INTERSHEET EDDY-CURRENT LOSSES IN LAMINATED CORE COVERED ON ONE SIDE WITH AN IDEAL CONDUCTOR

By

M. I. DABROWSKI

Technical University, Poznań, Poland,

Received February 9, 1981

Presented by Prof. Dr. Gy. RETTER

I. Introduction

In magnetic cores of the electromagnetic devices with a variable magnetic flux power loss appears due to eddy currents closed in single sheets as well as those passing through the imperfect sheet insulation. The method of analysis and computation of power loss in a single sheet are widely represented in literature and are not considered in this paper. Stray-eddy-current loss in laminated cores has been the subject of earlier works [1, 2]. In those considerations, it was assumed that the core with a rectangular cross section did not contact at the edges with other conducting parts of electromagnetic devices. In this paper, the other arrangement of the core is considered — Fig. 1 — that appears often in practice. In this structure, sheets of laminated core are connected on one side by the material with a greater conductivity. For example, the sheets can be electrically connected by the shaft or the case of an electrical machine.

In the paper, an analysis of stray eddy currents in the one side electrically connected laminated core is given and a method of computing additional power loss due to these currents is presented.

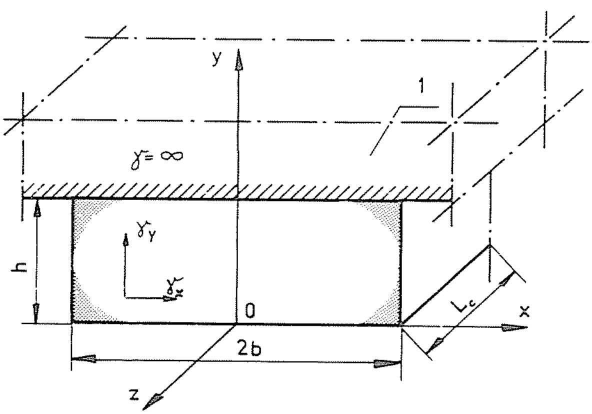

Fig. 1. Cross section of the laminated magnetic core covered with an ideal conductor 1

II. Mathematical formulation of the problem

The laminated magnetic core can be considered as an anisotropic body with respect to its conductivity. Conductivity in perpendicular direction to the sheets is less than that in parallel directions to the sheet surface, and substantially less than the conductivity of the material covering the core, i.e. than the conductivity in the region 1 — Fig 1. That is why, in the mathematical model, the core can be considered as an anisotropic homogeneous medium adherent on one side to the isotropic region with infinitely great conductivity. It has been assumed that the magnetic flux is oriented towards the axis and that the waveforms of all quantities describing the electromagnetic field are sinusoidal. It has been also assumed that the magnetic permeability of the core is constant.

On the basis of Maxwell equations, the following equations can be obtained for the magnetic field intensity H in the direction of axis and for the components of the electric field intensity K_x and K_y is the directions of x and y axes, respectively:

$$\rho_y \cdot \frac{\partial^2 H}{\partial x^2} + \rho_x \cdot \frac{\partial^2 H}{\partial y^2} = i \cdot \omega \cdot \mu \cdot H \tag{1a}$$

$$\rho_y \cdot \frac{\partial^2 K_x}{\partial x^2} + \rho_x \cdot \frac{\partial^2 K_x}{\partial y^2} = i \cdot \omega \cdot \mu \cdot K_x \tag{1b}$$

$$\rho_y \cdot \frac{\partial^2 K_y}{\partial x^2} + \rho_x \cdot \frac{\partial^2 K_y}{\partial y^2} = i \cdot \omega \cdot \mu \cdot K_y \tag{1c}$$

where: $i = \sqrt{-1}$; $\omega = 2\pi f$; f – frequency; ρ_x and ρ_y — resistivities of the core in the directions of x and y axes, respectively.

