the design with even the most sophisticated algorithm relies on trial, not less than does the fully intuitive one, and the published simulation results are more empirical recipes.

The main problem is that the majority of up-to-date design methods require to specify "a priori" criteria. Their reality or irreality can only be proved a posteriori. Thus, the design process is "trial and error" just as in the so-called classic methods, with the only difference that criterion parameters and not directly the control system parameters are searched, by iteration. The properties of a continuous single-output linear control system appear the simplest from the open-loop frequency transfer function, i.e. from its Bode diagram. In discrete systems, the frequency transfer function contains the frequency in exponential form, making the Bode diagram too complicated to plot. Taking into consideration, however, that frequency transfer properties are, first of all, interesting in ranges below the out-off frequency, simple modifications may facilitate the handling of the discrete frequency function.

2. Low-frequency approximation of the pulse transfer functions of a discrete system

The flow-chart of the discrete control system is seen in Fig. 1. $w_B^*(z)$ and $w_A^*(z)$ are pulse transfer functions of the process and the controller, respectively. The process being normally continuous in time, $w_B^*(z)$ also involves the pulse transfer function of a zero-order hold. Pulse transfer functions of some simple processes — including also the holding element — have been compiled in Table 1.

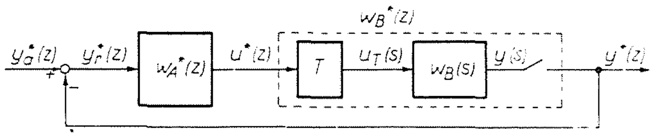


Fig. 1

The pulse transfer function is the rational fraction of z, both its numerator and denominator can be decomposed into the product of functions $(z-g_i)$, where g is a real or a complex number.

Provided the process contains no direct proportional channel, in the pulse transfer function of inertia processes the order of the numerator is less by one than the order of the denominator. The orders in a system containing dead time differ by more.

Let the frequency transfer function of the factor $(z-g)$ be:

$$w_g^*(j\omega)=e^{j\omega T}-g\,. \tag{1}$$

This function can be approximated in the $\omega \leqq 1/T$ frequency range by a more convenient form for practical applications. Validity of the following approximations depending on the type of g can be proved (see in the Appendix 1):

a) If g is a real number expressible as

$$g=e^{-T/T_g} \tag{2}$$

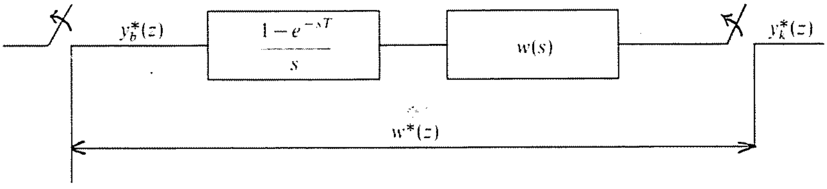
then the low-frequency approximation of the pulse transfer function

$$w_g^*(z)=z-e^{-T/T_g}=e^{sT}-e^{-T/T_g} \tag{3}$$

may be given in the form

$$\tilde{w}_g^*(j\omega)=(1-e^{-T/T_g})\,(1+j\omega T_g)e^{j\omega T_d} \tag{4}$$

Table 1



$w(s)$	$w^*(z)$
$\frac{1}{s}$	$\frac{T}{z-1}$
$\frac{1}{s^2}$	$\frac{T^2}{2}\cdot\frac{z+1}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{T^3}{6}\cdot\frac{z^2+4z+1}{(z-1)^3}$
$\frac{1}{1+sT_1}$	$\frac{1-e^{-T/T_1}}{z-e^{-T/T_1}}$
$\frac{1}{(1+sT_1)(1+sT_2)}$	$K\cdot\frac{z+\sigma_1}{(z-e^{-T/T_1})(z-e^{-T/T_2})}$ $K=\frac{(1-e^{-T/T_1})(1-e^{-T/T_2})}{1+\sigma_1}$ $\sigma_1=\frac{T_2e^{-T/T_1}(1-e^{-T/T_2})-T_1e^{-T/T_2}(1-e^{-T/T_1})}{T_1(1-e^{-T/T_1})-T_2(1-e^{-T/T_2})}$
$\frac{1}{(1+sT_1)^2}$	$K\cdot\frac{z+\sigma_1}{(z-e^{-T/T_1})^2}$ $K=1-e^{-T/T_1}(1+T/T_1)=\frac{(1-e^{-T/T_1})^2}{1+\sigma_1}$ $\sigma_1=\frac{e^{-2T/T_1}-e^{-T/T_1}(1-T/T_1)}{1-e^{-T/T_1}(1+T/T_1)}$

Table 1. (cont.)

$w(s)$	$w^*(z)$
$\dfrac{1}{s(1+sT_2)}$	$K \cdot \dfrac{z+\sigma_1}{(z-1)(z-e^{-T/T_2})}$ $K = T - T_2(1-e^{-T/T_2})$ $\sigma_1 = \dfrac{T_2(1-e^{-T/T_2}) - Te^{-T/T_2}}{T - T_2(1-e^{-T/T_2})}$
$\dfrac{1+s\tau_1}{1+sT_1}$	$\dfrac{\tau_1}{T_1} \cdot \dfrac{z - \left[1 - \dfrac{T_1}{\tau_1}(1-e^{-T/T_1})\right]}{z-e^{-T/T_1}}$
$\dfrac{1+s\tau_1}{(1+sT_1)(1+sT_2)}$	$K \cdot \dfrac{z+\sigma_1}{(z-e^{-T/T_1})(z-e^{-T/T_2})}$ $K = \dfrac{(1-e^{-T/T_1})(1-e^{-T/T_2})}{1+\sigma_1}$ $\sigma_1 = \dfrac{(T_2-\tau_1)e^{-T/T_1}[1-e^{-T/T_2}] - (T_1-\tau_1)e^{-T/T_2}(1-e^{-T/T_1})}{(T_1-\tau_1)(1-e^{-T/T_1}) - (T_2-\tau_1)(1-e^{-T/T_2})}$
$\dfrac{\omega_0^2}{s^2+2s\xi\omega_0+\omega_0^2}$ $\xi < 1$	$K \cdot \dfrac{z+\sigma_1}{z^2 - 2ze^{-aT}\cos bT + e^{-2aT}}$ $K = 1 - e^{-aT}\left(\cos bT + \dfrac{a}{b}\sin bT\right)$ $\sigma_1 = \dfrac{e^{-2aT} - e^{-aT}\left(\cos bT - \dfrac{a}{b}\sin bT\right)}{1 - e^{-aT}\left(\cos bT + \dfrac{a}{b}\sin bT\right)}$ $a = \xi\omega_0;\quad b = \omega_0\sqrt{1-\xi^2};\quad \omega_0^2 = a^2+b^2$

Table 1. (cont.)

$w(s)$	$w^*(z)$
$\dfrac{(1+s\tau_1)(1+s\tau_2)}{(1+sT_1)(1+sT_2)(1+sT_3)}$	$K\cdot\dfrac{z^2-\beta z+\alpha}{(z-e^{-T/T_1})(z-e^{-T/T_2})(z-e^{-T/T_3})}$ $K=a_1+a_2+a_3=\dfrac{(1-e^{-T/T_1})(1-e^{-T/T_2})(1-e^{-T/T_3})}{1-\beta+\alpha}$ $\beta=\dfrac{a_1(e^{-T/T_2}+e^{-T/T_3})+a_2(e^{-T/T_1}+e^{-T/T_3})+a_3(e^{-T/T_1}+e^{-T/T_2})}{a_1+a_2+a_3}$ $\alpha=\dfrac{a_1e^{-T/T_2}e^{-T/T_3}+a_2e^{-T/T_1}e^{-T/T_3}+a_3e^{-T/T_1}e^{-T/T_2}}{a_1+a_2+a_3}$ $a_1=\dfrac{(1-e^{-T/T_1})(T_1-\tau_1)(T_1-\tau_2)}{(T_1-T_2)(T_1-T_3)}$ $a_2=\dfrac{(1-e^{-T/T_2})(T_2-\tau_1)(T_2-\tau_2)}{(T_2-T_1)(T_2-T_3)}$ $a_3=\dfrac{(1-e^{-T/T_3})(T_3-\tau_1)(T_3-\tau_2)}{(T_3-T_1)(T_3-T_2)}$

where T_g may be either positive or negative, T is the sampling interval. T_d is an additional time shift, — reflecting the sampling effect — empirically described as a function of T_g by the following expression:

$$T_d=\frac{T}{1+\sqrt[3]{g}}=\frac{T}{1+\sqrt[3]{e^{-T/T_g}}} \tag{5}$$

In Fig. 2 the quotient of absolute values of the exact (1) and the approximate (4) frequency transfer functions:

$$\frac{|w_g^*(j\omega)|}{|\tilde{w}_g^*(j\omega)|}=\frac{e^{j\omega T}-e^{-T/T_g}}{(1-e^{-T/T_g})(1+j\omega T_g)}$$

and phase angle differences between $\varphi(j\omega)$ and $\tilde{\varphi}(j\omega)$ obtained from Eqs (1) and (4) have been plotted for some T/T_g values. In the frequency range $\omega T<1$ the error in the absolute value is less than 5%, and the angle difference is of 1° or 2°.

b) a special version of the former case is where $g=1$ or $T_g\to\infty$.

Then the low-frequency approximation of the function

$$w_g^*(z)=z-1 \quad \text{or} \quad w_g^*(j\omega)=e^{j\omega T}-1 \tag{6}$$

becomes:

$$\tilde{w}_g(j\omega)=j\omega T e^{j\omega T/2} \tag{7}$$

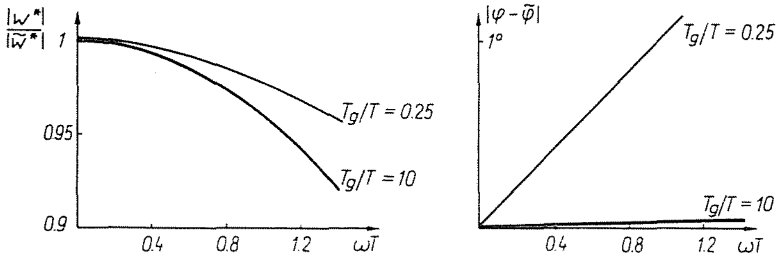


Fig. 2

c) If g is a complex number:

$$g=e^{-(a+jb)T}=e^{-aT}e^{-jbT} \tag{8}$$

Since coefficients of $w^*(z)$ are real, it always occurs with its conjugated g. Let us have therefore:

$$w_g^*(z)=(z-g)\,(z-\hat{g})=z^2-2ze^{-aT}\cos bT+e^{-2aT} \tag{9}$$

The approximate frequency function:

$$\tilde{w}_g^*(j\omega)=[1-2e^{-aT}\cos bT+e^{-2aT}]\cdot$$

$$\left[1+\frac{2aT}{(aT)^2+(bT)^2}j\omega T+\frac{(j\omega T)^2}{(aT)^2+(bT)^2}\right]e^{j\omega T_d} \tag{10}$$

Introducing the natural frequency ω_0 usual in the second-order expression of continuous signals and the damping coefficient ξ

$$\tilde{w}_g^*(j\omega)=\left[1-\left(\frac{\omega T}{\omega_0 T}\right)^2+2j\xi\frac{\omega T}{\omega_0 T}\right]e^{j\omega T_d} \tag{11}$$

where:

$$T_d=\frac{2T}{1+\sqrt[3]{e^{-aT}}} \tag{12}$$

$$\omega_0 T = \sqrt{(aT)^2 + (bT)^2} \qquad \xi = \frac{aT}{\sqrt{(aT)^2 + (bT)^2}}; \tag{13}$$

$$aT = \xi \omega_0 T, \qquad bT = \omega_0 T \sqrt{1 - \xi^2}$$

In case of $\omega T < 1$ and $bT < 2$, the approximation error is less than 10% in absolute value, the phase angle difference is 2° to 3°. Nevertheless, for $bT \to \pi$ the errors are increasing.

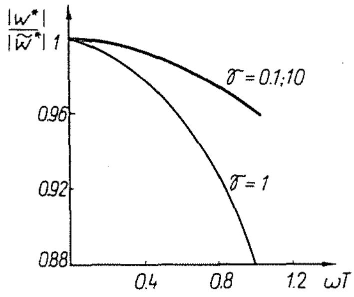


Fig. 3

d) Special case of the complex zero

$$bT = \pi$$

Now:

$$g = -e^{-aT} = -\gamma \tag{14}$$

$$w^*(z) = z + \gamma; \qquad w_g^*(j\omega) = e^{j\omega T} + \gamma \tag{15}$$

In this case, expressions

$$\tilde{w}^*(j\omega) = (1+\gamma) e^{j\omega T_d} \tag{16}$$

and

$$T_d = \frac{T}{1 + \gamma^{1.08}} \tag{17}$$

are better approximations than Eqs (11) and (22).

Ratio

$$\frac{|w_g^*(j\omega)|}{|\tilde{w}_g^*(j\omega)|}$$

computed for some γ values are seen in Fig. 3 the absolute error is less than 12%, the angle difference is 0.2 to 0.4°.

According to these approximations, in the discrete system, the resulting frequency transfer function of the process and the holding in the range $\omega T \leqq 1$ differs only by an additional dead-time from the continuous frequency transfer function.

Be for example the transfer function of a continuous process:

$$w(s)=\frac{1+5s}{(1+20s)(1+10s)(1+0.5s)} \tag{18}$$

In the discrete system the pulse transfer function with a sampling time $T=1$, including also the holding device:

$$w^*(z)=\frac{0.0144(z-0.819)(z+0.533)}{(z-0.951)(z-0.905)(z-0.1353)} \tag{19}$$

Replacing each factor by its low-frequency approximation Eqs (4 – 5) or (16 – 17) for $\omega T<1$ yields:

$$\tilde{w}^*(j\omega)=\frac{0.0144(1-0.819)(1+0.533)(1+5j\omega)\,e^{j\omega T_d}}{(1-0.951)(1-0.905)(1-0.1353)(1+20j\omega)(1+10j\omega)(1+0.5j\omega)}=$$

$$=\frac{(1+5j\omega)}{(1+20j\omega)(1+10j\omega)(1+0.5j\omega)}e^{-j0.491\omega}=w(j\omega)e^{-j0.49\omega} \tag{20}$$

The resultant additional dead-time being:

$$T_d=T\frac{1}{1+\sqrt[3]{0.819}}+\frac{1}{1+0.533^{1.08}}-\frac{1}{1+\sqrt[3]{0.951}}-$$

$$-\frac{1}{1+\sqrt[3]{0.905}}-\frac{1}{1+\sqrt[3]{0.135}}=-0.49T$$

Thus, the additional dead-time is $T_d \sim T/2$. This is not accidental. If the frequency transfer function of a stable process without dead-time strongly damps frequencies $\Omega \pm \omega$; $2\Omega+\omega$ etc., in comparison with the frequency band $\omega T<1$ — that is, it has at least one pole in the range $\omega T<1$ the high-frequency filtering effect of which is not compensated by the zeros — then in the low-frequency range in the discrete system, the sampling effect may always be taken into account with an additional deadtime $T_d=T/2$, irrespective of the distribution of the other poles and zeros $\Omega=2\pi/T$ is the sampling frequency.