Introducing symbols:

$$k = \frac{\rho_x}{\rho_y} = \frac{\gamma_y}{\gamma_x} \tag{2}$$

and

$$\alpha_y^2 = i \cdot \omega \cdot \mu \cdot \gamma_y \tag{3}$$

we obtain instead of the relation (1a)

$$\frac{\partial^2 H}{\partial x^2} + k \cdot \frac{\partial^2 H}{\partial y^2} = \alpha_y^2 \cdot H \tag{4}$$

In the midst of magnetic field intensity H and the components of the electric field intensity K_x and K_y the following relations occur:

$$K_x = \rho_x \cdot \frac{\partial H}{\partial y} \tag{5}$$

$$-K_y = \rho_y \cdot \frac{\partial H}{\partial x} \tag{6}$$

Therefore it is reasonable to solve first of all equation (4) and then determine K_x and K_y from the relations (5) and (6). The boundary problem for equation (4), with assumption of equal value of the field intensity H_0 on the boundary line of the core not adherent to a conductor was partly solved in work [1]. The following relationship was obtained

$$H(x, y) = H_0 \cdot \left[\frac{\cosh \alpha_x y}{\cosh \alpha_x h} + \frac{4}{\pi} \sum_n \left(\frac{\alpha_x}{\beta_{nk}}\right)^2 \cdot \frac{\sin n\pi/2}{n} \cdot \frac{\cosh \beta_{ny} \cdot x}{\cosh \beta_{ny} \cdot b} \cdot \sin \frac{n\pi y}{2h}\right] \tag{7}$$

where

$$\alpha_x^2 = i \cdot \omega \cdot \mu \cdot \gamma_x \tag{8a}$$

$$\beta_{nx}^2 = \alpha_x^2 + \left(\frac{n\pi}{2h}\right)^2 \tag{8b}$$

$$\beta_{ny}^2 = \alpha_y^2 + k\left(\frac{n \cdot \pi}{2h}\right)^2 \tag{8c}$$

α_y^2 — in reference Eq. (3); remaining denotations according to Fig. 2.

In works [2, 3] the real and imaginary parts were separated from relation (7) so the following expression for the distribution of the magnetic field intensity H had been obtained

$$H(x, y) = H_0 \cdot \left[\frac{\cosh m_x(h+y) \cdot \cos m_x(h-y) + \cos m_x(h-y) \cdot \cos m_x(h+y)}{\cosh 2m_x \cdot h + \cos 2m_x \cdot h} + \sum_n E_{Hn} \cdot d_n'\right] + \tag{9}$$

$$+iH_0\cdot\left[\frac{\sinh m_x(h+y)\cdot\sin m_x(h-y)+\sinh m_x(h-y)\cdot\sin m_x(h+y)}{\cosh 2m_xh+\cos 2m_x\cdot h}+\right.$$

$$\left.+\sum_n E_{Hn}\cdot d_n''\right]$$

where

$$m_x=\sqrt{\omega\cdot\mu\cdot\gamma_x/2} \tag{10a}$$

$$d_n' = \frac{1}{e_1}[e_2+e_3+F_n(e_4+e_5)] \tag{10b}$$

$$d_n'' = \frac{1}{e_1}\cdot[F_n\cdot(e_2+e_3)-e_4-e_5] \tag{10c}$$

$$e_1=(1+F_n^2)\cdot(\cosh 2\sqrt{k}\cdot m_x\sqrt{n}b+\cos 2\sqrt{k}m_x\cdot s_n\cdot b)$$

$$e_2=\cosh\sqrt{k}m_xr_n(b+x)\cdot\cos\sqrt{k}m_x\cdot s_n\cdot(b-x)$$

$$e_3=\cosh\sqrt{k}m_x\cdot r_n(b-x)\cdot\cos\sqrt{k}m_x\cdot s_n(b+x)$$

$$e_4=\sinh\sqrt{k}m_x\cdot r_n\cdot(b+x)\cdot\sin\sqrt{k}m_x\cdot s_n(b-x)$$

$$e_5=\sinh\sqrt{k}m_x\cdot r_n(b-x)\cdot\sin\sqrt{k}\cdot m_x\cdot s_n\cdot(b+x)$$

$$F_n = \left(\frac{n\cdot\pi}{2\sqrt{2}m_x\cdot h}\right)^2 \tag{11}$$

$$r_n = \sqrt{\sqrt{F_n^2+1}+F_n} \tag{12a}$$

$$s_n = \sqrt{\sqrt{F_n^2+1}-F_n} \tag{12b}$$

$$E_{Hn} = \frac{4}{\pi}\cdot\frac{\sin(\pi\cdot n/2)}{n}\cdot\cos\frac{n\pi y}{2h}. \tag{13}$$

Similar expressions can be obtained for the field intensities after differentiating in accordance with relations (5) and (6) [3].

III. Computation of power loss

The eddy-current loss in a length unit of the core in the direction of z axis — Fig. 2 — can be expressed, with the help of the complex Poynting vector, by the relation

$$P = Re\left[\frac{1}{2}\int_S K \times \tilde{H} \cdot dS\right]. \tag{14}$$

The complex vector K and the complex vector $\tilde{H}$ conjugate with H concern the points of the core surface S. After substituting the conjugate value $\tilde{H}$ obtained from equation (9) and the electric field intensity K to the formula (14) and after integrating we obtain the power loss in a volume unit of the core from Fig. 2 as

$$p = \frac{P}{4bh} = H_0^2 \cdot \rho_x \cdot m_x^2 \cdot \left\{\frac{1}{\xi_h} \cdot \frac{\sinh \xi_h - \sin \xi_h}{\cosh \xi_h + \cos \xi_h} + \right.$$

$$+ \frac{8}{\pi^2} \cdot \frac{1}{\xi_b} \sum_n \frac{s_n \cdot \sinh \xi_b \cdot r_n - r_n \cdot \sin \xi_b \cdot s_n}{n^2 \sqrt{1+F_n^2} \cdot (\cosh \xi_b \cdot r_n + \cos \xi_b \cdot s_n)} - \tag{15}$$

$$\left. - \frac{4}{\xi_b \cdot \xi_h^2} \sum_n \frac{(r_n + F_n \cdot s_n) \cdot \sinh \xi_b \cdot r_n + (s_n - F_n \cdot r_n) \cdot \sin \xi_b \cdot s_n}{\sqrt{(1+F_n^2)^3} \cdot (\cosh \xi_b \cdot r_n + \cos \xi_b \cdot s_n)} \right\}$$

in which the following symbols are used:

$$\xi_b = 2 \cdot m_x \cdot \sqrt{k} \cdot b \tag{16a}$$

$$\xi_h = 2m_x \cdot h \tag{16b}$$

Summation with respect to the index n concerns only subsequent odd numbers.

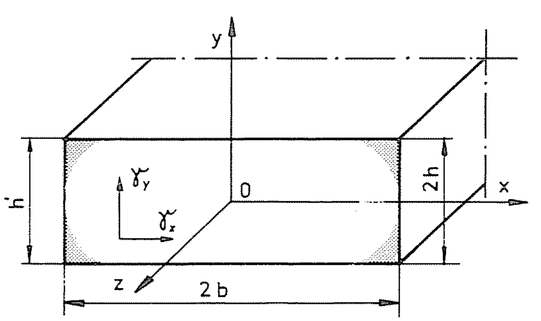

Fig. 2. Cross section of the laminated magnetic rectangular core

In the majority of practical problems, the field intensity H_0 on the core surface is not known at the beginning of considerations. Instead of that, the total magnetic flux in the core resulting from the average induction B_a is given.

$$\Phi = 4bhB_a = \mu \int_{-b}^{b} \int_{-h}^{h} H(x, y) \cdot \mathrm{d}x \cdot \mathrm{d}y \tag{17}$$