This theorem is simple to understand from Fig. 4. Assume that the input signal of the hold device $u^*(t)$ originates from the sampling of a sine wave $[u(t)]$

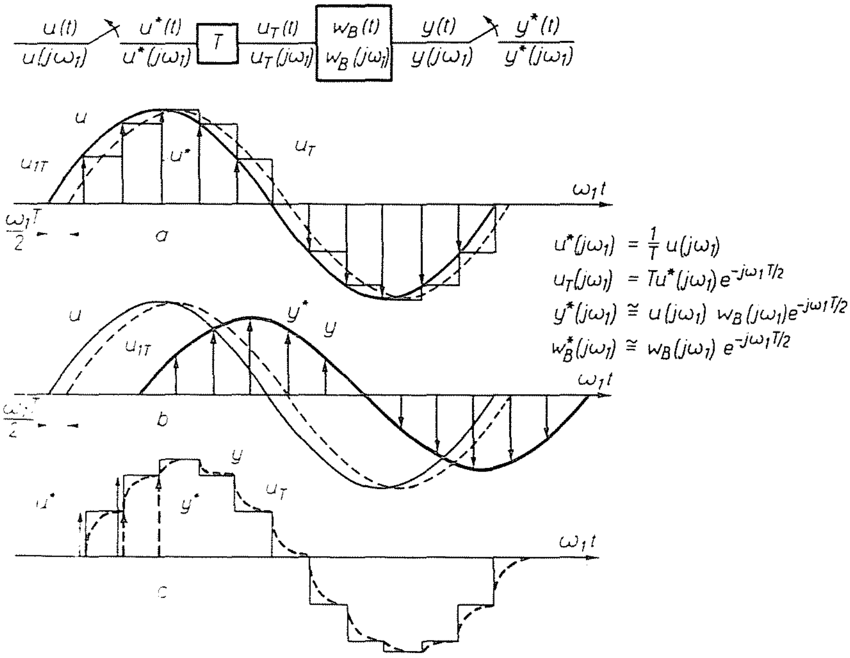


Fig. 4

of frequency ω_1, where $\omega_1 < 1/T$. The component of frequency ω_1 of the pulse sequences $u^*(t)$ is,

$$u^*(j\omega_1) = \frac{1}{T} u(j\omega_1) \tag{21}$$

The output signal of the hold device $[u_T]$ consists of the first harmonic $u_T(j\omega_1)$ and of higher frequency components. The phase shift of the first harmonic compared to signal $u^*(j\omega_1)$ is $\omega_1 T/2$. If the frequency transfer function $w_B(j\omega)$ damps the higher frequency components, then the output signal practically becomes a sine wave of frequency ω_1 where:

$$y(j\omega_1) = u_T(j\omega_1)w_B(j\omega_1) = Tu^*(j\omega_1)w_B(j\omega_1)e^{-j\omega T/2}$$

Sampling the continuous signal $y(t)$, the fundamental harmonic of the sampled signal $y^*(t)$ becomes:

$$y^*(j\omega_1) = \frac{1}{T} y(j\omega_1)$$

hence:

$$w_B^*(j\omega) = \frac{y^*(j\omega_1)}{u^*(j\omega_1)} = w_B(j\omega_1)e^{-j\omega_1 T/2} \tag{22}$$

The less the function $w_B(j\omega)$ damps the high-frequency components, the more follows the signal y the jumps of the signal u_T. In the extreme case transients exciting by jumps of function u_T vanish within one T sampling interval (Fig. 4). Then, pulse sequences $y^*(t)$ and $u^*(t)$ differ only by the time shift T, and the pulse transfer function becomes pure dead-time function.

In conclusion, the continuous process inserted in a discrete system may be described in the frequency range $\omega T < 1$ by its continuous transfer function and an additional time shift with a dead-time between $T/2$ and T, depending on the locus of the poles.

3. Some properties of the pulse transfer function of minimum phase continuous systems

Let be the poles of a transfer function $w(s)$ of a continuous system without dead-time in the left half-plane or in the origo, and its zeros in the left half-plane. The absolute value of the poles and zeros is less than $1/T$. Let be the order of numerator m, and the order of the denominator n, where $m<n$.

The pulse transfer function $w^*(z)$ including also the hold is rational function of z. Assume that it strongly damps the higher frequencies mentioned above, then its denominator consists of n factors type $z-g_i$. g_i is the exponential function of the i-th pole. In case of complex poles, two factors belonging to the conjugated poles may be reduced to one quadratic factor with real coefficients. The numerator always contains $(n-1)$ multiplying factors, m of them being of type $(z-\sigma_i)$, where σ_i is approximately an exponential function of the zeros in $w(s)$, and the remained factors of number $(n-1-m)$ are of type $z+\gamma_i$, where γ_i is positive real. These latter factors cause that the function $w_B^*(j\omega)$ differs only by a dead-time $T/2$ from the continuous frequency transfer function $w(j\omega)$ in the range $\omega T<1$, regardless of the m and n values. This will be illustrated by comparing the continuous transfer functions $w(s)$, pulse transfer functions and their low-frequency approximations in the case of three processes, with a sampling interval $T=1$.

a)

$$w_1^*(s) = \frac{1}{(1+10s)\,(1+7s)\,(1+s)} \tag{23}$$

$$w_1^*(z) = \frac{0.00177(z+0.193)\,(z+2.79)}{(z-0.905)\,(z-0.867)\,(z-0.368)} \tag{24}$$

$$\tilde{w}_1^*(j)=\frac{e^{-j\omega T_d}}{(1+10j\omega)(1+7j\omega)(1+j\omega)}=\frac{e^{-j0.49\omega}}{(1+10j\omega)(1+7j\omega)(1+j\omega)} \tag{25}$$

$$T_d=T[0.76+0.25-0.51-0.51-0.58]=-0.49T \tag{26}$$

The first two terms of the expression for T_d arise from the factors of the numerator, the others from those of the denominator.

b)

$$w_2(s)=\frac{(1+2js)}{(1+10s)(1+7s)(1+s)} \tag{27}$$

$$w_2^*(z)=\frac{0.011(z-0.606)(z+0.784)}{(z-0.905)(z-0.867)(z-0.368)} \tag{28}$$

$$\tilde{w}_2^*(j\omega)=\frac{1+2j\omega}{(1+10j\omega)(1+7j\omega)(1+j\omega)}e^{-j0.49\omega} \tag{29}$$

$$T_d=T[0.54+0.57-1.6]=-0.49T \tag{30}$$

c)

$$w_3(s)=\frac{(1+5s)(1+2s)}{(1+10s)(1+7s)(1+s)} \tag{31}$$

$$w_3^*(z)=\frac{0.1139(z-0.819)(z-0.613)}{(z-0.905)(z-0.867)(z-0.368)} \tag{32}$$

$$\tilde{w}_3^*(j\omega)=\frac{(1+5j\omega)(1+2.04j\omega)}{(1+10j\omega)(1+5j\omega)(1+j\omega)}e^{-j0.54\omega} \tag{33}$$

$$T_d=T(0.52+0.54-1.6)=-0.54T \tag{34}$$

In case of a), the numerator is of degree $m=0$. All three factors of the denominator of function $w^*(z)$ have additional time shifts somewhat over $0.5T$ causing a total additional time shift of $1.6T$. $w(s)$ having no zero, the numerator of $w^*(z)$ will contain two terms type $z+\gamma$ practically of no effect on the amplitude characteristic in the low-frequency range. Their resulting time shift of about $1.1T$ partly compensates the time shift of the denominator, the resultant additional dead time is about $T/2$.