After substitution H from Eq. (9) and some conversions we obtain the following relation

$$B_a = \mu \cdot H_0 \cdot (\varphi' - i \cdot \varphi'') \tag{18}$$

where:

$$\varphi' = \frac{1}{\xi_h} \cdot \frac{\sinh \xi_h - \sin \xi_h}{\cosh \xi_h + \cos \xi_h} + \frac{8}{\pi^2} \cdot \frac{1}{\xi_b} \sum_n \frac{1}{n^2} \cdot \tag{19a}$$

$$\cdot \frac{1}{\sqrt{(1+F_n^2)^3}} \cdot \frac{(r_n + F_n \cdot s_n) \cdot \sinh \xi_b \cdot r_n + (s_n - F_n \cdot r_n) \cdot \sin \xi_b \cdot s_n}{\cosh \xi_b r_n + \cos \xi_b \cdot s_n}$$

$$\varphi'' = \frac{1}{\xi_h} \cdot \frac{\sinh \xi_h - \sin \xi_h}{\cosh \xi_h + \cos \xi_h} +$$

$$+ \frac{8}{\pi^2} \cdot \frac{1}{\xi_b} \sum_n \frac{1}{n^2} \cdot \frac{1}{\sqrt{(1+F_n^2)^3}} \cdot \frac{(s_n + F_n \cdot r_n) \sinh \xi_b r_n - (r_n + F_n \cdot s_n) \sin \xi_b s_n}{\cosh \xi_b r_n + \cos \xi_b \cdot s_n} \tag{19b}$$

Computation of the field intensity H_0 from Eq. (18) and substitution into (15) allow, after some conversions, to obtain the unit stray-eddy-current loss in the core in which the average induction value is given.

$$p = \frac{B_a^2 \cdot \omega \cdot \gamma_x \cdot (2h)^2}{24} \cdot \left[\frac{6}{\xi_h^2 \varphi^2} \cdot \frac{\cosh \xi_h - \sin \xi_h}{\cosh \xi_h + \cos \xi_h} + \right.$$

$$+ \frac{48}{\pi^2 \xi_b \xi_h^2 \varphi^2} \sum_n \frac{s_n \cdot \sinh \xi_b r_n - r_n \cdot \sin \xi_b \cdot s_n}{n^2 \sqrt{1+F_n^2} \cdot (\cosh \xi_b r_n + \cos \xi_b s_n)} - \tag{20}$$

$$\left. - \frac{24}{\xi_b \cdot \xi_h^4 \cdot \varphi^2} \sum_n \frac{(r_n + F_n \cdot s_n) \cdot \sinh \xi_b r_n + (s_n - F_n \cdot r_n) \cdot \sin \xi_b s_n}{\sqrt{(1+F_n^2)^3} (\cosh \xi_b r_n + \cos \xi_b \cdot s_n)} \right]$$

where

$$\varphi^2 = \varphi'^2 + \varphi''^2 \tag{21}$$

The well known factor placed before the square brackets in Eq. (20) represents the losses in a volume unit of the homogeneous isotropic, infinitely wide and long plate, in which uniformly distributed flux is assumed [4]. In the expression in brackets, a finite length, anisotropy, and irregular distribution of the flux in the core, are taken into account. The expression in the brackets denoted by $\pi(\xi_b, \xi_h)$ was calculated by a computer and the results are shown in Fig. 3.

The results obtained in this way can be used to compute eddy-current loss in a more complex structure shown in Fig. 1. In order to do this, it is sufficient to notice that the current density component in the direction of axis x in points $y=0$ is equal to zero in both cases — Fig. 4. Therefore, on the basis of indentity of boundary conditions on the remaining edges, it can be stated that the eddy currents distribution in the core adherent on one side to an ideal conductor is identical to the distribution in the upper or lower half of the core of a double dimension $h'=2h$ in the direction of axis y — Fig. 4. Thus, the additional loss in

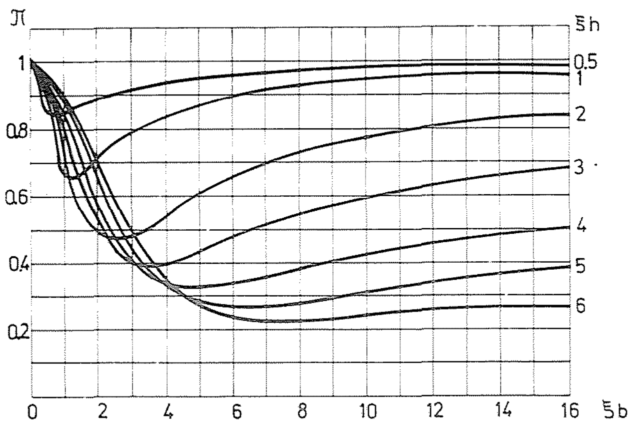

Fig. 3. Cores $\pi(\xi_b, \xi_h)$

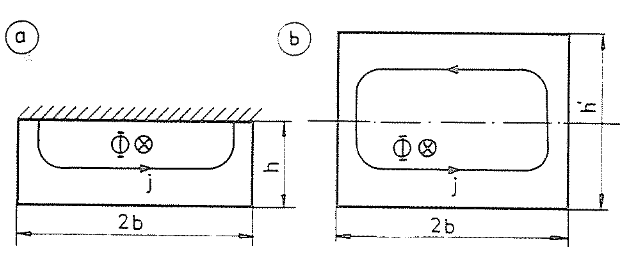

Fig. 4. Comparison of eddy-current lines in covered *a* and uncovered *b* core

a volume unit of the core shown in Fig. 1 should be computed from equation

$$p = \frac{B_a^2 \cdot \omega^2 \cdot \gamma_x \cdot h^2}{6} \cdot \pi' \qquad (22)$$

where the coefficient π' can be read from Fig. 3 for ξ_b in accordance with Eq. (16a) and for $\xi_h = 2m_x h$.

IV. Simplified relations

Expressions (15), (20) and (22) for the power loss are extremely complex. Therefore, by means of them, it is difficult to draw conclusions about the influence of the conductivity γ_x, the core dimensions and the one side potential connection of the lamina on the eddy-current loss. The expressions can be considerably simplified without significant computational accuracy decrease. When the remagnetization frequency $f \leqq 60$ Hz, the core magnetic permeability $\mu < 1500\,\mu_0$ and the resistivity $\rho_x > 0.5 \cdot 10^{-3}$ Ωm and when the core dimension $h < 1$ m — applied in practice, the parameter F_n value according to relation (11), fulfil the condition $F_n^2 \gg 1$ for $n = 1$, and even more for $n = 3; 5; 7$. Therefore

$$F_n^2 + 1 \cong F_n^2$$

and the expressions for the parameters r_n and s_n according to relation (12) are considerably simplified

$$r_n \approx \sqrt{2 \cdot F_n} \qquad (23a)$$

$$s_n \approx 0. \qquad (23b)$$

The second component in the square brackets in relations (15) and (20) becomes then equal to zero and the third component becomes negligibly small in comparison with the first one. Physically it means that the eddy-current loss is caused mainly by the current density component j_x. Taking these simplifications into consideration in relation (20), we obtain additional losses in a volume unit:

— of the core shown in Fig. 2

$$p = \frac{B_a^2 \cdot \omega^2 \cdot \gamma_x \cdot h^2}{24} \cdot \frac{3}{\xi_h} \cdot \frac{\sinh \xi_h - \sin \xi_h}{\cosh \xi_h - \cos \xi_h} \qquad (24)$$

— of the core shown in Fig. 1

$$p=\frac{B_a^2\cdot\omega^2\cdot\gamma_x\cdot(2h)^2}{24}\cdot\frac{3}{2\xi_h}\cdot\frac{\sinh 2\xi_h-\sin 2\xi_h}{\cosh 2\xi_h-\cos 2\xi_h}=$$

$$=\frac{B_a^2\cdot\omega^2\cdot\gamma_x\cdot h^2}{4}\cdot\frac{1}{\xi_h}\cdot\frac{\sinh 2\xi_h-\sin 2\xi_h}{\cosh 2\xi_h-\cos 2\xi_h} \tag{25}$$

It has been shown, by means of experimental works, that in the laminated cores of dimension $h>0.5$ m, the eddy-current loss caused by the current density component j_y is distinct. This concerns especially cores covered on one side with an ideal conductor. Moreover, it has been found that the component j_y is cosinusoidally distributed according to coordinate y [3]

$$j_y\approx H_a\cdot\frac{4}{\pi}\sqrt{k}\cdot\frac{\sqrt{2}m_x}{\sqrt{1+F_n^2}}\cos\frac{\pi}{n}y \tag{26}$$