In case of b), the zero in $w(s)$ causes a factor $(z-0.606)$ and a factor type $(z+\gamma)$ to appear in the numerator of $w^*(z)$. This latter causes the resulting dead-time to be again $T/2$ in the low-frequency range.

In case of c), $m=2$. Therefore the numerator of $w^*(z)$ will contain factors corresponding to the two zeros in $w(s)$.

If the system has also a dead-time $T_D=dT$, then the denominator of $w^*(z)$ will contain a multiplier of form z^d.

From case a) it is seen that the minimum phase system in plane s did not remain minimum phase type in plane z for the given sampling time, (one zero of $w^*(z)$ got outside the unit circle.)

It is not too difficult to show (Appendix 2) that if the order of denominator and numerator of the continuous transfer function $w(s)$ differ by $n-m=2$, then in many practical cases a minimum phase system in the plane $w(s)$ remains of minimum phase for any sampling time. For $(n-m)=3$ or $(n-m)=4$, if the absolute value of the poles and zeros of $w(s)$ is less than $1/T$, one zero of the system in z plane will get outside the unit circle.

Thus, the case where the pulse transfer function of a stable process has no minimum-phase character is by no means exceptional in the z domain.

Similar statements can be made on higher orders differences, but these are of little practical importance.

4. Digital compensation

Compensating means to form the frequency transfer properties of the control loop. Regardless of the actual design methods this will always be achieved by the transfer function of the controller inserting new poles and zeros into the open loop. In continuous system it will be made by the factors.

$$w_A=\frac{1+sT_1}{1+sT_1'}$$

If $1/T_1$ is chosen near to one of the poles of the process, w_A compensates the effect of this poles and instead of it inserts a new pole at $1/T_1'$. The effect is as if the original pole were shifted to another frequency range. Shifting a pole to lower frequency range means a PI compensation. The opposite case is the PD compensation. An ideal PI or PD compensation shifts the pole to $\omega\rightarrow 0$ or $\omega\rightarrow\infty$, respectively. Inserting a new pole without a new zero or a new zero without a new pole means pure I or D compensation, respectively.

The control system may always be considered as a filter transmitting at below the cut-off frequency of the open loop, and damping thereover. For stable, minimum phase continuous processes (in plane s) the phase angle of function $w(j\omega)$ at the cut-off frequency theoretically cannot be less than $-180°$ (practically $-120°$ to $-150°$). Since, sampling in the continuous process inserts a dead-time $0.5T-1T$ into the system, which is further increased by max. $0.5T$ upon each single compensation PD, the cut-off frequency of a discrete system practically cannot be more than $1/T$ [the possible maximum being about $1/2T$]. As a conclusion, the low-frequency relationship between functions $w(s)$ and $w^*(z)$ (chapter 2) is valid throughout the transmitting band. On this basis, the discrete compensation algorithms are easy to survey.

The discrete control algorithm inserts a pole-zero couple into the control system as terms type $(z-g)/(z-g')$. Such a term in the continuous system is equivalent to the effect of a definite pole-zero couple, and may cause an additional phase shift. Depending on g and g', also terms having no equivalent pole or zero in the continuous system may be generated. These are special possibilities of the discrete compensation, affecting mainly the phase characteristic. Any discrete optimization method with no equivalent in the continuous control e.g. dead-beat controller makes use of this possibility. The general algorithm

$$w_g^*(z)=A_g\frac{z-g}{z-g'}, \tag{35}$$

has the following versions:

a) If g and g' are positive real number, then

$$w_A^*(z)=A_g\frac{z-e^{-T/T_1}}{z-e^{-T/T_1'}} \tag{36}$$

Its low-frequency approximation

$$\tilde{w}_A^*(j\omega)=A_g\frac{1-e^{-T/T_1}}{1-e^{-T/T_1}}\frac{1+j\omega T_1}{1+j\omega T_1'}e^{j\omega(T_d-T_d')} \tag{37}$$

where additional time shifts T_d and T_d' are given by Eq. (5).

PI compensation is generally applied in the range below the cut-off frequency. Then $T_1'>T_1>T$, hence $g'>g$, but both are near unity, therefore $T_d'\div T_d=0$, so discrete PI compensation will cause no additional dead-time.

In an ideal PI algorithm $T_1'\to\infty$

$$w_A^*(z)=A_g\frac{z-e^{-T/T_1}}{z-1}=A_g\frac{z-g}{z-1} \tag{38}$$

$$\tilde{w}_A^*(j\omega)=A_g\frac{1-e^{-T/T_1}}{T}\frac{1+j\omega T_1}{j\omega} \tag{39}$$

For $T/T_1\ll 1$:

$$\tilde{w}_A^*(j\omega)=A_g\frac{1+j\omega T_1}{j\omega T_1} \tag{40}$$

The ideal PI member may be produced also by parallelly connecting a proportional element and a digital integrator. According to the simplest discrete integration formula:

$$w_A^*(z) = A_p \left[1 + \frac{T}{T_I} \frac{1}{z-1} \right] = A_P \frac{z-1+T/T_I}{z-1} \tag{41}$$

In case of $T_I = T_1$, and $T_1/T \ll 1$, it equals (38). It has no additional time shift, therefore even more accurate integration formulae are not better for this purpose.

PD compensation is applied in the range about the cut-off frequency. Now, $T_1' < T_1$ and usually $T_1' \leqq T$. Therefore $T_d' > T_d$ or $(T_d - T_d') < 0$, thus, the algorithm causes an additional time shift. PD algorithm requires the controller to produce dynamic gain. This may numerically be given e.g. as the quotient of initial and steady-state values of the step response.

For a unit-step controller input

$$y_i^*(z) = \frac{z}{z-1}$$

the output signal u^*

$$u^*(z) = \frac{z}{z-1} w_A^*(z) = A_g \frac{z}{z-1} \frac{z - e^{-T/T_1}}{z - e^{-T/T_1'}}$$

Hence:

$$u^*(t=0) = A_g; \quad u^*(t \to \infty) = A_g \frac{1 - e^{-T/T_1}}{1 - e^{-T/T_1'}}$$

$$A_{\text{din}}^* = \frac{u^*(t=0)}{u^*(t \to \infty)} = \frac{1 - e^{-T/T_1'}}{1 - e^{-T/T_1}} \tag{42}$$

For a continuous PD algorithm

$$A_{\text{din}} = \frac{T_1}{T_1'} \tag{43}$$

For $T_1 > 1$ and $T_1' > T$, the two expressions are identical, but for $T' < T$ they significantly differ.

For ideal PD compensation the continuous algorithm yields the non-realizable $A_{\text{din}} = \infty$. A discrete algorithm yields

$$w_A^*(z) = A_g \frac{z - e^{-T/T_1}}{z}$$

or

$$\tilde{w}_A^*(j\omega) = A_g(1 + j\omega)T_1 \; e^{-j\omega(T - T_d)} \tag{44}$$

equivalent in the low-frequency band to an ideal PD term with dead-time. The dynamic gain

$$A_{\text{din}}^* = \frac{1}{1 - e^{-T/T_1}} \tag{45}$$

In general, $T/T_1 < 1$. Then

$$A_{\text{din}}^* = \frac{T_1}{T}$$

and

$$(T - T_d) = T/2 \tag{46}$$

The dynamic gain enabling a continuous PD algorithm to achieve $T' = T$ for a given T_1, makes the discrete algorithm to act as if $T' \to 0$, and as if a dead-time $T/2$ would enter the system. In other words, the discrete PD algorithm exchanges the time-lag term of time constant T_1 for a term with dead-time $T/2$.