The equivalent penetration depth of this component in the core is:
— for the core, shown in Fig. 2

$$\Delta'\approx\frac{h}{\pi\cdot\sqrt{k}} \tag{27}$$

— for the core shown in Fig. 1

$$\Delta\approx\frac{2h}{\pi\cdot\sqrt{k}} \tag{28}$$

Power loss caused by the current component j_y in a length unit of the core in the direction of axis z can be expressed by the formulae:
— according to Fig. 2

$$P_y=2\rho_y\int\limits_{-h/2}^{h/2}\left(\frac{j_y}{\sqrt{2}}\right)^2\cdot\mathrm{d}y\cdot\Delta' \tag{29}$$

— according to Fig. 1

$$P_y=2\rho_y\int\limits_{0}^{h}\left(\frac{j_y}{\sqrt{2}}\right)^2\cdot\mathrm{d}y\cdot\Delta \tag{30}$$

Taking also relations (18), (24) and (25) into consideration we finally obtain simplified expressions for additional eddy-current loss in the core:

$$P = \frac{B_a^2 \cdot \omega^2 \cdot \gamma_x \cdot h^2}{24} \cdot \frac{3}{\xi_h} \cdot \frac{\sinh \xi_h - \sin \xi_h}{\cosh \xi_h - \cos \xi_h} V_c +$$

$$+ \frac{16 B_a^2 \cdot \omega^2 \cdot \gamma_x \cdot h^2}{\pi^3 \sqrt{k}} \cdot \frac{\xi_h}{\sqrt{4\xi^4 - \pi^4}} \cdot \frac{\cosh \xi_h + \cos \xi_h}{\cosh \xi_h - \cos \xi_h} \cdot L_c \qquad (31)$$

— for the core shown in Fig. 1

$$P = \frac{B_a^2 \cdot \omega^2 \cdot \gamma_x \cdot h^2}{6} \cdot \frac{3}{2\xi_h} \cdot \frac{\sinh 2\xi_h - \sin 2\xi_h}{\cosh 2\xi_h - \cos 2\xi_h} V_c +$$

$$+ \frac{256 B_a^2 \omega^2 \gamma_x h^4}{\pi^3 \cdot \sqrt{k}} \cdot \frac{2\xi_h}{\sqrt{64\xi_h^4 + \pi^4}} \cdot \frac{\cosh 2\xi_h + \cos 2\xi_h}{\cosh 2\xi_h - \cos 2\xi_h} L_c \quad . \qquad (32)$$

where: V_c — core volume, L_c — length of the core in the direction of axis z. The other symbols are identical to those in previous expressions.

V. Final conclusions

As an effect of the core sheets connection with a conductor the stray eddy-current loss, due to currents closing by the sheets insulation, considerably increases. The lossen can be especially great in the cores of wide lamina, i.e. of large dimension h — Fig. 1. The losses, due to the current density component j_y, are proportional to the fourth power of this dimension. As a consequence this results in the requirements with respect to the minimum value of the resistivity ρ_x of the core in the direction perpendicular to the lamina.

Summary

Intersheet eddy-currents in the laminated magnetic core with a rectangular cross section are investigated. It is assumed that the core is covered on one side with an ideal conductor. A method of computation of the additional stray-eddy-current loss, based on the Maxwell equations and results previously obtained by the author are presented. Parameters affecting power loss are examined.

References

1. BOWLY, L. V.—PORITSKY, H.: Trans. AIEE, 344 (1937).
2. DABROWSKI, M. I.: Rozprawy Elektrot., 501 (1965).
3. DABROWSKI, M. I.: Scientific Bull. of Lodz Tech. Univ. Elektryka, No 21 (1966).
4. STOLL, R. L.: The Analysis of Eddy Currents, Clarendon Press Oxford (1974).

Prof. Dr. M. I. DABROWSKI Technical University, Poznań, Poland