Accordingly, the discrete algorithm of a controller able to insert one pole-zero couple each on the low and high frequency ranges:

$$w_A^*(z) = A_0 \frac{z - g_1}{z - g_1'} \frac{z - g_2}{z - g_2'} = A_0 \frac{1 - (g_1 + g_2)z^{-1} + g_1 g_2 z^{-2}}{1 - (g_1' + g_2')z^{-1} + g_1' g_2' z^{-2}} =$$

$$= A_0 \frac{1 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}$$

includes five independent constants. With ideal PI and PD ($g' = 0$ and $g_1' = 1$) terms the number of independent constants is reduced to three.

$$w_A^*(z) = A \frac{1 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 - z^{-1}}$$

This is the counterpart of the continuous controller PID. It is usual to define the constants as the integration and differentiation times T_I and T_D of the continuous algorithm, but this is rather hampering than facilitating the design.

b) g and g' are negative real numbers,

$$g = -\gamma \quad \text{or} \quad g' = -\gamma'$$

Now, an algorithm of the form

$$w_A^*(z) = A_g \frac{z+\gamma_1}{z+\gamma_1'}$$

or

$$\tilde{w}_A^*(j\omega) = A_g \frac{1+\gamma_1}{1+\gamma_1'} e^{j\omega(T_d - T_d')} \tag{47}$$

acts in the low-frequency range as if inserting a pure time-shift into the system. For $\gamma_1 > \gamma$ the time shift $T_d - T_d'$ is positive, thereby algorithm (47) permits to somewhat reduce the dead-time in the system. In case of e.g. $\gamma_1 = 0$, connecting algorithm (44) and (47) in series yields the PD algorithm described by the following functions:

$$w_A^*(z) = A_g \frac{z - e^{-T/T_1}}{z} \frac{z}{z+\gamma_1'} = A_g \frac{z - e^{-T/T_1}}{z+\gamma_1'} \tag{48}$$

$$\tilde{w}_A^*(j\omega) = A_g \frac{1 - e^{-T/T_1}}{1+\gamma_1'} (1+\omega T_1) e^{-j\omega(T_d' - T_d)} \tag{49}$$

$$A_{\text{din}}^* = \frac{1+\gamma_1'}{1-e^{-T/T_1}} \tag{50}$$

where $(T' - T_d) < T/2$. Increasing the dynamic gain is seen to produce an equivalent ideal PD algorithm with an additional dead-time less than the $T/2$.

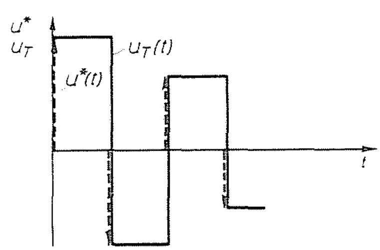


Fig. 5

But application of algorithm (49) is contraindicated by the possibility of a term type $1/(z+\gamma')$ to appear in the control signal u^* involving a pulse sequence of alternating sign (Fig. 5). This pulse sequence through the zero-order hold excites the process with an oscillating signal at a frequency $\Omega/2$. Oscillation of the same frequency appears in the output signal of the continuous process. They are observable between sampling points. Therefore a PD algorithm of this type may only be applied with a value γ causing no oscillation in the signal u^*.

c) A multiple PD compensation may involve also complex numbers g and g', giving with their conjugated a numerator or denominator of second order. If for an equivalent low-frequency approximation $\omega < 1/T$, these terms affect in the range $\omega T < 1$, mainly the phase characteristic so the effect may be similar to case b) but again undesirable vibrations may occur.

5. Synthesis of a sampled-data control system

5.1. *The role of the sampling time*

A fundamental characteristic of the synthesis of a discrete control system is the ratio of the sampling interval T and the cut-off frequency ω_c.

a) If $\omega_c \ll 1/T$, the additional time shifts due to sampling are negligible. The synthesis has no special character, but may be made by the same methods as in continuous systems. In low-frequency approximations according to Eqs (14–17) additional dead-times may first be neglected. Then exsists a mutual unambiguous relationship between the continuous and the discrete control algorithm, permitting to transform each to other.

b) Special discrete synthesis methods e.g. controllers with minimum variance or dead-beat controller are justified if ω_c and $1/T$ are of the same order of magnitude, generally $\omega_c \sim \frac{1}{T} : \frac{1}{3T}$. For the sake of an acceptable result, the sampling interval has to be chosen corresponding to the cut-off frequency that can be achieved by the control equipment. Thus, the speed of the control loop is the function of the factors determining the cut-off frequency e.g. power of the controller equipment, its linear operation range etc. The control algorithm itself exerts only a secondary effect on it. Often the difference between an optimum and an intelligent suboptimum control is of little importance. The optimum of discrete controls makes use of the special PD algorithm mentioned above (see point 4b), but these methods generally result in a system much more sensitive to the variation of the parameters or the input signal than systems of a more conservative design.

5.2. *Stochastic-deterministic equivalency*

Control systems are usually designed for deterministic or stationary stochastic signals. The two methods may be reduced to a common basis. From the theory of spectrum factorization it is known that the power density function of a stationary stochastic signal with a rational spectrum can be decomposed into the product of two conjugated complex rational functions, one of which has all poles in the continuous system in the left half-plane, in the discrete system inside the unit circle, and all zeros in the left half-plane or on the imaginary axis, or inside or on the unit circle, respectively. Thus, if the spectrum density of the stationary stochastic signal $y(t)$ in the continuous and discrete system is $r_{yy}(j\omega)$ and $r^*_{yy}(j\omega)$, respectively, then

$$r_{yy}(j\omega)=\eta(j\omega)\eta(-j\omega)$$

or

$$r^*_{yy}[e^{j\omega T}]=\eta^*(e^{j\omega T})\eta^*(e^{-j\omega T}) \tag{51}$$

Signals $\eta\ (j\omega)$ and $\eta^*(e^{j\omega T})$ may be interpreted as amplitude spectra of deterministic one-sided signals $\eta(t)$ and $\eta^*(t)$ thus, according to the Parseval theorem:

$$\operatorname{var} y=\frac{1}{2\pi}\int_{-\infty}^{\infty} r_{yy}\,\mathrm{d}\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta(j\omega)\eta(-j\omega)\,\mathrm{d}\omega=\frac{1}{2\pi}\int\eta^2(t)\,\mathrm{d}t$$

Consequently, in the control system, the square time integral in the range 0 to ∞ of any signal generated by the deterministic signal $\eta(t)$ is identical to the variance of the signal generated by the stochastic signal $y(t)$. In any case where the control circuit is designed for the optimum (suboptimum) of the variance of the error signal or the weighted variance sum of the error signal and control signal, it will also be optimum (suboptimum) for the deterministic signal $\eta(t)$ in the sense of the square time integral. Also the inverse of the theorem is valid, therefore in designing the control or simulating its operation, the stochastic signal may be replaced by a deterministic one. This is often advantageous, permitting to better observe specialities of the system operation.

5.3. *Minimum settling time control*

In control system shown in Fig. 1, let the z transform of the deterministic input signal in function of z^{-1}

$$y^*_a(z^{-1})=\frac{C(z^{-1})}{D(z^{-1})}=\frac{C'(z^{-1})C''(z^{-1})}{D(z^{-1})}z^{-a} \tag{52}$$

$C'(z^{-1})$ contains all zeros inside the unit circle, and $C''(z^{-1})$ on or outside it. All zeros in the denominator are inside of the unit circle or at $z=1$. The pulse transfer function of the process that may include also dead-time T_d:

$$w^*_B(z^{-1})=z^{-k}\frac{B(z^{-1})}{A(z^{-1})} \tag{53}$$

$k=1+T_d/T$, thus, if the system contains no dead-time $T_d/T=0$, $k=1$.

Let us find control algorithm $w_A^*(z^{-1})$ under the condition that error signal y_r^* has to vanish as soon as possible, thus, output signal y^* should follow y_a^* as truly as possible.

Hence, $y_r^*(z^{-1})$ must be a polynomial of finite degree, expressed in the form:

$$y_r^*(z^{-1}) = C''(z^{-1})h(z^{-1})z^{-a} = C''(z^{-1})z^{-a}(h_0 + h^{-1} + h_2 z^{-1} + \ldots) \quad (54)$$

Here $h(z^{-1})$ is a polynomial with still undefined order and coefficients. y_r^* may be expressed by the input signal and the error pulse transfer function

$$y_r^*(z^{-1}) = y_a(z^{-1})w_r^*(z^{-1}) = \frac{z^{-a}C'(z^{-1})C''(z^{-1})}{D(z^{-1})} w_r^*(z^{-1}).$$

Hence:

$$w_r^*(z^{-1}) = \frac{D(z^{-1})}{C'(z^{-1})} h(z^{-1}) \quad (55)$$

The form (54) of signal y_r^* ensures the stability of w_r^*.

A realizable algorithm $w_A^*(z^{-1})$ cannot make dead-time term z^{-k} of the discrete system disappear, therefore this term has to appear in the transfer function of the closed system. And since in a finite time the output signal takes the value y_a^*, also the denominator of $y^*(z^{-1})$ contains the polynomial $D(z^{-1})$. Hence $w^*(z^{-1})$ may be written as:

$$w^*(z^{-1}) = \frac{y^*(z^{-1})}{y_a^*(z^{-1})} = z^{-k}\frac{p(z^{-1})}{C'(z^{-1})} = z^{-k}\frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + \ldots}{C'(z^{-1})} \quad (56)$$

Here $p(z^{-1})$ is a polynomial of indefinite order and coefficients.

From the relationship between w^* and w_r^*:

$$1 - \frac{D(z^{-1})}{C'(z^{-1})} h(z^{-1}) = z^{-k}\frac{p(z^{-1})}{C'(z^{-1})}$$

hence:

$$C'(z^{-1}) - D(z^{-1})h(z^{-1}) = z^{-k}p(z^{-1}) \quad (57)$$

Comparing coefficients of powers of z^{-1} on either side yields a definite system of equations for the parameters h and p minimally required.

The pulse transfer function of the controller:

$$w_A^*(z^{-1}) = \frac{1}{w_B^*(z^{-1})}\frac{w^*(z^{-1})}{w_r^*(z^{-1})} = \frac{p(z^{-1})A(z^{-1})}{D(z^{-1})h(z^{-1})B(z^{-1})} \quad (58)$$

This equation includes also the following cases:

a) Minimum variance control

All zeros of $C(z^{-1})$ and $D(z^{-1})$ are inside the unit circle. $C'(z^{-1})=C(z^{-1})$ and $C''(z^{-1})=1$. Error signal y_r^* disappears latest at time $k+1$. Within this interval, error values appear by physical necessity dead-time, therefore the square integral of $y_r^*(t)$ or, according to the stochastic-deterministic equivalency, variance of the corresponding stochastic signal is minimum.

b) Signal $y_a^*(z^{-1})$ is z transform of deterministic signals $1(t)$; $1 \cdot t$; $1 \cdot t^2$. Then $w_A^*(z^{-1})$ is the minimum-type controller.

c) For $B/A=1$, the control is of pure dead-time dead-beat or minimum variance control.

In spite of the different mathematical apparata describing the different controls, they are based on the same compensation principle.

The algorithm (58) is, however, seldom applicable in original form. w_A^* contains the inverse of $w_B^*(z^{-1})$. If the process is no minimum phased in plane z, due either to zeros, or poles of $w_B^*(z^{-1})$, this compensation is unacceptable, similarly is not desirable if polynomial B contains factors type $1+\gamma z^{-1}$ because these in the denominator of function $w_A^*(z)$ lead to oscillation between sampling points in the continuous process. All these problems may be eliminated by suboptimum strategy, consisting essentially in permitting unstable poles of $w_B^*(z)$ in function $w_r^*(z^{-1})$, and unstable or oscillating zeros in function $w^*(z^{-1})$ to appear. Now, Eq. (57) becomes:

$$C'(z^{-1})-D(z^{-1})\,L(z^{-1})h(z^{-1})=z^{-k}R(z^{-1})p(z^{-1}) \tag{59}$$

where polynomial $R(z^{-1})$ contains the unstable or oscillating zeros, and $L(z^{-1})$ the unstable poles of $w_B^*(z)$. This strategy increases the settling time compared to (58).

Compensation mechanism of various design methods is illustrated by the following example.

Example.

Transfer function of the continuous process:

$$w(s) = \frac{1}{(1+4s)\,(1+2s)} \tag{60}$$

The asymptotic Bode diagram is seen in Fig. 6.

Let us consider the algorithm of the minimum settling time control. Let the input signal be the deterministic step function.

$$y_a^*(z^{-1}) = \frac{C(z^{-1})}{D(z^{-1})} = \frac{1}{1-z^{-1}} \tag{61}$$

$$w_B^*(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = 0.0489z^{-1}\frac{1+0.779z^{-1}}{(1-0.779z^{-1})(1-0.607z^{-1})} \tag{62}$$

In the following three cases will be compared.

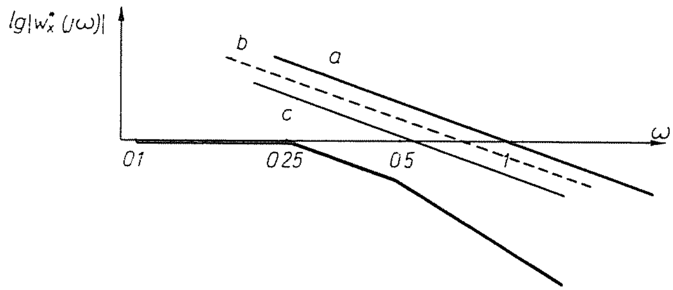


Fig. 6

a) The minimum type dead-beat control is, according to Eqs (57) and (58):

$$w_r^*(z^{-1}) = (1-z^{-1})\,; \qquad w^*(z^{-1}) = z^{-1} \tag{63}$$

$$w_A^*(z^{-1}) = \frac{20.45(1-0.779z^{-1})(1-0.607z^{-1})}{(1-z^{-1})(1+0.779z^{-1})} = 20.45\frac{(z-0.779)(z-0.607)}{(z-1)(z+0.779)} \tag{64}$$

Its equivalent low-frequency approximation:

$$\tilde{w}_A^*(j\omega) = \frac{(1+4j\omega)(1+2j\omega)}{j\omega}e^{-j0\omega} \tag{65}$$

The compensation algorithm consists of a PI and a PD part. PI part transfers pole at $z=0.779$, to $z=1$. The low-frequency effect of the PD algorithm is as if it would completely cancel the pole of the process at $s = -\frac{1}{2}$, without additional dead-time. The pulse transfer function of the open loop and its low-frequency approximation

$$w_x^*(z) = w_A^* w_B^* = \frac{1}{z-1} \tag{66}$$

$$\tilde{w}_x^*(j\omega) = \frac{1}{j\omega}e^{-j\omega/2} \tag{67}$$

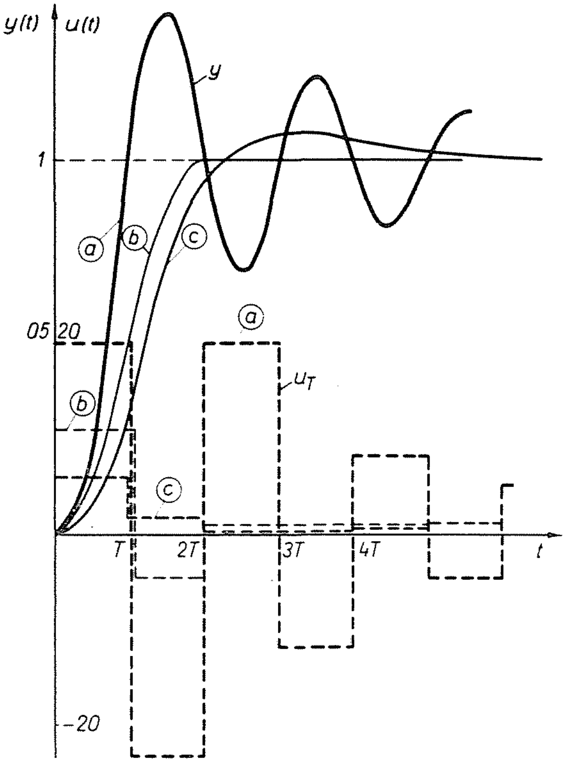


Fig. 7

Thus, the cut-off frequency is $\omega_c = 1$, the phase margin $\varphi_c = 61.35°$. The control signal:

$$u^*(z) = y_r^* w_A^* = 20.45 \frac{(z-0.779)(z-0.607)}{(z-1)(z+0.779)} \tag{68}$$

Time functions y and u are seen in Fig. 7 (a curves).

By the end of the first sampling period, the output signal reaches its steady-state value, but upon the effect of the very oscillating signal u_T arising from signal u^* it is strongly rippled between the sampling points. The input signal u_T of the process also contains an oscillation of frequency $\Omega/2$ damping with a time constant $T_1 = 4$, u having its maximum value at $t = T$, $u(T) = 23.82$. Oscillation of signal u and y makes this form of the algorithm practically useless.

In order to suppress oscillation of the control signal we apply the suboptimal Eq. (59)

$$L(z^{-1}) = 1 + 0.779z^{-1}$$

$$w_r^*(z^{-1}) = (1-z^{-1})(1+0.438z^{-1})\,; \quad w^*(z^{-1}) = 0.562z^{-1}(1+0.779z^{-1}) \tag{69}$$

$$w_A^*(z) = 11.5\,\frac{(z-0.779)(z-0.607)}{(z-1)(z+0.438)} \tag{70}$$

$$\tilde{w}_A^*(j\omega) = 0.695\,\frac{(1+4j\omega)(1+2j\omega)}{j\omega}\,e^{-0.147j\omega} \tag{71}$$

$$w_x^*(z) = \frac{0.562(z+0.779)}{(z-1)(z+0.4379)} \tag{72}$$

$$\tilde{w}_x^*(j\omega) = \frac{0.695}{j\omega}\,e^{-j0.64\omega} \tag{73}$$

The cut-off frequency $\omega_c \sim 0.695$; $\varphi_c = 64.42°$. From Eq. (70) it is obvious that the control algorithm again consists of an ideal PI algorithm and of PD algorithm shifting pole $z = 0.607$ to $z = -0.437$.

It differs from Eq. (64) in the PD part having an additional dead-time $0.174T$. This increases the additional dead-time of the open loop to $0.65T$. The about equal phase margin leads to a cut-off frequency lower than in case *a*. The error disappears during $2T$, without oscillating component in signals u_T and y.

$$u^*(z) = 11.5\,\frac{(z-0.779)(z-0.607)}{(z-1)z} \tag{74}$$

The maximum u value is $u(T) = 11.5$. Values y and u_T are seen in curves *b* of Fig. 7.

c) The most conventional solution complying with classic principles is a compensation PD introducing no new pole on the negative axis of plane z. Thereby it increases, however, the additional time shift due to the compensation PD, so that in order to maintain the about 60° phase margin, the cut-off frequency has to be reduced even compared to case *b*, to about $\omega_c = 0.55$.

$$w_A^*(z) = 6.32\,\frac{(z-0.779)(z-0.607)}{(z-1)z} \tag{75}$$

$$w_x^*(z) = 0.309\,\frac{(z+0.779)}{(z-1)z}\,; \quad \tilde{w}^*(j\omega) = \frac{0.55}{j\omega}\,e^{-0.93j\omega} \tag{76}$$

$$\omega_c = 0.55\,; \qquad \varphi_c = 60.7° \tag{77}$$

$$w_r^*(z) = \frac{z(z-1)}{z^2 - 0.691z + 0.241} \tag{78}$$

Output and input signals of the process y and u_T in the three cases have been compared in Fig. 7.

The optimal algorithm is practically useless because of the oscillations of the control and output signals. The other two suboptimal algorithms little differ from each other. From the aspect of signal y, case b) is the more favourable, but the peak of the control signal is about double of that in case c). Comparison with identical $u_{T\max}$ rather than identical sampling times would show a possibility to accelerate the case c) by reducing the sampling time.

Then with unaltered curve shape in the initial section, the difference between cases b and c should be reduced. Since case c is little sensitive to parameter changes, practically this solution is at least equivalent to case b. Were the input signal an exponentially decreasing function with time constant T_a rather than a step function, factor $(z-1)$ would be replaced by $(z-e^{-T/T_a})$ in Eqs (61) to (78). For $T/T_a \ll 1, e^{-T/T_a} \sim 1$ in the beginning there is little change in signals y and u compared to the former case. The only difference is that the signals y and u do not reach a constant value after the transient interval but are damping exponentially with time constant T_a. In compliance with the stochastic-deterministic equivalency, the optimal algorithm is at the same time a minimum variance control for the exponentially correlated input signal. Due not only to excessive parameter sensitivity but also to undesired oscillation in signal y and mainly u_T the optimal algorithm is unapplicable.

Otherwise, comparing optimal and suboptimal algorithm on the basis of equal control signal amplitudes rather than equal sampling times practically the same effect may be reached also by the vibrationless suboptimal algorithm, hence essentially, the minimum variance control has no advantage.

It is interesting that this phenomenon has been described in the literature that the minimum variance control fails in case of deterministic input signals. As a matter of fact, it fails also in case of stochastic signals input but in this case from the stochastic u_T and y signals are difficult to separate oscillations arising from the improper operation of the system. In this respect, most of the published results have been obtained with digital simulations where the process is replaced by a discrete model, so the oscillation seen in Fig. 7 cannot even be recorded at the sampling points. Curves recorded on a hybrid model are seen in Fig. 8. The strong pulsation is markedly seen on alternative a.

If the degrees of the denominator and the numerator of the process pulse transfer function differ by 3 or 4, the oscillation of the minimum variance control becomes unstable. Thereby only suboptimum alternatives may be taken into account. This fact, supported also by the experiences of the example above, means that the minimum variance algorithm is practically most seldom of use. But practical importance has the method minimizing the weighed sum of the variances of the output and control signals.

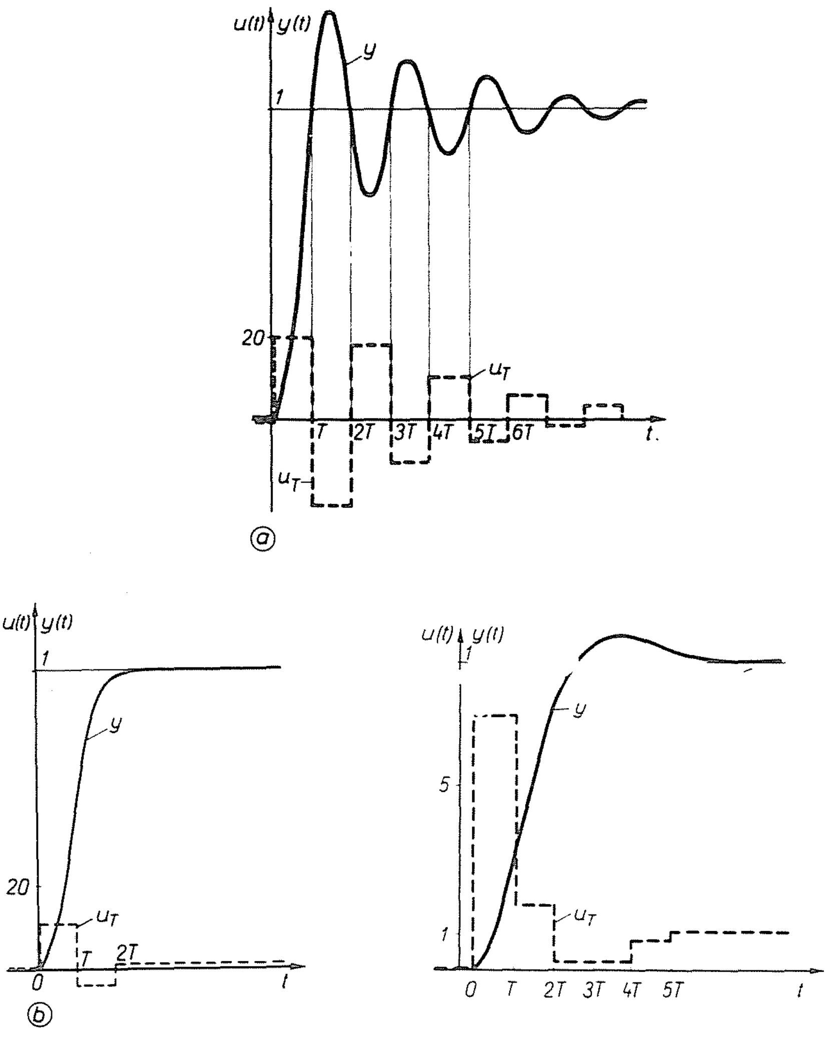


Fig. 6

Appendix 1

Let us divide Eq. (3) by $(1-g)$ and replace both numerator and denominator by their Taylor series. Denoting $a=1/T_g$:

$$\frac{w_g^*(z)}{1-g} = \frac{z-g}{1-g} = \frac{(s+a)T + (s^2+a^2)T^2/2 + (s^3+a^3)T^3/6 + \dots}{aT - aT^2/2 + a^3T^3/6 + \dots} =$$

$$= \frac{(s+a)T}{aT} \frac{1 + (s-a)T2 + (s^2-as+a^2)T^2/6 + \dots}{1-aT/2 + a^2T^2/6 + \dots} =$$

$$= \frac{s+a}{a} \frac{f(s;a)}{f(0;a)} = (1+sT_g)\frac{f(s;a)}{f(0;a)} = (1+sT_g)F(s;a)$$

$f(s;a)$ is the convergent power series of variable s, and $f(0;a)$ is its value at $s=0$. The frequency transfer function is obtained by replacing $s=j\omega$. The correction term $F(s;a)$ primarily affects the phase shift in the frequency range $\omega T<1$ therefore as a first approximation, it may be approximated by a pure shift term $\exp j\omega T_d$ yielding Eq. (4), T_d being an empirical value.

Appendix 2

Let us consider the behaviour of discrete frequency transfer function $w^*(j\omega)$ at frequency π/T. Here $z=-1$, hence $w^*(j\omega)$ is obtained from function $w^*(z)$ by substituting $z=-1$. Sampling a continuous signal y_{in} of frequency $\Omega/2$ yields a pulse series of constant amplitude and alternating sign [u^*], transformed by the holding element to a square wave u_T. The square wave consists of components of frequencies π/T; $3\pi/T$ etc. Since the transfer function $w(\omega)$ of the process filters out the higher harmonics, the output signal y of the process is identical with the component of frequency $\Omega/2$. Sampling this output sine wave yields a pulse sequence y^* with alternating sign in phase or in counter phase to pulse sequence u^*. Hence $w^*(z=-1)=w^*(\Omega/2)$ is either positive or negative. If the continuous transfer function $w(j\omega)$ has two uncompensated poles $(n-m=2)$ and at frequency $\Omega/2$, the phase angle is near to but above $-180°$ then according to Fig. 9a, y^* is counter-phased to u^* so $w^*(-1)$ is negative. In this case, numerator of function $w^*(z)$ contains a single factor type $z+\gamma$.

In addition, both the numerator and the denominator include an even number of factors, all being negative. Thus, $w^*(z)=-1$ can only be negative if factor type $z+\gamma$ is negative at $z=-1$, i.e. $\gamma<1$. Thus, all zeros of the numerator of $w^*(z)$ are inside the unit circle. On the other hand, for $n-m=3$ or $n-m=4$, if function $w(\Omega/2)$ causes a phase shift normally between $-180°$ and $-360°$ then u^* and y^* are in phase [Fig. 9b], $w^*(z=-1)$ is positive. This is possible if one

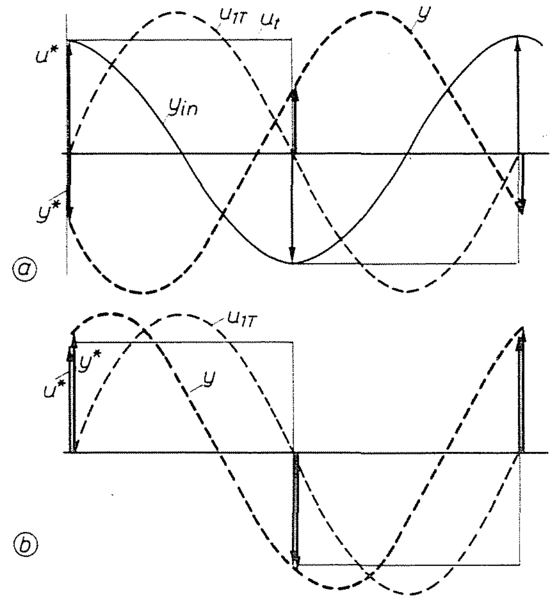


Fig. 9

from among factors of shape $z+\gamma_i$ in the numerator of $w(z)$ is positive at $z=-1$. Hence $\gamma_i>1$, thus, one zero of the numerator is outside the unit circle.

Summary

Based on the analysis of the pulse-transfer functions of continuous linear processes a general purpose design method is presented to give optimal or suboptimal DDC algorithms for deterministic and stochastic systems. The method eliminates the intersampling oscillation. It is valid for nonminimum phase and nonstable processes, as well.

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Prof. Dr. Róbert TUSCHÁK, H-1521 Budapest